

Computer Algebra Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.4-Cotangent/109-4.4.10-c+d-x^m-a+b-cotⁿ

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test. The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [61]. This is test number [109].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (61)	0.00 (0)
Mathematica	100.00 (61)	0.00 (0)
Fricas	100.00 (61)	0.00 (0)
Maple	95.08 (58)	4.92 (3)
Maxima	80.33 (49)	19.67 (12)
Giac	57.38 (35)	42.62 (26)
Mupad	45.90 (28)	54.10 (33)
Sympy	45.90 (28)	54.10 (33)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

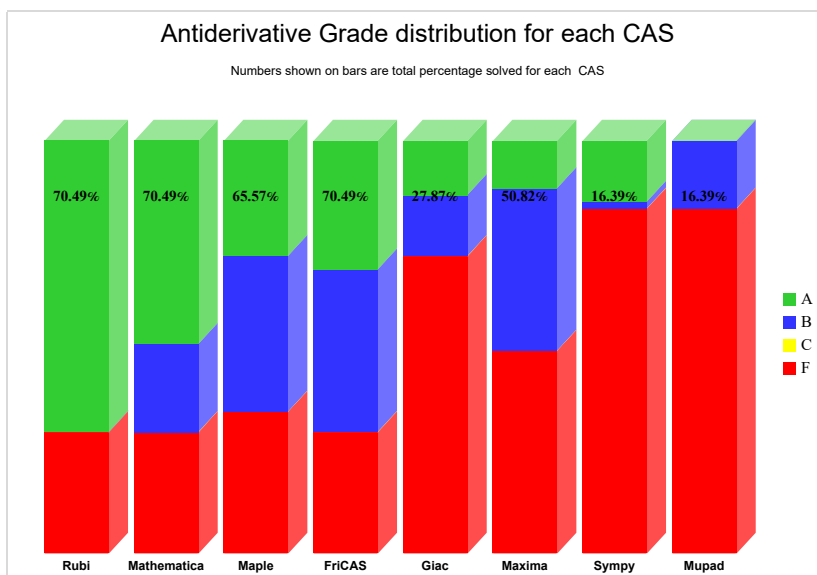
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

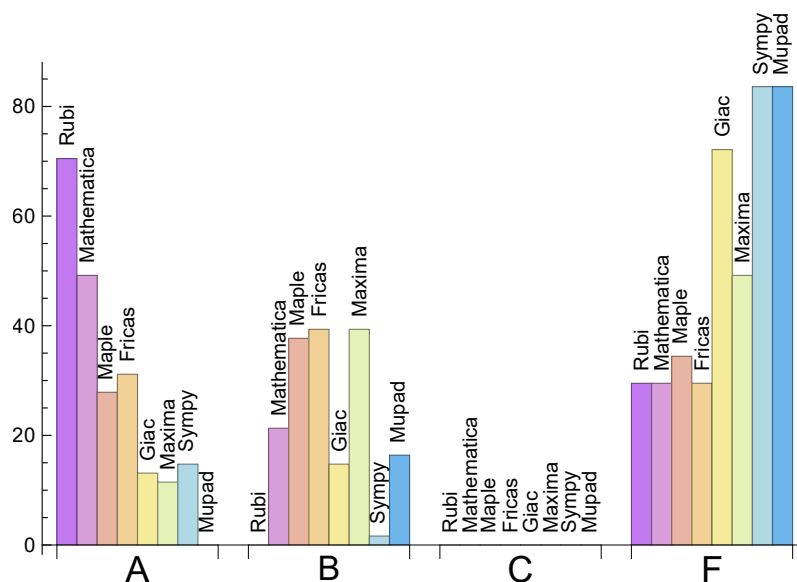
System	% A grade	% B grade	% C grade	% F grade
Rubi	70.492	0.000	0.000	29.508
Mathematica	49.180	21.311	0.000	29.508
Fricas	31.148	39.344	0.000	29.508
Maple	27.869	37.705	0.000	34.426
Sympy	14.754	1.639	0.000	83.607
Giac	13.115	14.754	0.000	72.131
Maxima	11.475	39.344	0.000	49.180
Mupad	0.000	16.393	0.000	83.607

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of

error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Maple	3	100.00	0.00	0.00
Maxima	12	25.00	0.00	75.00
Giac	26	100.00	0.00	0.00
Mupad	33	0.00	100.00	0.00
Sympy	33	93.94	6.06	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.27
Rubi	0.36
Maple	0.48
Giac	0.72
Sympy	0.99
Maxima	2.22
Mathematica	5.66
Mupad	10.04

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	91.86	1.16	22.00	1.10
Sympy	145.25	1.29	19.00	1.00
Rubi	174.25	1.00	126.00	1.00
Mathematica	332.89	1.67	182.00	1.26
Fricas	381.98	2.18	144.00	2.35
Giac	432.77	2.64	23.00	1.16
Maple	513.34	2.13	169.50	1.00
Maxima	1219.43	19.88	527.00	6.65

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

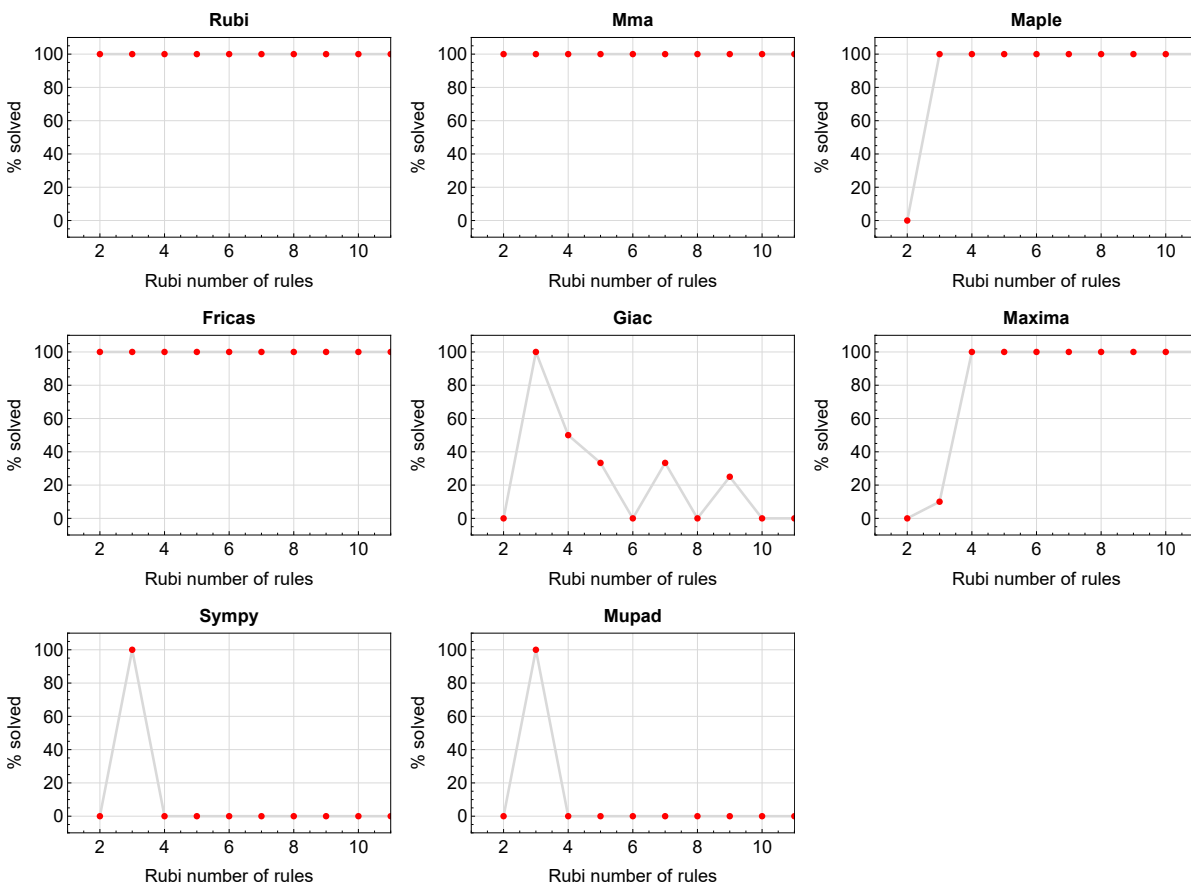


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

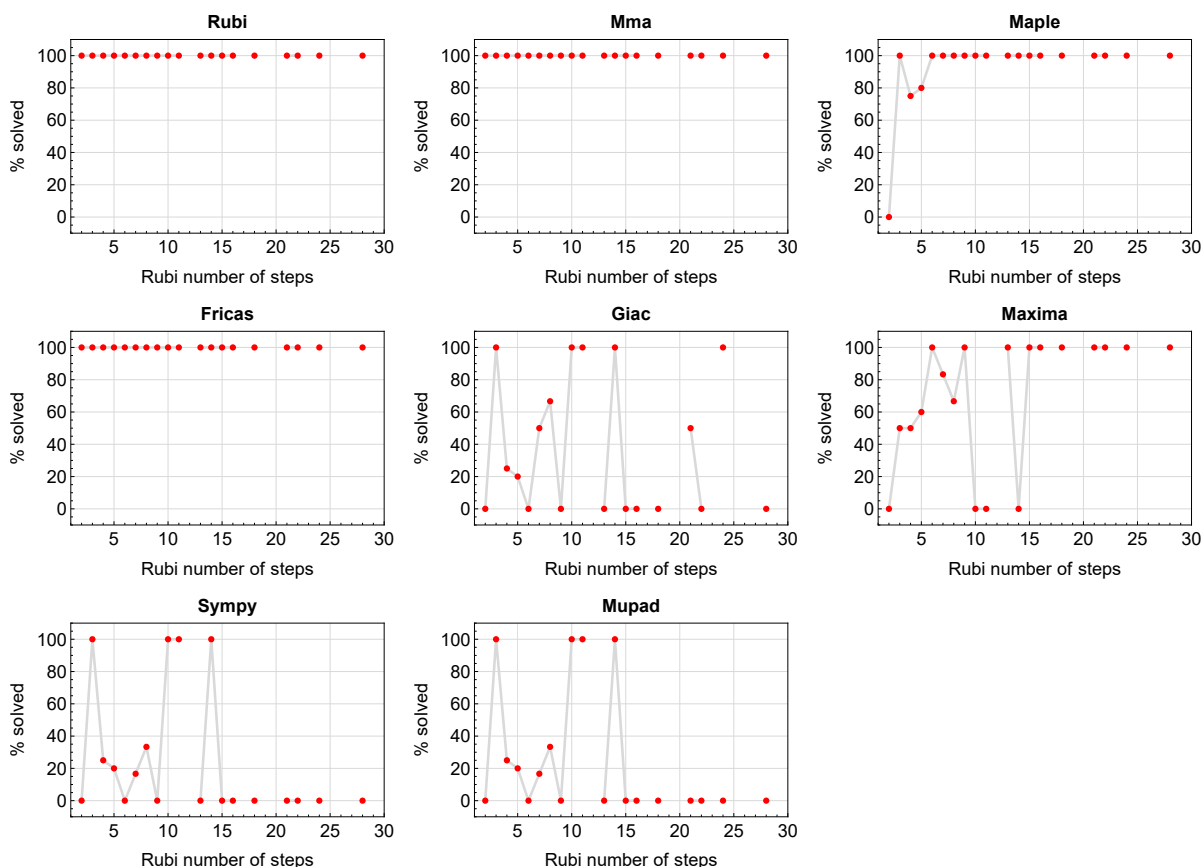


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram shows that the percentage of solved integrals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

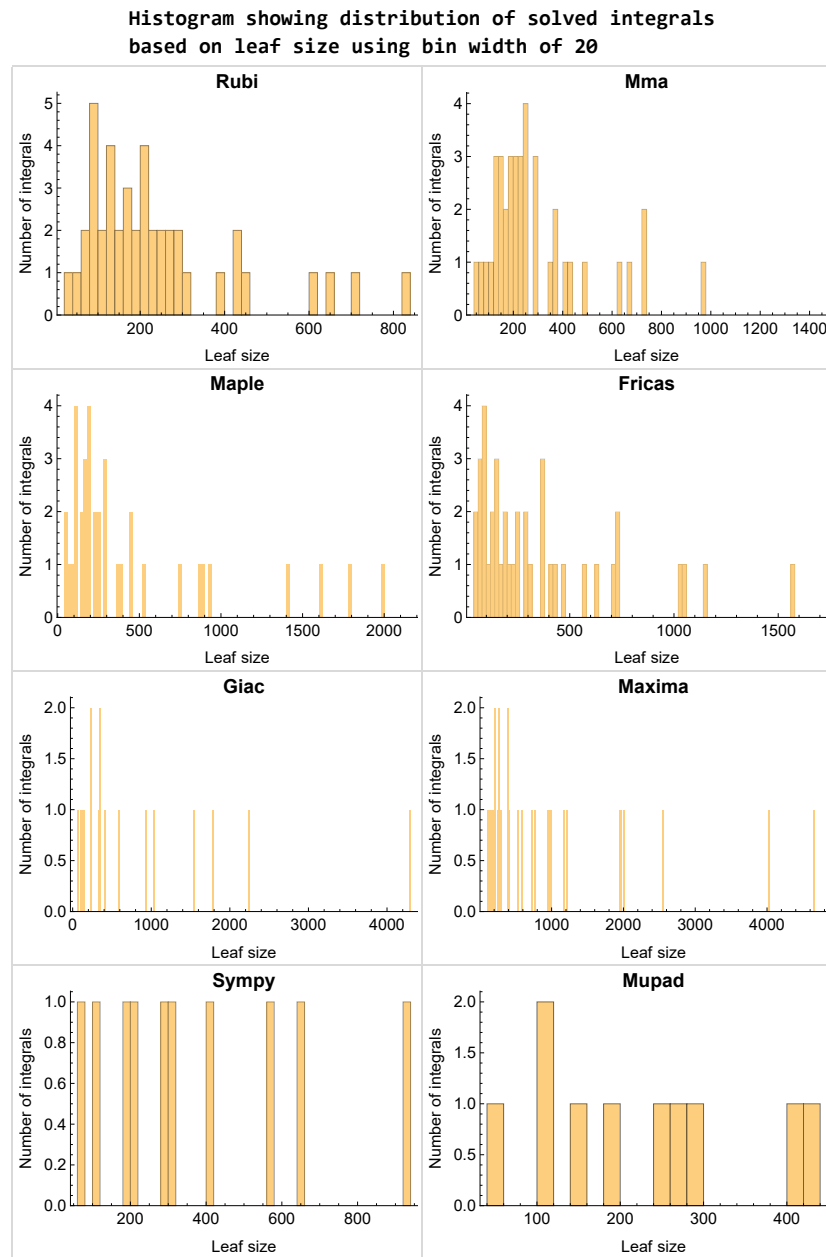


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

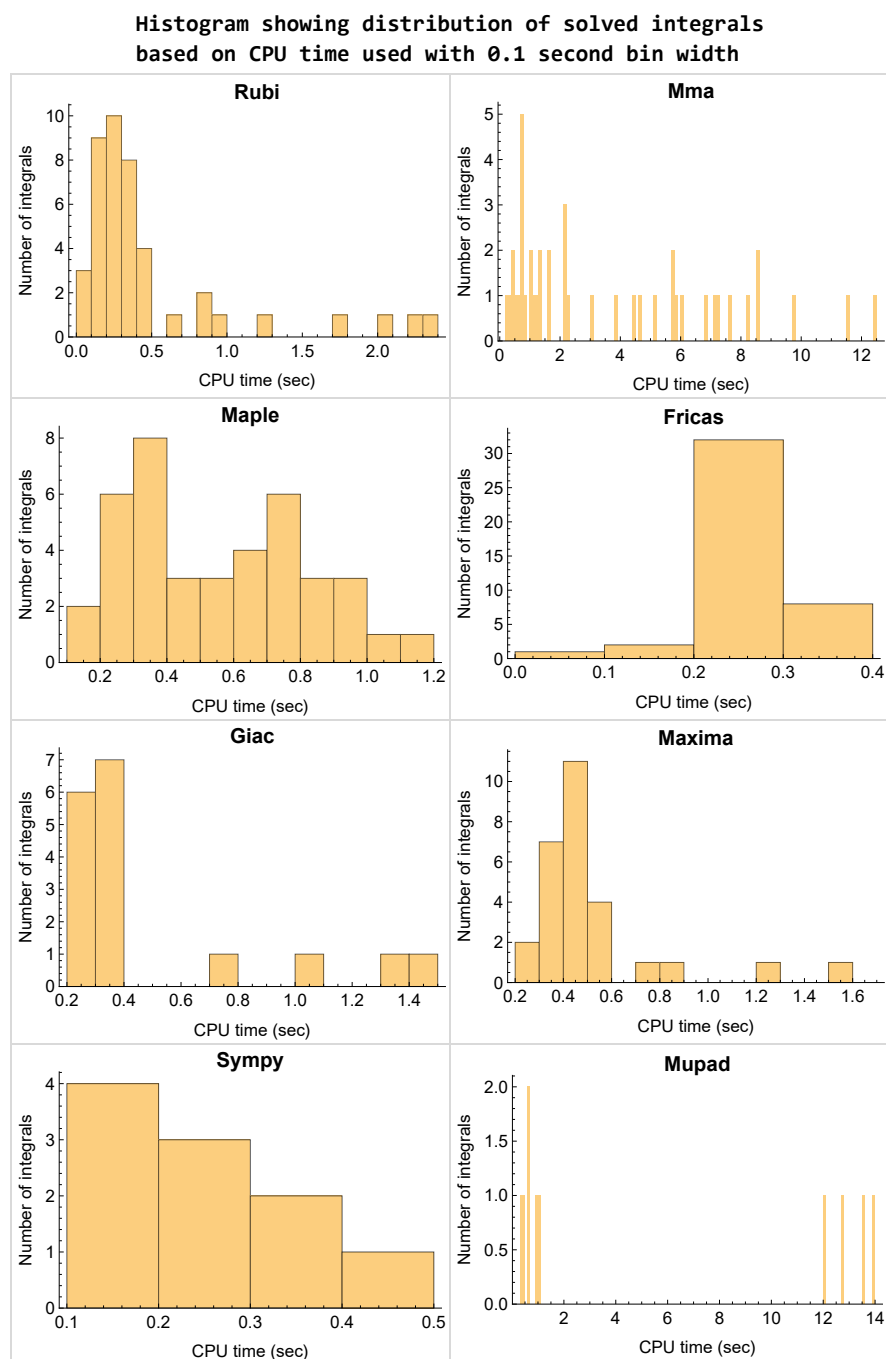


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fracas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

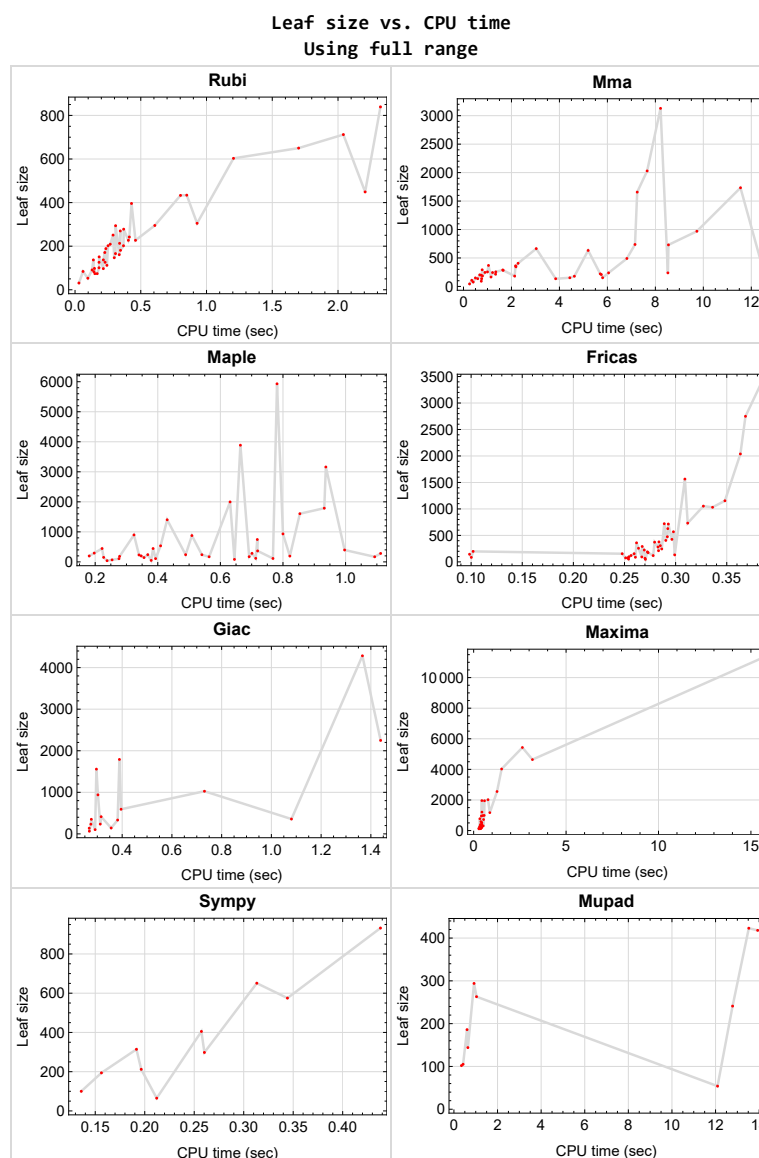


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{4, 5, 9, 10, 14, 15, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {42, 43, 47, 48, 57}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
x, aa = expr.operator(), expr.operands()
if x is None:
    return 1
else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x)^2/2$

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

The following fields are present only in *Rubi Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,...}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax

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June 27, 2023
Design v1.0a

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1	List of integrals sorted by grade for each CAS	22
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2.3	Detailed conclusion table specific for Rubi results	38

2.1 List of integrals sorted by grade for each CAS

Rubi	22
Mma	22
Maple	23
Fricas	23
Maxima	23
Giac	23
Mupad	24
Sympy	24

Rubi

A grade { 1, 2, 3, 6, 7, 8, 11, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 8, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 44, 49, 52, 53, 54, 58 }

B grade { 3, 6, 7, 11, 37, 38, 39, 42, 43, 47, 48, 57, 59 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 8, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31 }

B grade { 1, 2, 3, 6, 7, 11, 12, 13, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

C grade { }

F normal fail { 34, 35, 36 }

F(-1) timeout fail { }

F(-2) exception fail { }

Fricas

A grade { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36 }

B grade { 1, 2, 3, 6, 7, 8, 11, 12, 13, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

C grade { }

F normal fail { }

F(-1) timeout fail { }

F(-2) exception fail { }

Maxima

A grade { 19, 20, 21, 25, 26, 30, 31 }

B grade { 1, 2, 3, 6, 7, 8, 11, 12, 13, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

C grade { }

F normal fail { 34, 35, 36 }

F(-1) timeout fail { }

F(-2) exception fail { 16, 17, 18, 22, 23, 24, 27, 28, 29 }

Giac

A grade { 16, 17, 18, 23, 24, 27, 28, 29 }

B grade { 8, 19, 20, 21, 22, 25, 26, 30, 31 }

C grade { }

F normal fail { 1, 2, 3, 6, 7, 11, 12, 13, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

F(-1) timeout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 8, 16, 17, 18, 22, 23, 24, 27, 28, 29 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 6, 7, 11, 12, 13, 19, 20, 21, 25, 26, 30, 31, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

F(-2) exception fail { }

Sympy

A grade { 16, 17, 18, 22, 23, 24, 27, 28, 29 }

B grade { 8 }

C grade { }

F normal fail { 1, 2, 3, 6, 7, 11, 12, 13, 19, 20, 21, 25, 26, 30, 34, 35, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

F(-1) timedout fail { 31, 36 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column **N.S.** means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	184	240	391	306	0	0	0
N.S.	1	1.00	1.82	2.38	3.87	3.03	0.00	0.00	0.00
time (sec)	N/A	0.182	0.791	0.489	0.364	0.285	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	136	198	257	244	0	0	0
N.S.	1	1.00	1.84	2.68	3.47	3.30	0.00	0.00	0.00
time (sec)	N/A	0.169	0.593	0.347	0.322	0.286	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	135	150	140	174	0	0	0
N.S.	1	1.00	2.55	2.83	2.64	3.28	0.00	0.00	0.00
time (sec)	N/A	0.098	3.843	0.227	0.283	0.273	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.021	2.250	0.144	0.429	0.265	0.307	0.254	12.138

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.021	3.735	0.171	0.385	0.243	0.273	0.266	12.187

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	256	231	952	372	0	0	0
N.S.	1	1.00	2.64	2.38	9.81	3.84	0.00	0.00	0.00
time (sec)	N/A	0.215	1.346	0.340	0.418	0.279	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	153	183	386	281	0	0	0
N.S.	1	1.00	2.07	2.47	5.22	3.80	0.00	0.00	0.00
time (sec)	N/A	0.151	5.801	0.278	0.418	0.283	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	44	40	269	75	65	1026	54
N.S.	1	1.00	1.42	1.29	8.68	2.42	2.10	33.10	1.74
time (sec)	N/A	0.030	0.253	0.238	0.355	0.251	0.212	0.731	12.091

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	348	14	10	14	14
N.S.	1	1.00	1.17	1.00	29.00	1.17	0.83	1.17	1.17
time (sec)	N/A	0.035	7.198	0.242	0.428	0.243	0.351	0.288	12.196

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	364	14	12	14	14
N.S.	1	1.00	1.17	1.00	30.33	1.17	1.00	1.17	1.17
time (sec)	N/A	0.037	3.866	0.271	0.509	0.244	0.300	0.284	12.176

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	491	444	1960	569	0	0	0
N.S.	1	1.00	2.43	2.20	9.70	2.82	0.00	0.00	0.00
time (sec)	N/A	0.368	6.813	0.222	0.444	0.298	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	221	293	1208	425	0	0	0
N.S.	1	1.00	1.75	2.33	9.59	3.37	0.00	0.00	0.00
time (sec)	N/A	0.229	5.701	0.197	0.448	0.296	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	179	197	586	291	0	0	0
N.S.	1	1.00	1.97	2.16	6.44	3.20	0.00	0.00	0.00
time (sec)	N/A	0.130	4.624	0.181	0.391	0.267	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	763	14	10	14	14
N.S.	1	1.00	1.17	1.00	63.58	1.17	0.83	1.17	1.17
time (sec)	N/A	0.036	9.494	0.195	0.759	0.261	0.362	0.317	12.345

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	761	14	12	14	14
N.S.	1	1.00	1.17	1.00	63.42	1.17	1.00	1.17	1.17
time (sec)	N/A	0.034	6.451	0.142	0.865	0.252	0.317	0.324	12.311

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	246	170	0	154	314	233	423
N.S.	1	1.00	1.30	0.90	0.00	0.81	1.66	1.23	2.24
time (sec)	N/A	0.235	0.895	0.564	0.000	0.248	0.192	0.274	13.525

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	149	108	0	96	194	137	241
N.S.	1	1.00	1.09	0.79	0.00	0.70	1.42	1.00	1.76
time (sec)	N/A	0.139	0.498	0.393	0.000	0.254	0.156	0.268	12.781

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	107	50	0	48	100	64	105
N.S.	1	1.00	1.27	0.60	0.00	0.57	1.19	0.76	1.25
time (sec)	N/A	0.061	0.345	0.379	0.000	0.270	0.136	0.268	0.420

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	77	67	111	53	0	351	0
N.S.	1	1.00	0.48	0.42	0.69	0.33	0.00	2.18	0.00
time (sec)	N/A	0.336	0.405	0.254	0.288	0.254	0.000	0.276	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	215	105	121	74	0	357	0
N.S.	1	1.00	1.30	0.63	0.73	0.45	0.00	2.15	0.00
time (sec)	N/A	0.308	1.337	0.276	0.377	0.253	0.000	1.081	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	283	143	160	120	0	1558	0
N.S.	1	1.00	1.25	0.63	0.70	0.53	0.00	6.86	0.00
time (sec)	N/A	0.406	1.656	0.356	0.408	0.256	0.000	0.296	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	362	283	0	257	651	413	294
N.S.	1	1.00	1.34	1.05	0.00	0.95	2.41	1.53	1.09
time (sec)	N/A	0.344	2.171	0.701	0.000	0.264	0.313	0.315	0.923

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	255	173	0	154	405	235	186
N.S.	1	1.00	1.26	0.86	0.00	0.76	2.00	1.16	0.92
time (sec)	N/A	0.251	1.000	0.692	0.000	0.259	0.257	0.312	0.602

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	165	82	0	68	212	102	102
N.S.	1	1.00	1.09	0.54	0.00	0.45	1.40	0.68	0.68
time (sec)	N/A	0.183	1.147	0.645	0.000	0.270	0.196	0.291	0.346

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	136	118	194	87	0	939	0
N.S.	1	1.00	0.45	0.39	0.64	0.29	0.00	3.08	0.00
time (sec)	N/A	0.928	0.757	0.713	0.386	0.261	0.000	0.303	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	434	434	203	193	211	132	0	2249	0
N.S.	1	1.00	0.47	0.44	0.49	0.30	0.00	5.18	0.00
time (sec)	N/A	0.850	0.676	0.822	0.435	0.299	0.000	1.439	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	664	396	0	360	932	593	418
N.S.	1	1.00	1.68	1.00	0.00	0.91	2.35	1.50	1.06
time (sec)	N/A	0.430	3.033	0.997	0.000	0.262	0.438	0.396	13.931

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	369	238	0	211	575	333	263
N.S.	1	1.00	1.26	0.81	0.00	0.72	1.96	1.13	0.89
time (sec)	N/A	0.308	1.042	0.541	0.000	0.283	0.344	0.382	1.037

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	244	114	0	93	298	142	144
N.S.	1	1.00	1.17	0.55	0.00	0.44	1.43	0.68	0.69
time (sec)	N/A	0.268	1.211	0.768	0.000	0.267	0.260	0.356	0.640

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	197	169	275	119	0	1791	0
N.S.	1	1.00	0.44	0.38	0.61	0.27	0.00	3.99	0.00
time (sec)	N/A	2.206	0.740	1.093	0.419	0.278	0.000	0.389	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	712	712	292	281	299	188	0	4284	0
N.S.	1	1.00	0.41	0.39	0.42	0.26	0.00	6.02	0.00
time (sec)	N/A	2.041	0.779	1.112	0.517	0.272	0.000	1.366	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	496	110	51	23	24
N.S.	1	1.00	1.09	0.91	21.57	4.78	2.22	1.00	1.04
time (sec)	N/A	0.057	13.565	0.182	1.157	0.260	4.045	0.413	12.372

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	79	25	29	21	22
N.S.	1	1.00	1.10	0.90	3.76	1.19	1.38	1.00	1.05
time (sec)	N/A	0.033	6.586	0.123	0.503	0.292	1.833	0.299	12.402

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	91	0	0	85	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.148	0.742	0.000	0.000	0.100	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	171	152	0	0	144	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.226	4.433	0.000	0.000	0.098	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	251	251	238	0	0	198	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.289	6.049	0.000	0.000	0.102	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	632	876	978	628	0	0	0
N.S.	1	1.00	4.30	5.96	6.65	4.27	0.00	0.00	0.00
time (sec)	N/A	0.298	5.197	0.509	0.493	0.292	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	406	535	527	405	0	0	0
N.S.	1	1.00	3.62	4.78	4.71	3.62	0.00	0.00	0.00
time (sec)	N/A	0.242	2.276	0.409	0.439	0.290	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	209	240	209	224	0	0	0
N.S.	1	1.00	2.52	2.89	2.52	2.70	0.00	0.00	0.00
time (sec)	N/A	0.143	5.746	0.368	0.419	0.269	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	136	20	15	20	20
N.S.	1	1.00	1.11	1.00	7.56	1.11	0.83	1.11	1.11
time (sec)	N/A	0.034	3.655	0.189	0.415	0.248	0.670	0.270	12.325

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	242	31	17	20	20
N.S.	1	1.00	1.11	1.00	13.44	1.72	0.94	1.11	1.11
time (sec)	N/A	0.030	9.976	0.191	0.517	0.251	2.476	1.990	12.484

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	1657	1604	4024	1154	0	0	0
N.S.	1	1.00	5.62	5.44	13.64	3.91	0.00	0.00	0.00
time (sec)	N/A	0.607	7.243	0.854	1.512	0.348	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	737	932	1948	715	0	0	0
N.S.	1	1.00	3.25	4.11	8.58	3.15	0.00	0.00	0.00
time (sec)	N/A	0.460	7.151	0.800	0.586	0.293	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	238	365	774	378	0	0	0
N.S.	1	1.00	1.74	2.66	5.65	2.76	0.00	0.00	0.00
time (sec)	N/A	0.216	8.522	0.719	0.336	0.284	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	697	36	17	22	22
N.S.	1	1.00	1.10	1.00	34.85	1.80	0.85	1.10	1.10
time (sec)	N/A	0.068	25.480	0.364	1.175	0.273	1.066	0.367	12.206

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	910	47	19	22	22
N.S.	1	1.00	1.10	1.00	45.50	2.35	0.95	1.10	1.10
time (sec)	N/A	0.060	20.010	0.494	1.937	0.253	2.115	2.872	12.673

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	603	603	3129	3161	11252	2749	0	0	0
N.S.	1	1.00	5.19	5.24	18.66	4.56	0.00	0.00	0.00
time (sec)	N/A	1.205	8.216	0.937	15.554	0.368	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	433	433	2029	1788	5429	1564	0	0	0
N.S.	1	1.00	4.69	4.13	12.54	3.61	0.00	0.00	0.00
time (sec)	N/A	0.802	7.657	0.932	2.637	0.309	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	433	745	2017	721	0	0	0
N.S.	1	1.00	1.56	2.68	7.26	2.59	0.00	0.00	0.00
time (sec)	N/A	0.370	12.402	0.718	0.777	0.289	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	2585	52	17	22	22
N.S.	1	1.00	1.10	1.00	129.25	2.60	0.85	1.10	1.10
time (sec)	N/A	0.062	9.417	0.478	5.493	0.290	1.375	0.531	12.632

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	3017	63	19	22	22
N.S.	1	1.00	1.10	1.00	150.85	3.15	0.95	1.10	1.10
time (sec)	N/A	0.058	12.325	0.638	15.251	0.261	2.552	2.783	13.003

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	345	1402	990	1031	0	0	0
N.S.	1	1.00	1.43	5.79	4.09	4.26	0.00	0.00	0.00
time (sec)	N/A	0.413	2.185	0.430	0.570	0.336	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	289	897	720	732	0	0	0
N.S.	1	1.00	1.60	4.96	3.98	4.04	0.00	0.00	0.00
time (sec)	N/A	0.346	1.637	0.324	0.536	0.312	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	182	445	406	475	0	0	0
N.S.	1	1.00	1.44	3.53	3.22	3.77	0.00	0.00	0.00
time (sec)	N/A	0.183	2.132	0.385	0.446	0.292	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	279	27	17	22	22
N.S.	1	1.00	1.10	1.00	13.95	1.35	0.85	1.10	1.10
time (sec)	N/A	0.071	3.338	0.285	0.942	0.267	0.950	0.310	12.662

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	424	51	19	22	22
N.S.	1	1.00	1.10	1.00	21.20	2.55	0.95	1.10	1.10
time (sec)	N/A	0.071	5.301	0.294	2.496	0.258	1.651	2.216	12.161

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	839	839	1733	5931	4641	3363	0	0	0
N.S.	1	1.00	2.07	7.07	5.53	4.01	0.00	0.00	0.00
time (sec)	N/A	2.323	11.550	0.781	3.180	0.383	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	650	650	969	3886	2552	2038	0	0	0
N.S.	1	1.00	1.49	5.98	3.93	3.14	0.00	0.00	0.00
time (sec)	N/A	1.700	9.731	0.664	1.264	0.363	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	730	1997	1171	1053	0	0	0
N.S.	1	1.00	3.43	9.38	5.50	4.94	0.00	0.00	0.00
time (sec)	N/A	0.339	8.546	0.631	0.874	0.327	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	1494	55	19	22	22
N.S.	1	1.00	1.10	1.00	74.70	2.75	0.95	1.10	1.10
time (sec)	N/A	0.073	22.627	0.577	10.993	0.276	1.561	0.400	13.049

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	1975	96	20	22	22
N.S.	1	1.00	1.10	1.00	98.75	4.80	1.00	1.10	1.10
time (sec)	N/A	0.068	21.161	0.579	28.615	0.300	3.110	2.340	13.521

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. The column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [11] had the largest ratio of [.833300000000000041]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	6	1.00	10	0.600
2	A	5	5	1.00	10	0.500
3	A	4	4	1.00	8	0.500
4	N/A	0	0	1.00	10	0.000
5	N/A	0	0	1.00	10	0.000
6	A	7	7	1.00	12	0.583
7	A	6	6	1.00	12	0.500
8	A	3	3	1.00	10	0.300
9	N/A	0	0	1.00	12	0.000
10	N/A	0	0	1.00	12	0.000
11	A	13	10	1.00	12	0.833
12	A	9	8	1.00	12	0.667
13	A	7	7	1.00	10	0.700
14	N/A	0	0	1.00	12	0.000
15	N/A	0	0	1.00	12	0.000
16	A	5	3	1.00	23	0.130
17	A	4	3	1.00	23	0.130
18	A	3	3	1.00	21	0.143
19	A	7	4	1.00	23	0.174
20	A	7	4	1.00	23	0.174
21	A	8	5	1.00	23	0.217
22	A	10	3	1.00	23	0.130
23	A	8	3	1.00	23	0.130

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	7	3	1.00	21	0.143
25	A	21	5	1.00	23	0.217
26	A	24	7	1.00	23	0.304
27	A	14	3	1.00	23	0.130
28	A	11	3	1.00	23	0.130
29	A	11	3	1.00	21	0.143
30	A	53	7	1.00	23	0.304
31	A	60	9	1.00	23	0.391
32	N/A	0	0	1.00	23	0.000
33	N/A	0	0	1.00	21	0.000
34	A	2	2	1.00	23	0.087
35	A	4	2	1.00	23	0.087
36	A	5	2	1.00	23	0.087
37	A	8	7	1.00	18	0.389
38	A	7	6	1.00	18	0.333
39	A	6	5	1.00	16	0.312
40	N/A	0	0	1.00	18	0.000
41	N/A	0	0	1.00	18	0.000
42	A	15	9	1.00	20	0.450
43	A	13	10	1.00	20	0.500
44	A	9	7	1.00	18	0.389
45	N/A	0	0	1.00	20	0.000
46	N/A	0	0	1.00	20	0.000
47	A	28	11	1.00	20	0.550
48	A	22	11	1.00	20	0.550
49	A	16	9	1.00	18	0.500
50	N/A	0	0	1.00	20	0.000
51	N/A	0	0	1.00	20	0.000
52	A	6	6	1.00	20	0.300
53	A	5	5	1.00	20	0.250
54	A	4	4	1.00	18	0.222
55	N/A	0	0	1.00	20	0.000
56	N/A	0	0	1.00	20	0.000
57	A	21	9	1.00	20	0.450

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
58	A	18	10	1.00	20	0.500
59	A	5	5	1.00	18	0.278
60	N/A	0	0	1.00	20	0.000
61	N/A	0	0	1.00	20	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1	$\int x^3 \cot(a + bx) dx$	43
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3.3	$\int x \cot(a + bx) dx$	54
3.4	$\int \frac{\cot(a+bx)}{x} dx$	59
3.5	$\int \frac{\cot(a+bx)}{x^2} dx$	62
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3.9	$\int \frac{\cot^2(a+bx)}{x} dx$	81
3.10	$\int \frac{\cot^2(a+bx)}{x^2} dx$	84
3.11	$\int x^3 \cot^3(a + bx) dx$	87
3.12	$\int x^2 \cot^3(a + bx) dx$	95
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3.14	$\int \frac{\cot^3(a+bx)}{x} dx$	106
3.15	$\int \frac{\cot^3(a+bx)}{x^2} dx$	110
3.16	$\int \frac{(c+dx)^3}{a+ia \cot(e+fx)} dx$	114
3.17	$\int \frac{(c+dx)^2}{a+ia \cot(e+fx)} dx$	120
3.18	$\int \frac{c+dx}{a+ia \cot(e+fx)} dx$	125
3.19	$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))} dx$	129
3.20	$\int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))} dx$	134
3.21	$\int \frac{1}{(c+dx)^3(a+ia \cot(e+fx))} dx$	140
3.22	$\int \frac{(c+dx)^3}{(a+ia \cot(e+fx))^2} dx$	148
3.23	$\int \frac{(c+dx)^2}{(a+ia \cot(e+fx))^2} dx$	155
3.24	$\int \frac{c+dx}{(a+ia \cot(e+fx))^2} dx$	161

3.25	$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))^2} dx$	166
3.26	$\int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))^2} dx$	173
3.27	$\int \frac{(c+dx)^3}{(a+ia \cot(e+fx))^3} dx$	182
3.28	$\int \frac{(c+dx)^2}{(a+ia \cot(e+fx))^3} dx$	190
3.29	$\int \frac{c+dx}{(a+ia \cot(e+fx))^3} dx$	197
3.30	$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))^3} dx$	203
3.31	$\int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))^3} dx$	213
3.32	$\int (c+dx)^m (a+ia \cot(e+fx))^2 dx$	228
3.33	$\int (c+dx)^m (a+ia \cot(e+fx)) dx$	232
3.34	$\int \frac{(c+dx)^m}{a+ia \cot(e+fx)} dx$	235
3.35	$\int \frac{(c+dx)^m}{(a+ia \cot(e+fx))^2} dx$	239
3.36	$\int \frac{(c+dx)^m}{(a+ia \cot(e+fx))^3} dx$	243
3.37	$\int (c+dx)^3 (a+b \cot(e+fx)) dx$	248
3.38	$\int (c+dx)^2 (a+b \cot(e+fx)) dx$	255
3.39	$\int (c+dx) (a+b \cot(e+fx)) dx$	261
3.40	$\int \frac{a+b \cot(e+fx)}{c+dx} dx$	266
3.41	$\int \frac{a+b \cot(e+fx)}{(c+dx)^2} dx$	269
3.42	$\int (c+dx)^3 (a+b \cot(e+fx))^2 dx$	272
3.43	$\int (c+dx)^2 (a+b \cot(e+fx))^2 dx$	284
3.44	$\int (c+dx) (a+b \cot(e+fx))^2 dx$	294
3.45	$\int \frac{(a+b \cot(e+fx))^2}{c+dx} dx$	300
3.46	$\int \frac{(a+b \cot(e+fx))^2}{(c+dx)^2} dx$	304
3.47	$\int (c+dx)^3 (a+b \cot(e+fx))^3 dx$	308
3.48	$\int (c+dx)^2 (a+b \cot(e+fx))^3 dx$	330
3.49	$\int (c+dx) (a+b \cot(e+fx))^3 dx$	345
3.50	$\int \frac{(a+b \cot(e+fx))^3}{c+dx} dx$	354
3.51	$\int \frac{(a+b \cot(e+fx))^3}{(c+dx)^2} dx$	359
3.52	$\int \frac{(c+dx)^3}{a+b \cot(e+fx)} dx$	364
3.53	$\int \frac{(c+dx)^2}{a+b \cot(e+fx)} dx$	372
3.54	$\int \frac{c+dx}{a+b \cot(e+fx)} dx$	379
3.55	$\int \frac{1}{(c+dx)(a+b \cot(e+fx))} dx$	384
3.56	$\int \frac{1}{(c+dx)^2(a+b \cot(e+fx))} dx$	387
3.57	$\int \frac{(c+dx)^3}{(a+b \cot(e+fx))^2} dx$	391
3.58	$\int \frac{(c+dx)^2}{(a+b \cot(e+fx))^2} dx$	409
3.59	$\int \frac{c+dx}{(a+b \cot(e+fx))^2} dx$	422
3.60	$\int \frac{1}{(c+dx)(a+b \cot(e+fx))^2} dx$	430
3.61	$\int \frac{1}{(c+dx)^2(a+b \cot(e+fx))^2} dx$	434

3.1 $\int x^3 \cot(a + bx) dx$

Optimal result	43
Rubi [A] (verified)	43
Mathematica [A] (verified)	45
Maple [B] (verified)	46
Fricas [B] (verification not implemented)	46
Sympy [F]	47
Maxima [B] (verification not implemented)	47
Giac [F]	47
Mupad [F(-1)]	48

Optimal result

Integrand size = 10, antiderivative size = 101

$$\int x^3 \cot(a + bx) dx = -\frac{ix^4}{4} + \frac{x^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3ix^2 \text{PolyLog}(2, e^{2i(a+bx)})}{2b^2} + \frac{3x \text{PolyLog}(3, e^{2i(a+bx)})}{2b^3} + \frac{3i \text{PolyLog}(4, e^{2i(a+bx)})}{4b^4}$$

[Out] $-1/4*I*x^4+x^3*\ln(1-\exp(2*I*(b*x+a)))/b-3/2*I*x^2*\text{polylog}(2,\exp(2*I*(b*x+a)))/b^2+3/2*x*\text{polylog}(3,\exp(2*I*(b*x+a)))/b^3+3/4*I*\text{polylog}(4,\exp(2*I*(b*x+a)))/b^4$

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3798, 2221, 2611, 6744, 2320, 6724}

$$\int x^3 \cot(a + bx) dx = \frac{3i \text{PolyLog}(4, e^{2i(a+bx)})}{4b^4} + \frac{3x \text{PolyLog}(3, e^{2i(a+bx)})}{2b^3} - \frac{3ix^2 \text{PolyLog}(2, e^{2i(a+bx)})}{2b^2} + \frac{x^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{ix^4}{4}$$

[In] $\text{Int}[x^3*\text{Cot}[a + b*x],x]$

[Out] $(-1/4*I)*x^4 + (x^3*\text{Log}[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*x^2*\text{PolyLog}[2, E^((2*I)*(a + b*x))])/b^2 + (3*x*\text{PolyLog}[3, E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*\text{PolyLog}[4, E^((2*I)*(a + b*x))])/b^4$

Rule 2221

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*(f_) + (g_)
*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[(((c_) + (d_)*(x_))^(m_))*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol
] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[(((e_) + (f_)*(x_))^(m_))*PolyLog[n_, (d_)*((F_)^((c_)*((a_) + (b_
)*(x_)))^(p_)], x_Symbol] := Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ix^4}{4} - 2i \int \frac{e^{2i(a+bx)}x^3}{1 - e^{2i(a+bx)}} dx \\
&= -\frac{ix^4}{4} + \frac{x^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3 \int x^2 \log(1 - e^{2i(a+bx)}) dx}{b} \\
&= -\frac{ix^4}{4} + \frac{x^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3ix^2 \text{PolyLog}(2, e^{2i(a+bx)})}{2b^2} + \frac{(3i) \int x \text{PolyLog}(2, e^{2i(a+bx)}) dx}{b^2} \\
&= -\frac{ix^4}{4} + \frac{x^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3ix^2 \text{PolyLog}(2, e^{2i(a+bx)})}{2b^2} \\
&\quad + \frac{3x \text{PolyLog}(3, e^{2i(a+bx)})}{2b^3} - \frac{3 \int \text{PolyLog}(3, e^{2i(a+bx)}) dx}{2b^3} \\
&= -\frac{ix^4}{4} + \frac{x^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3ix^2 \text{PolyLog}(2, e^{2i(a+bx)})}{2b^2} \\
&\quad + \frac{3x \text{PolyLog}(3, e^{2i(a+bx)})}{2b^3} + \frac{(3i) \text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x} dx, x, e^{2i(a+bx)}\right)}{4b^4} \\
&= -\frac{ix^4}{4} + \frac{x^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{3ix^2 \text{PolyLog}(2, e^{2i(a+bx)})}{2b^2} \\
&\quad + \frac{3x \text{PolyLog}(3, e^{2i(a+bx)})}{2b^3} + \frac{3i \text{PolyLog}(4, e^{2i(a+bx)})}{4b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.82

$$\int x^3 \cot(a + bx) dx = \frac{ib^4x^4 + 4b^3x^3 \log(1 - e^{-i(a+bx)}) + 4b^3x^3 \log(1 + e^{-i(a+bx)}) + 12ib^2x^2 \text{PolyLog}(2, -e^{-i(a+bx)}) + 12ib^2x^2 \text{PolyLog}(2, e^{-i(a+bx)})}{4b^4}$$

[In] Integrate[x^3*Cot[a + b*x],x]

[Out] (I*b^4*x^4 + 4*b^3*x^3*Log[1 - E^((-I)*(a + b*x))] + 4*b^3*x^3*Log[1 + E^((-I)*(a + b*x))] + (12*I)*b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] + (12*I)*b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] + 24*b*x*PolyLog[3, -E^((-I)*(a + b*x))] + 24*b*x*PolyLog[3, E^((-I)*(a + b*x))] - (24*I)*PolyLog[4, -E^((-I)*(a + b*x))] - (24*I)*PolyLog[4, E^((-I)*(a + b*x))])/(4*b^4)

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(82) = 164$.

Time = 0.49 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.38

method	result
risch	$-\frac{ix^4}{4} - \frac{3i \operatorname{polylog}(2, -e^{i(bx+a)})x^2}{b^2} - \frac{3i \operatorname{polylog}(2, e^{i(bx+a)})x^2}{b^2} - \frac{2ia^3x}{b^3} + \frac{\ln(1+e^{i(bx+a)})x^3}{b} + \frac{\ln(1-e^{i(bx+a)})x^3}{b} + \frac{a^3 \ln(1+e^{i(bx+a)})}{b}$

[In] `int(x^3*cot(b*x+a),x,method=_RETURNVERBOSE)`

[Out] $-1/4*I*x^4-3*I/b^2*\operatorname{polylog}(2, -\exp(I*(b*x+a)))*x^2-3*I/b^2*\operatorname{polylog}(2, \exp(I*(b*x+a)))*x^2-2*I/b^3*a^3*x+1/b*\ln(1+\exp(I*(b*x+a)))*x^3+1/b*\ln(1-\exp(I*(b*x+a)))*x^3+1/b^4*a^3*\ln(1-\exp(I*(b*x+a)))+2/b^4*a^3*\ln(\exp(I*(b*x+a)))-1/b^4*a^3*\ln(\exp(I*(b*x+a))-1)-3/2*I/b^4*a^4+6*I/b^4*\operatorname{polylog}(4, -\exp(I*(b*x+a)))+6*I/b^4*\operatorname{polylog}(4, \exp(I*(b*x+a)))+6/b^3*\operatorname{polylog}(3, -\exp(I*(b*x+a)))*x+6/b^3*\operatorname{polylog}(3, \exp(I*(b*x+a)))*x$

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(78) = 156$.

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.03

$$\int x^3 \cot(a + bx) dx$$

$$= \frac{-6i b^2 x^2 \operatorname{Li}_2(\cos(2bx + 2a) + i \sin(2bx + 2a)) + 6i b^2 x^2 \operatorname{Li}_2(\cos(2bx + 2a) - i \sin(2bx + 2a)) - 4a^3 \log(\cos(2bx + 2a) + i \sin(2bx + 2a)) - 4a^3 \log(\cos(2bx + 2a) - i \sin(2bx + 2a))}{1}$$

[In] `integrate(x^3*cot(b*x+a),x, algorithm="fricas")`

[Out] $1/8*(-6*I*b^2*x^2*\operatorname{dilog}(\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a)) + 6*I*b^2*x^2*\operatorname{dilog}(\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a)) - 4*a^3*\log(-1/2*\cos(2*b*x + 2*a) + 1/2*I*\sin(2*b*x + 2*a) + 1/2) - 4*a^3*\log(-1/2*\cos(2*b*x + 2*a) - 1/2*I*\sin(2*b*x + 2*a) + 1/2) + 6*b*x*\operatorname{polylog}(3, \cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a)) + 6*b*x*\operatorname{polylog}(3, \cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a)) + 4*(b^3*x^3 + a^3)*\log(-\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a) + 1) + 4*(b^3*x^3 + a^3)*\log(-\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a) + 1) + 3*I*\operatorname{polylog}(4, \cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a)) - 3*I*\operatorname{polylog}(4, \cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a)))/b^4$

Sympy [F]

$$\int x^3 \cot(a + bx) dx = \int x^3 \cot(a + bx) dx$$

```
[In] integrate(x**3*cot(b*x+a),x)
```

```
[Out] Integral(x**3*cot(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(78) = 156.

Time = 0.36 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.87

$$\int x^3 \cot(a + bx) dx = \frac{i (bx + a)^4 - 4i (bx + a)^3 a + 6i (bx + a)^2 a^2 + 4 a^3 \log(\sin(bx + a)) - 24 bx \operatorname{Li}_3(-e^{i(bx+a)}) - 24 bx \operatorname{Li}_3(e^{i(bx+a)})}{b^4}$$

```
[In] integrate(x^3*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*(I*(b*x + a)^4 - 4*I*(b*x + a)^3*a + 6*I*(b*x + a)^2*a^2 + 4*a^3*log(sin(b*x + a)) - 24*b*x*polylog(3, -e^(I*b*x + I*a)) - 24*b*x*polylog(3, e^(I*b*x + I*a)) + 4*(-I*(b*x + a)^3 + 3*I*(b*x + a)^2*a - 3*I*(b*x + a)*a^2)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 4*(I*(b*x + a)^3 - 3*I*(b*x + a)^2*a + 3*I*(b*x + a)*a^2)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 12*(I*(b*x + a)^2 - 2*I*(b*x + a)*a + I*a^2)*dilog(-e^(I*b*x + I*a)) + 12*(I*(b*x + a)^2 - 2*I*(b*x + a)*a + I*a^2)*dilog(e^(I*b*x + I*a)) - 2*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 2*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 24*I*polylog(4, -e^(I*b*x + I*a)) - 24*I*polylog(4, e^(I*b*x + I*a)))/b^4
```

Giac [F]

$$\int x^3 \cot(a + bx) dx = \int x^3 \cot(bx + a) dx$$

```
[In] integrate(x^3*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*cot(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cot(a + bx) dx = \int x^3 \cot(a + bx) dx$$

```
[In] int(x^3*cot(a + b*x),x)
```

```
[Out] int(x^3*cot(a + b*x), x)
```


3.2 $\int x^2 \cot(a + bx) dx$

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Optimal result

Integrand size = 10, antiderivative size = 74

$$\int x^2 \cot(a + bx) dx = -\frac{ix^3}{3} + \frac{x^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{ix \operatorname{PolyLog}(2, e^{2i(a+bx)})}{b^2} + \frac{\operatorname{PolyLog}(3, e^{2i(a+bx)})}{2b^3}$$

[Out] $-1/3*I*x^3+x^2*\ln(1-\exp(2*I*(b*x+a)))/b-I*x*\operatorname{polylog}(2,\exp(2*I*(b*x+a)))/b^2+1/2*\operatorname{polylog}(3,\exp(2*I*(b*x+a)))/b^3$

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3798, 2221, 2611, 2320, 6724}

$$\int x^2 \cot(a + bx) dx = \frac{\operatorname{PolyLog}(3, e^{2i(a+bx)})}{2b^3} - \frac{ix \operatorname{PolyLog}(2, e^{2i(a+bx)})}{b^2} + \frac{x^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{ix^3}{3}$$

[In] $\operatorname{Int}[x^2*\operatorname{Cot}[a + b*x], x]$

[Out] $(-1/3*I)*x^3 + (x^2*\operatorname{Log}[1 - E^((2*I)*(a + b*x))])/b - (I*x*\operatorname{PolyLog}[2, E^((2*I)*(a + b*x))])/b^2 + \operatorname{PolyLog}[3, E^((2*I)*(a + b*x))]/(2*b^3)$

Rule 2221

$\operatorname{Int}[\frac{((F_.)^{((g_.)*((e_.) + (f_.)*(x_)))})^{(n_.)}*((c_.) + (d_.)*(x_))^{(m_.)}}{((a_.) + (b_.)*((F_.)^{((g_.)*((e_.) + (f_.)*(x_)))})^{(n_.)})}, x_Symbol] \rightarrow \operatorname{Simp}$

```

[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

Rule 2320

```

Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

Rule 2611

```

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :=> Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

```

Rule 3798

```

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] :=> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

```

Rule 6724

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :=> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ix^3}{3} - 2i \int \frac{e^{2i(a+bx)} x^2}{1 - e^{2i(a+bx)}} dx \\
&= -\frac{ix^3}{3} + \frac{x^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{2 \int x \log(1 - e^{2i(a+bx)}) dx}{b} \\
&= -\frac{ix^3}{3} + \frac{x^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{ix \text{PolyLog}(2, e^{2i(a+bx)})}{b^2} + \frac{i \int \text{PolyLog}(2, e^{2i(a+bx)}) dx}{b^2} \\
&= -\frac{ix^3}{3} + \frac{x^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{ix \text{PolyLog}(2, e^{2i(a+bx)})}{b^2} + \frac{\text{Subst}\left(\int \frac{\text{PolyLog}(2, x)}{x} dx, x, e^{2i(a+bx)}\right)}{2b^3} \\
&= -\frac{ix^3}{3} + \frac{x^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{ix \text{PolyLog}(2, e^{2i(a+bx)})}{b^2} + \frac{\text{PolyLog}(3, e^{2i(a+bx)})}{2b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.84

$$\int x^2 \cot(a + bx) dx$$

$$= \frac{ib^3 x^3 + 3b^2 x^2 \log(1 - e^{-i(a+bx)}) + 3b^2 x^2 \log(1 + e^{-i(a+bx)}) + 6ibx \operatorname{PolyLog}(2, -e^{-i(a+bx)}) + 6ibx \operatorname{PolyLog}(2, e^{-i(a+bx)}) + 6i \operatorname{PolyLog}(3, -E^{(-I)(a+bx)}) + 6i \operatorname{PolyLog}(3, E^{(-I)(a+bx)})}{3b^3}$$

[In] Integrate[x^2*Cot[a + b*x],x]

[Out] (I*b^3*x^3 + 3*b^2*x^2*Log[1 - E^((-I)*(a + b*x))] + 3*b^2*x^2*Log[1 + E^((-I)*(a + b*x))] + (6*I)*b*x*PolyLog[2, -E^((-I)*(a + b*x))] + (6*I)*b*x*PolyLog[2, E^((-I)*(a + b*x))] + 6*PolyLog[3, -E^((-I)*(a + b*x))] + 6*PolyLog[3, E^((-I)*(a + b*x))])/(3*b^3)

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(62) = 124.

Time = 0.35 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.68

method	result
risch	$-\frac{ix^3}{3} + \frac{4ia^3}{3b^3} + \frac{2ia^2x}{b^2} + \frac{\ln(1+e^{i(bx+a)})x^2}{b} - \frac{2i \operatorname{polylog}(2, -e^{i(bx+a)})x}{b^2} + \frac{2 \operatorname{polylog}(3, -e^{i(bx+a)})}{b^3} + \frac{\ln(1-e^{i(bx+a)})x^2}{b} - \frac{2i \operatorname{polylog}(2, e^{i(bx+a)})x}{b^2} + \frac{2 \operatorname{polylog}(3, e^{i(bx+a)})}{b^3} + \frac{\ln(1-e^{-i(bx+a)})x^2}{b} - \frac{2i \operatorname{polylog}(2, e^{-i(bx+a)})x}{b^2} + \frac{2 \operatorname{polylog}(3, e^{-i(bx+a)})}{b^3} + \frac{\ln(1+e^{-i(bx+a)})x^2}{b}$

[In] int(x^2*cot(b*x+a),x,method=_RETURNVERBOSE)

[Out] -1/3*I*x^3+4/3*I/b^3*a^3+2*I/b^2*a^2*x+1/b*ln(1+exp(I*(b*x+a)))*x^2-2*I/b^2*polylog(2,-exp(I*(b*x+a)))*x+2/b^3*polylog(3,-exp(I*(b*x+a)))+1/b*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*a^2*ln(1-exp(I*(b*x+a)))-2*I/b^2*polylog(2,exp(I*(b*x+a)))*x+2/b^3*polylog(3,exp(I*(b*x+a)))-2/b^3*a^2*ln(exp(I*(b*x+a)))+1/b^3*a^2*ln(exp(I*(b*x+a))-1)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(59) = 118.

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.30

$$\int x^2 \cot(a + bx) dx$$

$$= \frac{-2i bx \operatorname{Li}_2(\cos(2bx + 2a) + i \sin(2bx + 2a)) + 2i bx \operatorname{Li}_2(\cos(2bx + 2a) - i \sin(2bx + 2a)) + 2a^2 \log(-1 + e^{i(bx+a)}) + 2a^2 \log(-1 - e^{i(bx+a)}) + 2a^2 \log(-1 + e^{-i(bx+a)}) + 2a^2 \log(-1 - e^{-i(bx+a)})}{3b^3}$$

[In] integrate(x^2*cot(b*x+a),x, algorithm="fricas")

[Out] $\frac{1}{4}*(-2*I*b*x*dilog(\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a)) + 2*I*b*x*dilog(\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a)) + 2*a^2*\log(-1/2*\cos(2*b*x + 2*a) + 1/2*I*\sin(2*b*x + 2*a) + 1/2) + 2*a^2*\log(-1/2*\cos(2*b*x + 2*a) - 1/2*I*\sin(2*b*x + 2*a) + 1/2) + 2*(b^2*x^2 - a^2)*\log(-\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a) + 1) + 2*(b^2*x^2 - a^2)*\log(-\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a) + 1) + \text{polylog}(3, \cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a)) + \text{polylog}(3, \cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a)))/b^3$

Sympy [F]

$$\int x^2 \cot(a + bx) dx = \int x^2 \cot(a + bx) dx$$

[In] integrate(x**2*cot(b*x+a),x)

[Out] Integral(x**2*cot(a + b*x), x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(59) = 118.

Time = 0.32 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.47

$$\int x^2 \cot(a + bx) dx = \frac{2i (bx + a)^3 - 6i (bx + a)^2 a + 12i bx \text{Li}_2(-e^{i(bx+ia)}) + 12i bx \text{Li}_2(e^{i(bx+ia)}) - 6a^2 \log(\sin(bx + a)) + 6(\dots)}{\dots}$$

[In] integrate(x^2*cot(b*x+a),x, algorithm="maxima")

[Out] $\frac{-1/6*(2*I*(b*x + a)^3 - 6*I*(b*x + a)^2*a + 12*I*b*x*dilog(-e^{(I*b*x + I*a)}) + 12*I*b*x*dilog(e^{(I*b*x + I*a)}) - 6*a^2*\log(\sin(b*x + a)) + 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*\arctan2(\sin(b*x + a), \cos(b*x + a) + 1) + 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*\arctan2(\sin(b*x + a), -\cos(b*x + a) + 1) - 3*((b*x + a)^2 - 2*(b*x + a)*a)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 + 2*\cos(b*x + a) + 1) - 3*((b*x + a)^2 - 2*(b*x + a)*a)*\log(\cos(b*x + a)^2 + \sin(b*x + a)^2 - 2*\cos(b*x + a) + 1) - 12*\text{polylog}(3, -e^{(I*b*x + I*a)}) - 12*\text{polylog}(3, e^{(I*b*x + I*a)})}{b^3}$

Giac [F]

$$\int x^2 \cot(a + bx) dx = \int x^2 \cot(bx + a) dx$$

[In] integrate(x^2*cot(b*x+a),x, algorithm="giac")

[Out] integrate(x^2*cot(b*x + a), x)

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot(a + bx) dx = \int x^2 \cot(a + bx) dx$$

[In] int(x^2*cot(a + b*x),x)

[Out] int(x^2*cot(a + b*x), x)

3.3 $\int x \cot(a + bx) dx$

Optimal result	54
Rubi [A] (verified)	54
Mathematica [B] (verified)	55
Maple [B] (verified)	56
Fricas [B] (verification not implemented)	56
Sympy [F]	57
Maxima [B] (verification not implemented)	57
Giac [F]	57
Mupad [F(-1)]	58

Optimal result

Integrand size = 8, antiderivative size = 53

$$\int x \cot(a + bx) dx = -\frac{ix^2}{2} + \frac{x \log(1 - e^{2i(a+bx)})}{b} - \frac{i \operatorname{PolyLog}(2, e^{2i(a+bx)})}{2b^2}$$

[Out] $-1/2*I*x^2+x*\ln(1-\exp(2*I*(b*x+a)))/b-1/2*I*\operatorname{polylog}(2,\exp(2*I*(b*x+a)))/b^2$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3798, 2221, 2317, 2438}

$$\int x \cot(a + bx) dx = -\frac{i \operatorname{PolyLog}(2, e^{2i(a+bx)})}{2b^2} + \frac{x \log(1 - e^{2i(a+bx)})}{b} - \frac{ix^2}{2}$$

[In] `Int[x*Cot[a + b*x],x]`

[Out] $(-1/2*I)*x^2 + (x*\operatorname{Log}[1 - E^{((2*I)*(a + b*x))}])/b - ((I/2)*\operatorname{PolyLog}[2, E^{((2*I)*(a + b*x))}])/b^2$

Rule 2221

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_)))^(n_))), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m *E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{ix^2}{2} - 2i \int \frac{e^{2i(a+bx)}x}{1 - e^{2i(a+bx)}} dx \\
 &= -\frac{ix^2}{2} + \frac{x \log(1 - e^{2i(a+bx)})}{b} - \frac{\int \log(1 - e^{2i(a+bx)}) dx}{b} \\
 &= -\frac{ix^2}{2} + \frac{x \log(1 - e^{2i(a+bx)})}{b} + \frac{i \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2i(a+bx)}\right)}{2b^2} \\
 &= -\frac{ix^2}{2} + \frac{x \log(1 - e^{2i(a+bx)})}{b} - \frac{i \text{PolyLog}(2, e^{2i(a+bx)})}{2b^2}
 \end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 135 vs. $2(53) = 106$.

Time = 3.84 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.55

$$\int x \cot(a + bx) dx = \frac{1}{2} \left(x^2 \cot(a) - \frac{-ibx(\pi - 2 \arctan(\tan(a))) - \pi \log(1 + e^{-2ibx}) - 2(bx + \arctan(\tan(a))) \log(1 - e^{2i(bx + \arctan(\tan(a))))}{b} - e^{i \arctan(\tan(a))} x^2 \cot(a) \sqrt{\sec^2(a)}}{2} \right)$$

```
[In] Integrate[x*Cot[a + b*x], x]
```

```
[Out] (x^2*Cot[a] - ((-I)*b*x*(Pi - 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)
] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]])]) + Pi
*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]) + I*PolyLo
g[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))]/b^2 - E^(I*ArcTan[Tan[a]])*x^2*Cot[
a]*Sqrt[Sec[a]^2])/2
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(43) = 86$.

Time = 0.23 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.83

method	result
risch	$-\frac{ix^2}{2} - \frac{2iax}{b} - \frac{ia^2}{b^2} + \frac{\ln(1+e^{i(bx+a)})x}{b} - \frac{i \operatorname{polylog}(2, -e^{i(bx+a)})}{b^2} + \frac{\ln(1-e^{i(bx+a)})x}{b} + \frac{\ln(1-e^{i(bx+a)})a}{b^2} - \frac{i \operatorname{polylog}(2, e^{i(bx+a)})}{b^2}$
parts	$-\frac{\ln(\cot(bx+a)^2+1)x}{2b} + \frac{i \left(\ln(\cot(bx+a)-i) \ln(\cot(bx+a)^2+1) - \frac{\ln(\cot(bx+a)-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(\cot(bx+a)+i)}{2}\right) - \ln(\cot(bx+a)-i) \ln\left(-\frac{i(\cot(bx+a)+i)}{2}\right) \right)}{2}$

```
[In] int(x*cot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*x^2-2*I/b*a*x-I/b^2*a^2+1/b*ln(1+exp(I*(b*x+a)))*x-I/b^2*polylog(2,-
exp(I*(b*x+a)))+1/b*ln(1-exp(I*(b*x+a)))*x+1/b^2*ln(1-exp(I*(b*x+a)))*a-I/b
^2*polylog(2,exp(I*(b*x+a)))+2/b^2*a*ln(exp(I*(b*x+a)))-1/b^2*a*ln(exp(I*(b
*x+a)))-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.28

$$\int x \cot(a + bx) dx = \frac{2a \log\left(-\frac{1}{2} \cos(2bx + 2a) + \frac{1}{2}i \sin(2bx + 2a) + \frac{1}{2}\right) + 2a \log\left(-\frac{1}{2} \cos(2bx + 2a) - \frac{1}{2}i \sin(2bx + 2a) + \frac{1}{2}\right)}{2}$$

```
[In] integrate(x*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*a*1
og(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) - 2*(b*x + a)*log(
-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) - 2*(b*x + a)*log(-cos(2*b*x +
2*a) - I*sin(2*b*x + 2*a) + 1) + I*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2
*a)) - I*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)))/b^2
```


Sympy [F]

$$\int x \cot(a + bx) dx = \int x \cot(a + bx) dx$$

[In] integrate(x*cot(b*x+a),x)

[Out] Integral(x*cot(a + b*x), x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(40) = 80$.

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.64

$$\int x \cot(a + bx) dx$$

$$= \frac{-i b^2 x^2 + 2i bx \arctan(\sin(bx + a), \cos(bx + a) + 1) - 2i bx \arctan(\sin(bx + a), -\cos(bx + a) + 1) + b^2 x^2 \arctan(\sin(bx + a), \cos(bx + a) + 1) \arctan(\sin(bx + a), -\cos(bx + a) + 1) + b^2 x^2 \log(\cos(bx + a)^2 + \sin(bx + a)^2 + 2\cos(bx + a) + 1) + b^2 x^2 \log(\cos(bx + a)^2 + \sin(bx + a)^2 - 2\cos(bx + a) + 1) - 2i \operatorname{dilog}(-e^{(Ibx + I)a}) - 2i \operatorname{dilog}(e^{(Ibx + I)a})}{b^2}$$

[In] integrate(x*cot(b*x+a),x, algorithm="maxima")

[Out] 1/2*(-I*b^2*x^2 + 2*I*b*x*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*I*b*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + b*x*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + b*x*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 2*I*dilog(-e^(I*b*x + I*a)) - 2*I*dilog(e^(I*b*x + I*a)))/b^2

Giac [F]

$$\int x \cot(a + bx) dx = \int x \cot(bx + a) dx$$

[In] integrate(x*cot(b*x+a),x, algorithm="giac")

[Out] integrate(x*cot(b*x + a), x)

Mupad [F(-1)]

Timed out.

$$\int x \cot(a + bx) dx = \int x \cot(a + bx) dx$$

```
[In] int(x*cot(a + b*x),x)
```

```
[Out] int(x*cot(a + b*x), x)
```

3.4 $\int \frac{\cot(a+bx)}{x} dx$

Optimal result	59
Rubi [N/A]	59
Mathematica [N/A]	60
Maple [N/A] (verified)	60
Fricas [N/A]	60
Sympy [N/A]	60
Maxima [N/A]	61
Giac [N/A]	61
Mupad [N/A]	61

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\cot(a+bx)}{x} dx = \text{Int}\left(\frac{\cot(a+bx)}{x}, x\right)$$

[Out] Unintegrable(cot(b*x+a)/x,x)

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot(a+bx)}{x} dx = \int \frac{\cot(a+bx)}{x} dx$$

[In] Int[Cot[a + b*x]/x,x]

[Out] Defer[Int][Cot[a + b*x]/x, x]

Rubi steps

$$\text{integral} = \int \frac{\cot(a+bx)}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a + bx)}{x} dx = \int \frac{\cot(a + bx)}{x} dx$$

[In] Integrate[Cot[a + b*x]/x,x]

[Out] Integrate[Cot[a + b*x]/x, x]

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cot(bx + a)}{x} dx$$

[In] int(cot(b*x+a)/x,x)

[Out] int(cot(b*x+a)/x,x)

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a + bx)}{x} dx = \int \frac{\cot(bx + a)}{x} dx$$

[In] integrate(cot(b*x+a)/x,x, algorithm="fricas")

[Out] integral(cot(b*x + a)/x, x)

Sympy [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cot(a + bx)}{x} dx = \int \frac{\cot(a + bx)}{x} dx$$

[In] integrate(cot(b*x+a)/x,x)

[Out] Integral(cot(a + b*x)/x, x)

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a + bx)}{x} dx = \int \frac{\cot(bx + a)}{x} dx$$

[In] integrate(cot(b*x+a)/x,x, algorithm="maxima")

[Out] integrate(cot(b*x + a)/x, x)

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a + bx)}{x} dx = \int \frac{\cot(bx + a)}{x} dx$$

[In] integrate(cot(b*x+a)/x,x, algorithm="giac")

[Out] integrate(cot(b*x + a)/x, x)

Mupad [N/A]

Not integrable

Time = 12.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a + bx)}{x} dx = \int \frac{\cot(a + bx)}{x} dx$$

[In] int(cot(a + b*x)/x,x)

[Out] int(cot(a + b*x)/x, x)

3.5 $\int \frac{\cot(a+bx)}{x^2} dx$

Optimal result	62
Rubi [N/A]	62
Mathematica [N/A]	63
Maple [N/A] (verified)	63
Fricas [N/A]	63
Sympy [N/A]	63
Maxima [N/A]	64
Giac [N/A]	64
Mupad [N/A]	64

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\cot(a+bx)}{x^2} dx = \text{Int}\left(\frac{\cot(a+bx)}{x^2}, x\right)$$

[Out] Unintegrable(cot(b*x+a)/x^2,x)

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot(a+bx)}{x^2} dx = \int \frac{\cot(a+bx)}{x^2} dx$$

[In] Int[Cot[a + b*x]/x^2,x]

[Out] Defer[Int][Cot[a + b*x]/x^2, x]

Rubi steps

$$\text{integral} = \int \frac{\cot(a+bx)}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 3.73 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a + bx)}{x^2} dx = \int \frac{\cot(a + bx)}{x^2} dx$$

[In] Integrate[Cot[a + b*x]/x^2,x]

[Out] Integrate[Cot[a + b*x]/x^2, x]

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cot(bx + a)}{x^2} dx$$

[In] int(cot(b*x+a)/x^2,x)

[Out] int(cot(b*x+a)/x^2,x)

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a + bx)}{x^2} dx = \int \frac{\cot(bx + a)}{x^2} dx$$

[In] integrate(cot(b*x+a)/x^2,x, algorithm="fricas")

[Out] integral(cot(b*x + a)/x^2, x)

Sympy [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cot(a + bx)}{x^2} dx = \int \frac{\cot(a + bx)}{x^2} dx$$

[In] integrate(cot(b*x+a)/x**2,x)

[Out] Integral(cot(a + b*x)/x**2, x)

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a + bx)}{x^2} dx = \int \frac{\cot(bx + a)}{x^2} dx$$

[In] integrate(cot(b*x+a)/x^2,x, algorithm="maxima")

[Out] integrate(cot(b*x + a)/x^2, x)

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a + bx)}{x^2} dx = \int \frac{\cot(bx + a)}{x^2} dx$$

[In] integrate(cot(b*x+a)/x^2,x, algorithm="giac")

[Out] integrate(cot(b*x + a)/x^2, x)

Mupad [N/A]

Not integrable

Time = 12.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a + bx)}{x^2} dx = \int \frac{\cot(a + b x)}{x^2} dx$$

[In] int(cot(a + b*x)/x^2,x)

[Out] int(cot(a + b*x)/x^2, x)

3.6 $\int x^3 \cot^2(a + bx) dx$

Optimal result	65
Rubi [A] (verified)	65
Mathematica [B] (verified)	67
Maple [B] (verified)	68
Fricas [B] (verification not implemented)	68
Sympy [F]	69
Maxima [B] (verification not implemented)	69
Giac [F]	70
Mupad [F(-1)]	70

Optimal result

Integrand size = 12, antiderivative size = 97

$$\int x^3 \cot^2(a + bx) dx = -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3 \cot(a + bx)}{b} + \frac{3x^2 \log(1 - e^{2i(a+bx)})}{b^2} - \frac{3ix \operatorname{PolyLog}(2, e^{2i(a+bx)})}{b^3} + \frac{3 \operatorname{PolyLog}(3, e^{2i(a+bx)})}{2b^4}$$

[Out] $-I*x^3/b - 1/4*x^4 - x^3*\cot(b*x+a)/b + 3*x^2*\ln(1 - \exp(2*I*(b*x+a)))/b^2 - 3*I*x*\operatorname{polylog}(2, \exp(2*I*(b*x+a)))/b^3 + 3/2*\operatorname{polylog}(3, \exp(2*I*(b*x+a)))/b^4$

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3801, 3798, 2221, 2611, 2320, 6724, 30}

$$\int x^3 \cot^2(a + bx) dx = \frac{3 \operatorname{PolyLog}(3, e^{2i(a+bx)})}{2b^4} - \frac{3ix \operatorname{PolyLog}(2, e^{2i(a+bx)})}{b^3} + \frac{3x^2 \log(1 - e^{2i(a+bx)})}{b^2} - \frac{x^3 \cot(a + bx)}{b} - \frac{ix^3}{b} - \frac{x^4}{4}$$

[In] $\operatorname{Int}[x^3*\operatorname{Cot}[a + b*x]^2, x]$

[Out] $((-I)*x^3)/b - x^4/4 - (x^3*\operatorname{Cot}[a + b*x])/b + (3*x^2*\operatorname{Log}[1 - E^((2*I)*(a + b*x))])/b^2 - ((3*I)*x*\operatorname{PolyLog}[2, E^((2*I)*(a + b*x))])/b^3 + (3*\operatorname{PolyLog}[3, E^((2*I)*(a + b*x))])/(2*b^4)$

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol
] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))))], x],
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_) + (d_)*(x_))^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symb
ol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{x^3 \cot(a+bx)}{b} + \frac{3 \int x^2 \cot(a+bx) dx}{b} - \int x^3 dx \\
 &= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3 \cot(a+bx)}{b} - \frac{(6i) \int \frac{e^{2i(a+bx)} x^2}{1-e^{2i(a+bx)}} dx}{b} \\
 &= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3 \cot(a+bx)}{b} + \frac{3x^2 \log(1-e^{2i(a+bx)})}{b^2} - \frac{6 \int x \log(1-e^{2i(a+bx)}) dx}{b^2} \\
 &= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3 \cot(a+bx)}{b} + \frac{3x^2 \log(1-e^{2i(a+bx)})}{b^2} \\
 &\quad - \frac{3ix \text{PolyLog}(2, e^{2i(a+bx)})}{b^3} + \frac{(3i) \int \text{PolyLog}(2, e^{2i(a+bx)}) dx}{b^3} \\
 &= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3 \cot(a+bx)}{b} + \frac{3x^2 \log(1-e^{2i(a+bx)})}{b^2} \\
 &\quad - \frac{3ix \text{PolyLog}(2, e^{2i(a+bx)})}{b^3} + \frac{3 \text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x} dx, x, e^{2i(a+bx)}\right)}{2b^4} \\
 &= -\frac{ix^3}{b} - \frac{x^4}{4} - \frac{x^3 \cot(a+bx)}{b} + \frac{3x^2 \log(1-e^{2i(a+bx)})}{b^2} \\
 &\quad - \frac{3ix \text{PolyLog}(2, e^{2i(a+bx)})}{b^3} + \frac{3 \text{PolyLog}(3, e^{2i(a+bx)})}{2b^4}
 \end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 256 vs. 2(97) = 194.

Time = 1.35 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.64

$$\begin{aligned}
 \int x^3 \cot^2(a+bx) dx &= -\frac{x^4}{4} \\
 &\quad - \frac{ie^{2ia}(2b^3 e^{-2ia} x^3 + 3ib^2(1-e^{-2ia})x^2 \log(1-e^{-i(a+bx)}) + 3ib^2(1-e^{-2ia})x^2 \log(1+e^{-i(a+bx)}) - 6b(1-e^{-2ia})x \log(1-e^{-i(a+bx)}))}{b^4} \\
 &\quad + \frac{x^3 \csc(a) \csc(a+bx) \sin(bx)}{b}
 \end{aligned}$$

[In] Integrate[x^3*Cot[a + b*x]^2,x]

[Out] -1/4*x^4 - (I*E^((2*I)*a))*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))]) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*L

$\log[1 + E^{((-I)*(a + b*x))}] - 6*b*(1 - E^{((-2*I)*a)})*x*PolyLog[2, -E^{((-I)*(a + b*x))}] - 6*b*(1 - E^{((-2*I)*a)})*x*PolyLog[2, E^{((-I)*(a + b*x))}] + (6*I)*(1 - E^{((-2*I)*a)})*PolyLog[3, -E^{((-I)*(a + b*x))}] + (6*I)*(1 - E^{((-2*I)*a)})*PolyLog[3, E^{((-I)*(a + b*x))}]/(b^4*(-1 + E^{(2*I)*a})) + (x^3*Csc[a]*Csc[a + b*x]*Sin[b*x])/b$

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(85) = 170$.

Time = 0.34 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.38

method	result
risch	$-\frac{x^4}{4} - \frac{2ix^3}{b(e^{2i(bx+a)}-1)} + \frac{6ia^2x}{b^3} - \frac{2ix^3}{b} + \frac{4ia^3}{b^4} + \frac{3\ln(1+e^{i(bx+a)})x^2}{b^2} - \frac{6i \operatorname{polylog}(2, e^{i(bx+a)})x}{b^3} + \frac{6 \operatorname{polylog}(3, -e^{i(bx+a)})}{b^4}$

[In] int(x^3*cot(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] $-1/4*x^4-2*I*x^3/b/(\exp(2*I*(b*x+a))-1)+6*I/b^3*a^2*x-2*I/b*x^3+4*I/b^4*a^3+3/b^2*\ln(1+\exp(I*(b*x+a)))*x^2-6*I/b^3*\operatorname{polylog}(2,\exp(I*(b*x+a)))*x+6/b^4*\operatorname{polylog}(3,-\exp(I*(b*x+a)))+3/b^2*\ln(1-\exp(I*(b*x+a)))*x^2-3/b^4*a^2*\ln(1-\exp(I*(b*x+a)))-6*I/b^3*\operatorname{polylog}(2,-\exp(I*(b*x+a)))*x+6/b^4*\operatorname{polylog}(3,\exp(I*(b*x+a)))-6/b^4*a^2*\ln(\exp(I*(b*x+a)))+3/b^4*a^2*\ln(\exp(I*(b*x+a)))-1$

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(82) = 164$.

Time = 0.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.84

$$\int x^3 \cot^2(a + bx) dx = \frac{b^4 x^4 \sin(2bx + 2a) + 4b^3 x^3 \cos(2bx + 2a) + 4b^3 x^3 + 6i bx \operatorname{Li}_2(\cos(2bx + 2a) + i \sin(2bx + 2a)) \sin(2bx + 2a)}{b^4}$$

[In] integrate(x^3*cot(b*x+a)^2,x, algorithm="fricas")

[Out] $-1/4*(b^4*x^4*\sin(2*b*x + 2*a) + 4*b^3*x^3*\cos(2*b*x + 2*a) + 4*b^3*x^3 + 6*I*b*x*\operatorname{dilog}(\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a))*\sin(2*b*x + 2*a) - 6*I*b*x*\operatorname{dilog}(\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a))*\sin(2*b*x + 2*a) - 6*a^2*\log(-1/2*\cos(2*b*x + 2*a) + 1/2*I*\sin(2*b*x + 2*a) + 1/2)*\sin(2*b*x + 2*a) - 6*a^2*\log(-1/2*\cos(2*b*x + 2*a) - 1/2*I*\sin(2*b*x + 2*a) + 1/2)*\sin(2*b*x + 2*a) - 6*(b^2*x^2 - a^2)*\log(-\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a) + 1)*\sin(2*b*x + 2*a) - 6*(b^2*x^2 - a^2)*\log(-\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a) + 1)*\sin(2*b*x + 2*a)$

```
2*a) + 1)*sin(2*b*x + 2*a) - 3*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x +
2*a))*sin(2*b*x + 2*a) - 3*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)
)*sin(2*b*x + 2*a))/(b^4*sin(2*b*x + 2*a))
```

Sympy [F]

$$\int x^3 \cot^2(a + bx) dx = \int x^3 \cot^2(a + bx) dx$$

```
[In] integrate(x**3*cot(b*x+a)**2,x)
```

```
[Out] Integral(x**3*cot(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(82) = 164$.

Time = 0.42 (sec) , antiderivative size = 952, normalized size of antiderivative = 9.81

$$\int x^3 \cot^2(a + bx) dx = \text{Too large to display}$$

```
[In] integrate(x^3*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a + 1/tan(b*x + a))*a^3 - 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 +
(b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a
)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(
b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^2/(cos(2
*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1) + 2*(-I*(b*x +
a)^4 + 4*I*(b*x + a)^3*a - 12*((b*x + a)^2 - 2*(b*x + a)*a - ((b*x + a)^2
- 2*(b*x + a)*a)*cos(2*b*x + 2*a) + (-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*sin(
2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 12*((b*x + a)^2 - 2
*(b*x + a)*a - ((b*x + a)^2 - 2*(b*x + a)*a)*cos(2*b*x + 2*a) - (I*(b*x + a
)^2 - 2*I*(b*x + a)*a)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a
) + 1) + (I*(b*x + a)^4 - 4*(b*x + a)^3*(I*a + 2) + 24*(b*x + a)^2*a*cos(2
*b*x + 2*a) - 24*(b*x*cos(2*b*x + 2*a) + I*b*x*sin(2*b*x + 2*a) - b*x)*dilo
g(-e^(I*b*x + I*a)) - 24*(b*x*cos(2*b*x + 2*a) + I*b*x*sin(2*b*x + 2*a) - b
*x)*dilog(e^(I*b*x + I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a + (I*(b*x
+ a)^2 - 2*I*(b*x + a)*a)*cos(2*b*x + 2*a) - ((b*x + a)^2 - 2*(b*x + a)*a)*
sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
- 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a + (I*(b*x + a)^2 - 2*I*(b*x + a)*a)*
```

```

cos(2*b*x + 2*a) - ((b*x + a)^2 - 2*(b*x + a)*a)*sin(2*b*x + 2*a))*log(cos(
b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 24*(I*cos(2*b*x + 2*a)
- sin(2*b*x + 2*a) - I)*polylog(3, -e^(I*b*x + I*a)) - 24*(I*cos(2*b*x + 2*
a) - sin(2*b*x + 2*a) - I)*polylog(3, e^(I*b*x + I*a)) - ((b*x + a)^4 - 4*(
b*x + a)^3*(a - 2*I) - 24*I*(b*x + a)^2*a)*sin(2*b*x + 2*a))/(-4*I*cos(2*b*
x + 2*a) + 4*sin(2*b*x + 2*a) + 4*I))/b^4

```

Giac [F]

$$\int x^3 \cot^2(a + bx) dx = \int x^3 \cot (bx + a)^2 dx$$

```
[In] integrate(x^3*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cot(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cot^2(a + bx) dx = \int x^3 \cot(a + bx)^2 dx$$

```
[In] int(x^3*cot(a + b*x)^2,x)
```

```
[Out] int(x^3*cot(a + b*x)^2, x)
```

3.7 $\int x^2 \cot^2(a + bx) dx$

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Optimal result

Integrand size = 12, antiderivative size = 74

$$\int x^2 \cot^2(a + bx) dx = -\frac{ix^2}{b} - \frac{x^3}{3} - \frac{x^2 \cot(a + bx)}{b} + \frac{2x \log(1 - e^{2i(a+bx)})}{b^2} - \frac{i \operatorname{PolyLog}(2, e^{2i(a+bx)})}{b^3}$$

[Out] $-I*x^2/b - 1/3*x^3 - x^2*\cot(b*x+a)/b + 2*x*\ln(1-\exp(2*I*(b*x+a)))/b^2 - I*polylog(2, \exp(2*I*(b*x+a)))/b^3$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3801, 3798, 2221, 2317, 2438, 30}

$$\int x^2 \cot^2(a + bx) dx = -\frac{i \operatorname{PolyLog}(2, e^{2i(a+bx)})}{b^3} + \frac{2x \log(1 - e^{2i(a+bx)})}{b^2} - \frac{x^2 \cot(a + bx)}{b} - \frac{ix^2}{b} - \frac{x^3}{3}$$

[In] $\operatorname{Int}[x^2*\cot[a + b*x]^2, x]$

[Out] $((-I)*x^2)/b - x^3/3 - (x^2*\cot[a + b*x])/b + (2*x*\log[1 - E^((2*I)*(a + b*x))])/b^2 - (I*\operatorname{PolyLog}[2, E^((2*I)*(a + b*x))])/b^3$

Rule 30

$\operatorname{Int}[(x_)^m, x_Symbol] := \operatorname{Simp}[x^{(m + 1)}/(m + 1), x] /; \operatorname{FreeQ}[m, x] \ \&\& \operatorname{NeQ}[m, -1]$

Rule 2221

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[(((c_) + (d_)*(x_))^(m_))*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol
] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))], x],
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[(((c_) + (d_)*(x_))^(m_))*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symb
ol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2 \cot(a + bx)}{b} + \frac{2 \int x \cot(a + bx) dx}{b} - \int x^2 dx \\
&= -\frac{ix^2}{b} - \frac{x^3}{3} - \frac{x^2 \cot(a + bx)}{b} - \frac{(4i) \int \frac{e^{2i(a+bx)} x}{1 - e^{2i(a+bx)}} dx}{b} \\
&= -\frac{ix^2}{b} - \frac{x^3}{3} - \frac{x^2 \cot(a + bx)}{b} + \frac{2x \log(1 - e^{2i(a+bx)})}{b^2} - \frac{2 \int \log(1 - e^{2i(a+bx)}) dx}{b^2} \\
&= -\frac{ix^2}{b} - \frac{x^3}{3} - \frac{x^2 \cot(a + bx)}{b} + \frac{2x \log(1 - e^{2i(a+bx)})}{b^2} + \frac{i \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2i(a+bx)}\right)}{b^3}
\end{aligned}$$

$$= -\frac{ix^2}{b} - \frac{x^3}{3} - \frac{x^2 \cot(a+bx)}{b} + \frac{2x \log(1 - e^{2i(a+bx)})}{b^2} - \frac{i \operatorname{PolyLog}(2, e^{2i(a+bx)})}{b^3}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 153 vs. $2(74) = 148$.

Time = 5.80 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.07

$$\int x^2 \cot^2(a+bx) dx = -\frac{x^3}{3} + \frac{ibx(\pi - 2 \arctan(\tan(a))) + \pi \log(1 + e^{-2ibx}) + 2(bx + \arctan(\tan(a))) \log(1 - e^{2i(bx + \arctan(\tan(a)))})}{b} + \frac{x^2 \csc(a) \csc(a+bx) \sin(bx)}{b}$$

[In] Integrate[x^2*Cot[a + b*x]^2,x]

[Out] $-1/3*x^3 + (I*b*x*(\pi - 2*ArcTan[Tan[a]]) + \pi*Log[1 + E^{((-2*I)*b*x]}) + 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^{((2*I)*(b*x + ArcTan[Tan[a]])}]] - \pi*Log[Cos[b*x]] - 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] - I*PolyLog[2, E^{((2*I)*(b*x + ArcTan[Tan[a]])}]] - b^2*E^{(I*ArcTan[Tan[a]])}*x^2*Cot[a]*Sqrt[Sec[a]^2])/b^3 + (x^2*Csc[a]*Csc[a + b*x]*Sin[b*x])/b$

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(66) = 132$.

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.47

method	result
risch	$-\frac{x^3}{3} - \frac{2ix^2}{b(e^{2i(bx+a)}-1)} - \frac{2ix^2}{b} - \frac{4iax}{b^2} - \frac{2ia^2}{b^3} + \frac{2 \ln(1+e^{i(bx+a)})x}{b^2} - \frac{2i \operatorname{polylog}(2, -e^{i(bx+a)})}{b^3} + \frac{2 \ln(1-e^{i(bx+a)})x}{b^2} +$

[In] int(x^2*cot(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] $-1/3*x^3-2*I*x^2/b/(\exp(2*I*(b*x+a))-1)-2*I/b*x^2-4*I/b^2*a*x-2*I/b^3*a^2+2/b^2*\ln(1+\exp(I*(b*x+a)))*x-2*I/b^3*\operatorname{polylog}(2,-\exp(I*(b*x+a)))+2/b^2*\ln(1-\exp(I*(b*x+a)))*x+2/b^3*\ln(1-\exp(I*(b*x+a)))*a-2*I/b^3*\operatorname{polylog}(2,\exp(I*(b*x+a)))+4/b^3*a*\ln(\exp(I*(b*x+a)))-2/b^3*a*\ln(\exp(I*(b*x+a))-1)$

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(63) = 126$.

Time = 0.28 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.80

$$\int x^2 \cot^2(a + bx) dx = \frac{2b^3x^3 \sin(2bx + 2a) + 6b^2x^2 \cos(2bx + 2a) + 6b^2x^2 + 6a \log\left(-\frac{1}{2} \cos(2bx + 2a) + \frac{1}{2}i \sin(2bx + 2a)\right)}{b^3 \sin(2bx + 2a)}$$

```
[In] integrate(x^2*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*x^3*sin(2*b*x + 2*a) + 6*b^2*x^2*cos(2*b*x + 2*a) + 6*b^2*x^2 + 6*a*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) + 6*a*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b*x + a)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) - 6*(b*x + a)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) + 3*I*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 3*I*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a))/(b^3*sin(2*b*x + 2*a))
```

Sympy [F]

$$\int x^2 \cot^2(a + bx) dx = \int x^2 \cot^2(a + bx) dx$$

```
[In] integrate(x**2*cot(b*x+a)**2,x)
```

```
[Out] Integral(x**2*cot(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(63) = 126$.

Time = 0.42 (sec) , antiderivative size = 386, normalized size of antiderivative = 5.22

$$\int x^2 \cot^2(a + bx) dx = \frac{-i b^3 x^3 + 6 (bx \cos(2bx + 2a) + i bx \sin(2bx + 2a) - bx) \arctan(\sin(bx + a), \cos(bx + a) + 1) - 6 (bx \cos(2bx + 2a) + i bx \sin(2bx + 2a) - bx)}{b^3 \sin(2bx + 2a)}$$

```
[In] integrate(x^2*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (-I*b^3*x^3 + 6*(b*x*cos(2*b*x + 2*a) + I*b*x*sin(2*b*x + 2*a) - b*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 6*(b*x*cos(2*b*x + 2*a) + I*b*x*sin(2*b*x + 2*a) - b*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*b^3*x^3 - 6*b^2*x^2)*cos(2*b*x + 2*a) - 6*(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) - 1)*dilog(-e^(I*b*x + I*a)) - 6*(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) - 1)*dilog(e^(I*b*x + I*a)) - 3*(I*b*x*cos(2*b*x + 2*a) - b*x*sin(2*b*x + 2*a) - I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*(I*b*x*cos(2*b*x + 2*a) - b*x*sin(2*b*x + 2*a) - I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (b^3*x^3 + 6*I*b^2*x^2)*sin(2*b*x + 2*a))/(-3*I*b^3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) + 3*I*b^3)
```

Giac [F]

$$\int x^2 \cot^2(a + bx) dx = \int x^2 \cot(bx + a)^2 dx$$

```
[In] integrate(x^2*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cot(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^2(a + bx) dx = \int x^2 \cot(a + bx)^2 dx$$

```
[In] int(x^2*cot(a + b*x)^2,x)
```

```
[Out] int(x^2*cot(a + b*x)^2, x)
```

3.8 $\int x \cot^2(a + bx) dx$

Optimal result	76
Rubi [A] (verified)	76
Mathematica [A] (verified)	77
Maple [A] (verified)	77
Fricas [B] (verification not implemented)	78
Sympy [B] (verification not implemented)	78
Maxima [B] (verification not implemented)	78
Giac [B] (verification not implemented)	79
Mupad [B] (verification not implemented)	80

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int x \cot^2(a + bx) dx = -\frac{x^2}{2} - \frac{x \cot(a + bx)}{b} + \frac{\log(\sin(a + bx))}{b^2}$$

[Out] $-1/2*x^2 - x*\cot(b*x+a)/b + \ln(\sin(b*x+a))/b^2$

Rubi [A] (verified)

Time = 0.03 (sec), antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3801, 3556, 30}

$$\int x \cot^2(a + bx) dx = \frac{\log(\sin(a + bx))}{b^2} - \frac{x \cot(a + bx)}{b} - \frac{x^2}{2}$$

[In] $\text{Int}[x*\text{Cot}[a + b*x]^2, x]$

[Out] $-1/2*x^2 - (x*\text{Cot}[a + b*x])/b + \text{Log}[\text{Sin}[a + b*x]]/b^2$

Rule 30

$\text{Int}[(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[x^{(m + 1)}/(m + 1), x] \text{ ; FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

Rule 3556

$\text{Int}[\tan[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow \text{Simp}[-\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] \text{ ; FreeQ}\{c, d\}, x]$

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Dist[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{x \cot(a + bx)}{b} + \frac{\int \cot(a + bx) dx}{b} - \int x dx \\ &= -\frac{x^2}{2} - \frac{x \cot(a + bx)}{b} + \frac{\log(\sin(a + bx))}{b^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int x \cot^2(a + bx) dx = -\frac{x^2}{2} - \frac{x \cot(a)}{b} + \frac{\log(\sin(a + bx))}{b^2} + \frac{x \csc(a) \csc(a + bx) \sin(bx)}{b}$$

[In] Integrate[x*Cot[a + b*x]^2,x]

[Out] -1/2*x^2 - (x*Cot[a])/b + Log[Sin[a + b*x]]/b^2 + (x*Csc[a]*Csc[a + b*x]*Sin[b*x])/b

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

method	result	size
default	$-\frac{x^2}{2} + \frac{-(bx+a) \cot(bx+a) + \ln(\sin(bx+a)) + a \cot(bx+a)}{b^2}$	40
parallelrisch	$\frac{-x^2 b^2 - 2bx \cot(bx+a) + 2 \ln(\tan(bx+a)) - \ln(\sec(bx+a)^2)}{2b^2}$	45
norman	$\frac{-\frac{x}{b} - \frac{\tan(bx+a)x^2}{2}}{\tan(bx+a)} + \frac{\ln(\tan(bx+a))}{b^2} - \frac{\ln(1 + \tan(bx+a)^2)}{2b^2}$	56
risch	$-\frac{x^2}{2} - \frac{2ix}{b} - \frac{2ia}{b^2} - \frac{2ix}{b(e^{2i(bx+a)} - 1)} + \frac{\ln(e^{2i(bx+a)} - 1)}{b^2}$	57

[In] int(x*cot(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/2*x^2+1/b^2*(-(b*x+a)*cot(b*x+a)+ln(sin(b*x+a))+a*cot(b*x+a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(29) = 58.

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.42

$$\int x \cot^2(a + bx) dx = \frac{b^2 x^2 \sin(2bx + 2a) + 2bx \cos(2bx + 2a) + 2bx - \log\left(-\frac{1}{2} \cos(2bx + 2a) + \frac{1}{2}\right) \sin(2bx + 2a)}{2b^2 \sin(2bx + 2a)}$$

[In] integrate(x*cot(b*x+a)^2,x, algorithm="fricas")

[Out] -1/2*(b^2*x^2*sin(2*b*x + 2*a) + 2*b*x*cos(2*b*x + 2*a) + 2*b*x - log(-1/2*cos(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a))/(b^2*sin(2*b*x + 2*a))

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(26) = 52.

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.10

$$\int x \cot^2(a + bx) dx = \begin{cases} \tilde{\infty} x^2 & \text{for } a = 0 \wedge b = 0 \\ \frac{x^2 \cot^2(a)}{2} & \text{for } b = 0 \\ \tilde{\infty} x^2 & \text{for } a = -bx \\ -\frac{x^2}{2} - \frac{x}{b \tan(a + bx)} - \frac{\log(\tan^2(a + bx) + 1)}{2b^2} + \frac{\log(\tan(a + bx))}{b^2} & \text{otherwise} \end{cases}$$

[In] integrate(x*cot(b*x+a)**2,x)

[Out] Piecewise((zoo*x**2, Eq(a, 0) & Eq(b, 0)), (x**2*cot(a)**2/2, Eq(b, 0)), (zoo*x**2, Eq(a, -b*x)), (-x**2/2 - x/(b*tan(a + b*x)) - log(tan(a + b*x)**2 + 1)/(2*b**2) + log(tan(a + b*x))/b**2, True))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(29) = 58.

Time = 0.36 (sec) , antiderivative size = 269, normalized size of antiderivative = 8.68

$$\int x \cot^2(a + bx) dx = \frac{2 \left(bx + a + \frac{1}{\tan(bx+a)} \right) a - \frac{(bx+a)^2 \cos(2bx+2a)^2 + (bx+a)^2 \sin(2bx+2a)^2 - 2(bx+a)^2 \cos(2bx+2a) + (bx+a)^2 - (\cos(2bx+2a)^2 + \sin(2bx+2a)^2)}{2b^2 \sin(2bx+2a)}}{2b^2 \sin(2bx+2a)}$$

[In] integrate(x*cot(b*x+a)^2,x, algorithm="maxima")

$$\frac{1/2*b*x)^2*\tan(1/2*a)^2 + \tan(1/2*a)^4 + 2*\tan(1/2*b*x)^2 + 2*\tan(1/2*a)^2 + 1))*\tan(1/2*a))/(b^2*\tan(1/2*b*x)^2*\tan(1/2*a) + b^2*\tan(1/2*b*x)*\tan(1/2*a)^2 - b^2*\tan(1/2*b*x) - b^2*\tan(1/2*a))$$

Mupad [B] (verification not implemented)

Time = 12.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int x \cot^2(a + bx) dx = \frac{\ln(e^{a+bx} - 1)}{b^2} - \frac{x}{b} - \frac{x^2}{2} - \frac{x}{b(e^{a+bx} - 1)}$$

[In] int(x*cot(a + b*x)^2,x)

[Out] log(exp(a+b*x)-1)/b^2 - (x)/b - x^2/2 - (x)/(b*(exp(a+b*x)-1))

3.9 $\int \frac{\cot^2(a+bx)}{x} dx$

Optimal result	81
Rubi [N/A]	81
Mathematica [N/A]	82
Maple [N/A] (verified)	82
Fricas [N/A]	82
Sympy [N/A]	82
Maxima [N/A]	83
Giac [N/A]	83
Mupad [N/A]	83

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\cot^2(a+bx)}{x} dx = \text{Int}\left(\frac{\cot^2(a+bx)}{x}, x\right)$$

[Out] Unintegrable(cot(b*x+a)^2/x,x)

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^2(a+bx)}{x} dx = \int \frac{\cot^2(a+bx)}{x} dx$$

[In] Int[Cot[a + b*x]^2/x,x]

[Out] Defer[Int][Cot[a + b*x]^2/x, x]

Rubi steps

$$\text{integral} = \int \frac{\cot^2(a+bx)}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 7.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^2(a + bx)}{x} dx = \int \frac{\cot^2(a + bx)}{x} dx$$

[In] Integrate[Cot[a + b*x]^2/x,x]

[Out] Integrate[Cot[a + b*x]^2/x, x]

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cot (bx + a)^2}{x} dx$$

[In] int(cot(b*x+a)^2/x,x)

[Out] int(cot(b*x+a)^2/x,x)

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^2(a + bx)}{x} dx = \int \frac{\cot (bx + a)^2}{x} dx$$

[In] integrate(cot(b*x+a)^2/x,x, algorithm="fricas")

[Out] integral(cot(b*x + a)^2/x, x)

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\cot^2(a + bx)}{x} dx = \int \frac{\cot^2(a + bx)}{x} dx$$

[In] integrate(cot(b*x+a)**2/x,x)

[Out] Integral(cot(a + b*x)**2/x, x)

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 348, normalized size of antiderivative = 29.00

$$\int \frac{\cot^2(a + bx)}{x} dx = \int \frac{\cot(bx + a)^2}{x} dx$$

[In] integrate(cot(b*x+a)^2/x,x, algorithm="maxima")

```
[Out] -(b*x*cos(2*b*x + 2*a)^2*log(x) + b*x*log(x)*sin(2*b*x + 2*a)^2 - 2*b*x*cos(2*b*x + 2*a)*log(x) + b*x*log(x) - (b^2*x*cos(2*b*x + 2*a)^2 + b^2*x*sin(2*b*x + 2*a)^2 - 2*b^2*x*cos(2*b*x + 2*a) + b^2*x)*integrate(sin(b*x + a)/(b^2*x^2*cos(b*x + a)^2 + b^2*x^2*sin(b*x + a)^2 + 2*b^2*x^2*cos(b*x + a) + b^2*x^2), x) + (b^2*x*cos(2*b*x + 2*a)^2 + b^2*x*sin(2*b*x + 2*a)^2 - 2*b^2*x*cos(2*b*x + 2*a) + b^2*x)*integrate(sin(b*x + a)/(b^2*x^2*cos(b*x + a)^2 + b^2*x^2*sin(b*x + a)^2 - 2*b^2*x^2*cos(b*x + a) + b^2*x^2), x) + 2*sin(2*b*x + 2*a))/(b*x*cos(2*b*x + 2*a)^2 + b*x*sin(2*b*x + 2*a)^2 - 2*b*x*cos(2*b*x + 2*a) + b*x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^2(a + bx)}{x} dx = \int \frac{\cot(bx + a)^2}{x} dx$$

[In] integrate(cot(b*x+a)^2/x,x, algorithm="giac")

[Out] integrate(cot(b*x + a)^2/x, x)

Mupad [N/A]

Not integrable

Time = 12.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^2(a + bx)}{x} dx = \int \frac{\cot(a + bx)^2}{x} dx$$

[In] int(cot(a + b*x)^2/x,x)

[Out] int(cot(a + b*x)^2/x, x)

3.10 $\int \frac{\cot^2(a+bx)}{x^2} dx$

Optimal result	84
Rubi [N/A]	84
Mathematica [N/A]	85
Maple [N/A] (verified)	85
Fricas [N/A]	85
Sympy [N/A]	85
Maxima [N/A]	86
Giac [N/A]	86
Mupad [N/A]	86

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\cot^2(a+bx)}{x^2} dx = \text{Int}\left(\frac{\cot^2(a+bx)}{x^2}, x\right)$$

[Out] Unintegrable(cot(b*x+a)^2/x^2,x)

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^2(a+bx)}{x^2} dx = \int \frac{\cot^2(a+bx)}{x^2} dx$$

[In] Int[Cot[a + b*x]^2/x^2,x]

[Out] Defer[Int][Cot[a + b*x]^2/x^2, x]

Rubi steps

$$\text{integral} = \int \frac{\cot^2(a+bx)}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 3.87 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^2(a + bx)}{x^2} dx = \int \frac{\cot^2(a + bx)}{x^2} dx$$

[In] Integrate[Cot[a + b*x]^2/x^2,x]

[Out] Integrate[Cot[a + b*x]^2/x^2, x]

Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cot^2(bx + a)}{x^2} dx$$

[In] int(cot(b*x+a)^2/x^2,x)

[Out] int(cot(b*x+a)^2/x^2,x)

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^2(a + bx)}{x^2} dx = \int \frac{\cot^2(bx + a)}{x^2} dx$$

[In] integrate(cot(b*x+a)^2/x^2,x, algorithm="fricas")

[Out] integral(cot(b*x + a)^2/x^2, x)

Sympy [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cot^2(a + bx)}{x^2} dx = \int \frac{\cot^2(a + bx)}{x^2} dx$$

[In] integrate(cot(b*x+a)**2/x**2,x)

[Out] Integral(cot(a + b*x)**2/x**2, x)

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 364, normalized size of antiderivative = 30.33

$$\int \frac{\cot^2(a + bx)}{x^2} dx = \int \frac{\cot(bx + a)^2}{x^2} dx$$

[In] integrate(cot(b*x+a)^2/x^2,x, algorithm="maxima")

```
[Out] (b*x*cos(2*b*x + 2*a)^2 + b*x*sin(2*b*x + 2*a)^2 - 2*b*x*cos(2*b*x + 2*a) +
b*x + 2*(b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(2*b*x + 2*a)^2 - 2*b^2*x
^2*cos(2*b*x + 2*a) + b^2*x^2)*integrate(sin(b*x + a)/(b^2*x^3*cos(b*x + a)
^2 + b^2*x^3*sin(b*x + a)^2 + 2*b^2*x^3*cos(b*x + a) + b^2*x^3), x) - 2*(b^
2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(2*b*x + 2*a)^2 - 2*b^2*x^2*cos(2*b*x
+ 2*a) + b^2*x^2)*integrate(sin(b*x + a)/(b^2*x^3*cos(b*x + a)^2 + b^2*x^3
*sin(b*x + a)^2 - 2*b^2*x^3*cos(b*x + a) + b^2*x^3), x) - 2*sin(2*b*x + 2*a
))/ (b*x^2*cos(2*b*x + 2*a)^2 + b*x^2*sin(2*b*x + 2*a)^2 - 2*b*x^2*cos(2*b*x
+ 2*a) + b*x^2)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^2(a + bx)}{x^2} dx = \int \frac{\cot(bx + a)^2}{x^2} dx$$

[In] integrate(cot(b*x+a)^2/x^2,x, algorithm="giac")

[Out] integrate(cot(b*x + a)^2/x^2, x)

Mupad [N/A]

Not integrable

Time = 12.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^2(a + bx)}{x^2} dx = \int \frac{\cot(a + bx)^2}{x^2} dx$$

[In] int(cot(a + b*x)^2/x^2,x)

[Out] int(cot(a + b*x)^2/x^2, x)

3.11 $\int x^3 \cot^3(a + bx) dx$

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Optimal result

Integrand size = 12, antiderivative size = 202

$$\int x^3 \cot^3(a + bx) dx = -\frac{3ix^2}{2b^2} - \frac{x^3}{2b} + \frac{ix^4}{4} - \frac{3x^2 \cot(a + bx)}{2b^2} - \frac{x^3 \cot^2(a + bx)}{2b}$$

$$+ \frac{3x \log(1 - e^{2i(a+bx)})}{b^3} - \frac{x^3 \log(1 - e^{2i(a+bx)})}{b}$$

$$- \frac{3i \operatorname{PolyLog}(2, e^{2i(a+bx)})}{2b^4} + \frac{3ix^2 \operatorname{PolyLog}(2, e^{2i(a+bx)})}{2b^2}$$

$$- \frac{3x \operatorname{PolyLog}(3, e^{2i(a+bx)})}{2b^3} - \frac{3i \operatorname{PolyLog}(4, e^{2i(a+bx)})}{4b^4}$$

[Out] $-3/2*I*x^2/b^2-1/2*x^3/b+1/4*I*x^4-3/2*x^2*\cot(b*x+a)/b^2-1/2*x^3*\cot(b*x+a)^2/b+3*x*\ln(1-\exp(2*I*(b*x+a)))/b^3-x^3*\ln(1-\exp(2*I*(b*x+a)))/b-3/2*I*\operatorname{polylog}(2,\exp(2*I*(b*x+a)))/b^4+3/2*I*x^2*\operatorname{polylog}(2,\exp(2*I*(b*x+a)))/b^2-3/2*x*\operatorname{polylog}(3,\exp(2*I*(b*x+a)))/b^3-3/4*I*\operatorname{polylog}(4,\exp(2*I*(b*x+a)))/b^4$

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3801, 3798, 2221, 2317, 2438, 30, 2611, 6744, 2320, 6724}

$$\int x^3 \cot^3(a + bx) dx = -\frac{3i \operatorname{PolyLog}(2, e^{2i(a+bx)})}{2b^4} - \frac{3i \operatorname{PolyLog}(4, e^{2i(a+bx)})}{4b^4}$$

$$- \frac{3x \operatorname{PolyLog}(3, e^{2i(a+bx)})}{2b^3} + \frac{3x \log(1 - e^{2i(a+bx)})}{b^3}$$

$$+ \frac{3ix^2 \operatorname{PolyLog}(2, e^{2i(a+bx)})}{2b^2} - \frac{3x^2 \cot(a + bx)}{2b^2}$$

$$- \frac{x^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{x^3 \cot^2(a + bx)}{2b} - \frac{3ix^2}{2b^2} - \frac{x^3}{2b} + \frac{ix^4}{4}$$

[In] Int[x^3*Cot[a + b*x]^3,x]

[Out] (((-3*I)/2)*x^2)/b^2 - x^3/(2*b) + (I/4)*x^4 - (3*x^2*Cot[a + b*x])/(2*b^2) - (x^3*Cot[a + b*x]^2)/(2*b) + (3*x*Log[1 - E^((2*I)*(a + b*x))])/b^3 - (x^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 + (((3*I)/2)*x^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*x*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*PolyLog[4, E^((2*I)*(a + b*x))])/b^4

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2221

Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2317

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2320

Int[u, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2438

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 2611

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,

f, g, n}, x] && GtQ[m, 0]

Rule 3798

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 3801

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Dist[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 6724

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 6744

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{x^3 \cot^2(a + bx)}{2b} + \frac{3 \int x^2 \cot^2(a + bx) dx}{2b} - \int x^3 \cot(a + bx) dx \\
 &= \frac{ix^4}{4} - \frac{3x^2 \cot(a + bx)}{2b^2} - \frac{x^3 \cot^2(a + bx)}{2b} \\
 &\quad + 2i \int \frac{e^{2i(a+bx)} x^3}{1 - e^{2i(a+bx)}} dx + \frac{3 \int x \cot(a + bx) dx}{b^2} - \frac{3 \int x^2 dx}{2b} \\
 &= -\frac{3ix^2}{2b^2} - \frac{x^3}{2b} + \frac{ix^4}{4} - \frac{3x^2 \cot(a + bx)}{2b^2} - \frac{x^3 \cot^2(a + bx)}{2b} \\
 &\quad - \frac{x^3 \log(1 - e^{2i(a+bx)})}{b} - \frac{(6i) \int \frac{e^{2i(a+bx)} x}{1 - e^{2i(a+bx)}} dx}{b^2} + \frac{3 \int x^2 \log(1 - e^{2i(a+bx)}) dx}{b}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{3ix^2}{2b^2} - \frac{x^3}{2b} + \frac{ix^4}{4} - \frac{3x^2 \cot(a+bx)}{2b^2} - \frac{x^3 \cot^2(a+bx)}{2b} \\
&+ \frac{3x \log(1-e^{2i(a+bx)})}{b^3} - \frac{x^3 \log(1-e^{2i(a+bx)})}{b} + \frac{3ix^2 \text{PolyLog}(2, e^{2i(a+bx)})}{2b^2} \\
&- \frac{3 \int \log(1-e^{2i(a+bx)}) dx}{b^3} - \frac{(3i) \int x \text{PolyLog}(2, e^{2i(a+bx)}) dx}{b^2} \\
&= -\frac{3ix^2}{2b^2} - \frac{x^3}{2b} + \frac{ix^4}{4} - \frac{3x^2 \cot(a+bx)}{2b^2} - \frac{x^3 \cot^2(a+bx)}{2b} + \frac{3x \log(1-e^{2i(a+bx)})}{b^3} \\
&- \frac{x^3 \log(1-e^{2i(a+bx)})}{b} + \frac{3ix^2 \text{PolyLog}(2, e^{2i(a+bx)})}{2b^2} - \frac{3x \text{PolyLog}(3, e^{2i(a+bx)})}{2b^3} \\
&+ \frac{(3i) \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2i(a+bx)}\right)}{2b^4} + \frac{3 \int \text{PolyLog}(3, e^{2i(a+bx)}) dx}{2b^3} \\
&= -\frac{3ix^2}{2b^2} - \frac{x^3}{2b} + \frac{ix^4}{4} - \frac{3x^2 \cot(a+bx)}{2b^2} - \frac{x^3 \cot^2(a+bx)}{2b} + \frac{3x \log(1-e^{2i(a+bx)})}{b^3} \\
&- \frac{x^3 \log(1-e^{2i(a+bx)})}{b} - \frac{3i \text{PolyLog}(2, e^{2i(a+bx)})}{2b^4} + \frac{3ix^2 \text{PolyLog}(2, e^{2i(a+bx)})}{2b^2} \\
&- \frac{3x \text{PolyLog}(3, e^{2i(a+bx)})}{2b^3} - \frac{(3i) \text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x} dx, x, e^{2i(a+bx)}\right)}{4b^4} \\
&= -\frac{3ix^2}{2b^2} - \frac{x^3}{2b} + \frac{ix^4}{4} - \frac{3x^2 \cot(a+bx)}{2b^2} - \frac{x^3 \cot^2(a+bx)}{2b} \\
&+ \frac{3x \log(1-e^{2i(a+bx)})}{b^3} - \frac{x^3 \log(1-e^{2i(a+bx)})}{b} - \frac{3i \text{PolyLog}(2, e^{2i(a+bx)})}{2b^4} \\
&+ \frac{3ix^2 \text{PolyLog}(2, e^{2i(a+bx)})}{2b^2} - \frac{3x \text{PolyLog}(3, e^{2i(a+bx)})}{2b^3} - \frac{3i \text{PolyLog}(4, e^{2i(a+bx)})}{4b^4}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 491 vs. $2(202) = 404$.

Time = 6.81 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.43

$$\begin{aligned}
\int x^3 \cot^3(a+bx) dx &= -\frac{1}{4}x^4 \cot(a) - \frac{x^3 \csc^2(a+bx)}{2b} \\
&+ \frac{e^{ia} \csc(a) (b^4 e^{-2ia} x^4 + 2ib^3 (1-e^{-2ia}) x^3 \log(1-e^{-i(a+bx)}) + 2ib^3 (1-e^{-2ia}) x^3 \log(1+e^{-i(a+bx)}) - 6b^2}{b^3} \\
&+ \frac{3x^2 \csc(a) \csc(a+bx) \sin(bx)}{2b^2} \\
&- \frac{3 \csc(a) \sec(a) \left(b^2 e^{i \arctan(\tan(a))} x^2 + \frac{(ibx(-\pi+2 \arctan(\tan(a))) - \pi \log(1+e^{-2ibx}) - 2(bx + \arctan(\tan(a))) \log(1-e^{2i(bx + \arctan(\tan(a))))}{2b^4} \sqrt{\sec^2(a) (\cos^2(a) + \sin^2(a))} \right)}{2b^4}
\end{aligned}$$

[In] Integrate[x^3*Cot[a + b*x]^3,x]

```
[Out] -1/4*(x^4*Cot[a]) - (x^3*Csc[a + b*x]^2)/(2*b) + (E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*a))*x*PolyLog[3, -E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*a))*x*PolyLog[3, E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, -E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, E^((-I)*(a + b*x))])/(4*b^4) + (3*x^2*Csc[a]*Csc[a + b*x]*Sin[b*x])/(2*b^2) - (3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x]) - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]])]) + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]])])]*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(167) = 334$.

Time = 0.22 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.20

method	result
risch	$\frac{3i \operatorname{polylog}(2, -e^{i(bx+a)})x^2}{b^2} + \frac{3i \operatorname{polylog}(2, e^{i(bx+a)})x^2}{b^2} + \frac{2ia^3x}{b^3} - \frac{\ln(1+e^{i(bx+a)})x^3}{b} - \frac{\ln(1-e^{i(bx+a)})x^3}{b} - \frac{a^3 \ln(1-e^{i(bx+a)})}{b^4}$

```
[In] int(x^3*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*x^4-1/b*ln(1+exp(I*(b*x+a)))*x^3-1/b*ln(1-exp(I*(b*x+a)))*x^3-1/b^4*a^3*ln(1-exp(I*(b*x+a)))+x^2*(2*x*exp(2*I*(b*x+a))*b-3*I*exp(2*I*(b*x+a))+3*I)/b^2/(exp(2*I*(b*x+a))-1)^2-2/b^4*a^3*ln(exp(I*(b*x+a)))+1/b^4*a^3*ln(exp(I*(b*x+a))-1)-6/b^3*polylog(3,-exp(I*(b*x+a)))*x-6/b^3*polylog(3,exp(I*(b*x+a)))*x+3*I/b^2*polylog(2,-exp(I*(b*x+a)))*x^2+3*I/b^2*polylog(2,exp(I*(b*x+a)))*x^2+2*I/b^3*a^3*x-6*I/b^3*a*x+6/b^4*a*ln(exp(I*(b*x+a)))-3/b^4*a*ln(exp(I*(b*x+a))-1)+3/b^4*ln(1-exp(I*(b*x+a)))*a+3/b^3*ln(1+exp(I*(b*x+a)))*x+3/b^3*ln(1-exp(I*(b*x+a)))*x-3*I/b^4*polylog(2,exp(I*(b*x+a)))+3/2*I/b^4*a^4-6*I/b^4*polylog(4,-exp(I*(b*x+a)))-6*I/b^4*polylog(4,exp(I*(b*x+a)))-3*I/b^2*x^2-3*I/b^4*a^2-3*I/b^4*polylog(2,-exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(160) = 320$.

Time = 0.30 (sec) , antiderivative size = 569, normalized size of antiderivative = 2.82

$$\int x^3 \cot^3(a + bx) dx$$

$$= \frac{8b^3x^3 + 12b^2x^2 \sin(2bx + 2a) - 6(ib^2x^2 + (-ib^2x^2 + i) \cos(2bx + 2a) - i) \text{Li}_2(\cos(2bx + 2a) + i \sin(2bx + 2a))}{b^4 \cos(2bx + 2a) - b^4}$$

[In] integrate(x^3*cot(b*x+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{8}*(8*b^3*x^3 + 12*b^2*x^2*\sin(2*b*x + 2*a) - 6*(I*b^2*x^2 + (-I*b^2*x^2 + I)*\cos(2*b*x + 2*a) - I)*\text{dilog}(\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a)) - 6*(-I*b^2*x^2 + (I*b^2*x^2 - I)*\cos(2*b*x + 2*a) + I)*\text{dilog}(\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a)) - 4*(a^3 - (a^3 - 3*a)*\cos(2*b*x + 2*a) - 3*a)*\log(-1/2*\cos(2*b*x + 2*a) + 1/2*I*\sin(2*b*x + 2*a) + 1/2) - 4*(a^3 - (a^3 - 3*a)*\cos(2*b*x + 2*a) - 3*a)*\log(-1/2*\cos(2*b*x + 2*a) - 1/2*I*\sin(2*b*x + 2*a) + 1/2) + 4*(b^3*x^3 + a^3 - 3*b*x - (b^3*x^3 + a^3 - 3*b*x - 3*a)*\cos(2*b*x + 2*a) - 3*a)*\log(-\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a) + 1) + 4*(b^3*x^3 + a^3 - 3*b*x - (b^3*x^3 + a^3 - 3*b*x - 3*a)*\cos(2*b*x + 2*a) - 3*a)*\log(-\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a) + 1) - 3*(I*\cos(2*b*x + 2*a) - I)*\text{polylog}(4, \cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a)) - 3*(-I*\cos(2*b*x + 2*a) + I)*\text{polylog}(4, \cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a)) - 6*(b*x*\cos(2*b*x + 2*a) - b*x)*\text{polylog}(3, \cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a)) - 6*(b*x*\cos(2*b*x + 2*a) - b*x)*\text{polylog}(3, \cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a)))/(b^4*\cos(2*b*x + 2*a) - b^4)$

Sympy [F]

$$\int x^3 \cot^3(a + bx) dx = \int x^3 \cot^3(a + bx) dx$$

[In] integrate(x**3*cot(b*x+a)**3,x)

[Out] Integral(x**3*cot(a + b*x)**3, x)


```

3 - 3*I*(b*x + a)^2*a + 3*(I*a^2 - I)*(b*x + a) + 3*I*a)*cos(4*b*x + 4*a) +
2*(-I*(b*x + a)^3 + 3*I*(b*x + a)^2*a + 3*(-I*a^2 + I)*(b*x + a) - 3*I*a)*
cos(2*b*x + 2*a) - ((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^2 - 1)*(b*x + a) +
3*a)*sin(4*b*x + 4*a) + 2*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(a^2 - 1)*(b*
x + a) + 3*a)*sin(2*b*x + 2*a) + 3*I*a)*log(cos(b*x + a)^2 + sin(b*x + a)^2
- 2*cos(b*x + a) + 1) - 24*(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + I*sin(
4*b*x + 4*a) - 2*I*sin(2*b*x + 2*a) + 1)*polylog(4, -e^(I*b*x + I*a)) - 24*
(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + I*sin(4*b*x + 4*a) - 2*I*sin(2*b*x
+ 2*a) + 1)*polylog(4, e^(I*b*x + I*a)) + 24*(I*b*x*cos(4*b*x + 4*a) - 2*I
*b*x*cos(2*b*x + 2*a) - b*x*sin(4*b*x + 4*a) + 2*b*x*sin(2*b*x + 2*a) + I*b
*x)*polylog(3, -e^(I*b*x + I*a)) + 24*(I*b*x*cos(4*b*x + 4*a) - 2*I*b*x*cos
(2*b*x + 2*a) - b*x*sin(4*b*x + 4*a) + 2*b*x*sin(2*b*x + 2*a) + I*b*x)*poly
log(3, e^(I*b*x + I*a)) - (-I*(b*x + a)^4 + 4*I*(b*x + a)^3*a - 6*(I*a^2 -
2*I)*(b*x + a)^2 - 24*I*(b*x + a)*a)*sin(4*b*x + 4*a) + 2*(-I*(b*x + a)^4 +
4*(b*x + a)^3*(I*a + 1) + 6*(-I*a^2 - 2*a + I)*(b*x + a)^2 + 12*(a^2 - I*a
)*(b*x + a) - 6*I*a^2)*sin(2*b*x + 2*a))/(-4*I*cos(4*b*x + 4*a) + 8*I*cos(2
*b*x + 2*a) + 4*sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a) - 4*I))/b^4

```

Giac [F]

$$\int x^3 \cot^3(a + bx) dx = \int x^3 \cot(bx + a)^3 dx$$

```
[In] integrate(x^3*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cot(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cot^3(a + bx) dx = \int x^3 \cot(a + bx)^3 dx$$

```
[In] int(x^3*cot(a + b*x)^3,x)
```

```
[Out] int(x^3*cot(a + b*x)^3, x)
```

3.12 $\int x^2 \cot^3(a + bx) dx$

Optimal result	95
Rubi [A] (verified)	95
Mathematica [A] (verified)	97
Maple [B] (verified)	98
Fricas [B] (verification not implemented)	98
Sympy [F]	99
Maxima [B] (verification not implemented)	99
Giac [F]	100
Mupad [F(-1)]	100

Optimal result

Integrand size = 12, antiderivative size = 126

$$\int x^2 \cot^3(a + bx) dx = -\frac{x^2}{2b} + \frac{ix^3}{3} - \frac{x \cot(a + bx)}{b^2} - \frac{x^2 \cot^2(a + bx)}{2b} - \frac{x^2 \log(1 - e^{2i(a+bx)})}{b} + \frac{\log(\sin(a + bx))}{b^3} + \frac{ix \operatorname{PolyLog}(2, e^{2i(a+bx)})}{b^2} - \frac{\operatorname{PolyLog}(3, e^{2i(a+bx)})}{2b^3}$$

[Out] $-1/2*x^2/b+1/3*I*x^3-x*\cot(b*x+a)/b^2-1/2*x^2*\cot(b*x+a)^2/b-x^2*\ln(1-\exp(2*I*(b*x+a)))/b+\ln(\sin(b*x+a))/b^3+I*x*\operatorname{polylog}(2,\exp(2*I*(b*x+a)))/b^2-1/2*\operatorname{polylog}(3,\exp(2*I*(b*x+a)))/b^3$

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3801, 3556, 30, 3798, 2221, 2611, 2320, 6724}

$$\int x^2 \cot^3(a + bx) dx = -\frac{\operatorname{PolyLog}(3, e^{2i(a+bx)})}{2b^3} + \frac{\log(\sin(a + bx))}{b^3} + \frac{ix \operatorname{PolyLog}(2, e^{2i(a+bx)})}{b^2} - \frac{x \cot(a + bx)}{b^2} - \frac{x^2 \log(1 - e^{2i(a+bx)})}{b} - \frac{x^2 \cot^2(a + bx)}{2b} - \frac{x^2}{2b} + \frac{ix^3}{3}$$

[In] $\operatorname{Int}[x^2*\cot[a + b*x]^3,x]$

[Out] $-1/2*x^2/b + (I/3)*x^3 - (x*\cot[a + b*x])/b^2 - (x^2*\cot[a + b*x]^2)/(2*b) - (x^2*\log[1 - E^((2*I)*(a + b*x))])/b + \log[\sin[a + b*x]]/b^3 + (I*x*\operatorname{PolyLog}[2, E^((2*I)*(a + b*x))])/b^2 - \operatorname{PolyLog}[3, E^((2*I)*(a + b*x))]/(2*b^3)$

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2221

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2611

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 3556

Int[tan[(c_) + (d_)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3798

Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 3801

Int[((c_) + (d_)*(x_))^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Dist[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 6724

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{x^2 \cot^2(a + bx)}{2b} + \frac{\int x \cot^2(a + bx) dx}{b} - \int x^2 \cot(a + bx) dx \\
 &= \frac{ix^3}{3} - \frac{x \cot(a + bx)}{b^2} - \frac{x^2 \cot^2(a + bx)}{2b} + 2i \int \frac{e^{2i(a+bx)} x^2}{1 - e^{2i(a+bx)}} dx + \frac{\int \cot(a + bx) dx}{b^2} - \frac{\int x dx}{b} \\
 &= -\frac{x^2}{2b} + \frac{ix^3}{3} - \frac{x \cot(a + bx)}{b^2} - \frac{x^2 \cot^2(a + bx)}{2b} - \frac{x^2 \log(1 - e^{2i(a+bx)})}{b} \\
 &\quad + \frac{\log(\sin(a + bx))}{b^3} + \frac{2 \int x \log(1 - e^{2i(a+bx)}) dx}{b} \\
 &= -\frac{x^2}{2b} + \frac{ix^3}{3} - \frac{x \cot(a + bx)}{b^2} - \frac{x^2 \cot^2(a + bx)}{2b} - \frac{x^2 \log(1 - e^{2i(a+bx)})}{b} \\
 &\quad + \frac{\log(\sin(a + bx))}{b^3} + \frac{ix \text{PolyLog}(2, e^{2i(a+bx)})}{b^2} - \frac{i \int \text{PolyLog}(2, e^{2i(a+bx)}) dx}{b^2} \\
 &= -\frac{x^2}{2b} + \frac{ix^3}{3} - \frac{x \cot(a + bx)}{b^2} - \frac{x^2 \cot^2(a + bx)}{2b} - \frac{x^2 \log(1 - e^{2i(a+bx)})}{b} \\
 &\quad + \frac{\log(\sin(a + bx))}{b^3} + \frac{ix \text{PolyLog}(2, e^{2i(a+bx)})}{b^2} - \frac{\text{Subst}\left(\int \frac{\text{PolyLog}(2, x)}{x} dx, x, e^{2i(a+bx)}\right)}{2b^3} \\
 &= -\frac{x^2}{2b} + \frac{ix^3}{3} - \frac{x \cot(a + bx)}{b^2} - \frac{x^2 \cot^2(a + bx)}{2b} - \frac{x^2 \log(1 - e^{2i(a+bx)})}{b} \\
 &\quad + \frac{\log(\sin(a + bx))}{b^3} + \frac{ix \text{PolyLog}(2, e^{2i(a+bx)})}{b^2} - \frac{\text{PolyLog}(3, e^{2i(a+bx)})}{2b^3}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 5.70 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.75

$$\int x^2 \cot^3(a + bx) dx = \frac{6bx \cot(a) + 2b^3 x^3 \cot(a) + 3b^2 x^2 \csc^2(a + bx) - 6 \log(\sin(a + bx)) + 2e^{-ia}(i + \cot(a)) (ib^3 x^3 - b^3 x^3 \cot(a))}{b^3}$$

[In] Integrate[x^2*Cot[a + b*x]^3, x]

[Out] -1/6*(6*b*x*Cot[a] + 2*b^3*x^3*Cot[a] + 3*b^2*x^2*Csc[a + b*x]^2 - 6*Log[Sin[a + b*x]]) + (2*(I + Cot[a])*(I*b^3*x^3 - b^3*x^3*Cot[a] + 3*b^2*x^2*Log[1

$$- E^{((-I)*(a + b*x))} + 3*b^2*x^2*\text{Log}[1 + E^{((-I)*(a + b*x))}] + (6*I)*b*x* \text{PolyLog}[2, -E^{((-I)*(a + b*x))}] + (6*I)*b*x*\text{PolyLog}[2, E^{((-I)*(a + b*x))}] + 6*\text{PolyLog}[3, -E^{((-I)*(a + b*x))}] + 6*\text{PolyLog}[3, E^{((-I)*(a + b*x))}]*\text{Sin}[a])/E^{(I*a)} - 6*b*x*\text{Csc}[a]*\text{Csc}[a + b*x]*\text{Sin}[b*x])/b^3$$

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(110) = 220$.

Time = 0.20 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.33

method	result
risch	$-\frac{4ia^3}{3b^3} + \frac{2x(xe^{2i(bx+a)}b - ie^{2i(bx+a)} + i)}{b^2(e^{2i(bx+a)} - 1)^2} + \frac{a^2 \ln(1 - e^{i(bx+a)})}{b^3} - \frac{2ia^2x}{b^2} + \frac{2a^2 \ln(e^{i(bx+a)})}{b^3} - \frac{a^2 \ln(e^{i(bx+a)} - 1)}{b^3} + \frac{ix^3}{3} - \dots$

[In] int(x^2*cot(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] $-4/3*I/b^3*a^3+2*x*(x*\exp(2*I*(b*x+a))*b-I*\exp(2*I*(b*x+a))+I)/b^2/(\exp(2*I*(b*x+a))-1)^2+1/b^3*a^2*\ln(1-\exp(I*(b*x+a)))-2*I/b^2*a^2*x+2/b^3*a^2*\ln(\exp(I*(b*x+a)))-1/b^3*a^2*\ln(\exp(I*(b*x+a))-1)+1/3*I*x^3-1/b*\ln(1+\exp(I*(b*x+a)))*x^2+2*I/b^2*\text{polylog}(2,\exp(I*(b*x+a)))*x-1/b*\ln(1-\exp(I*(b*x+a)))*x^2+2*I/b^2*\text{polylog}(2,-\exp(I*(b*x+a)))*x-2/b^3*\text{polylog}(3,-\exp(I*(b*x+a)))-2/b^3*\text{polylog}(3,\exp(I*(b*x+a)))+1/b^3*\ln(1+\exp(I*(b*x+a)))-2/b^3*\ln(\exp(I*(b*x+a)))+1/b^3*\ln(\exp(I*(b*x+a))-1)$

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(107) = 214$.

Time = 0.30 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.37

$$\int x^2 \cot^3(a + bx) dx = \frac{4b^2x^2 + 4bx \sin(2bx + 2a) - 2(-ibx \cos(2bx + 2a) + ibx) \text{Li}_2(\cos(2bx + 2a) + i \sin(2bx + 2a)) - 2(\dots)}{\dots}$$

[In] integrate(x^2*cot(b*x+a)^3,x, algorithm="fricas")

[Out] $1/4*(4*b^2*x^2 + 4*b*x*\sin(2*b*x + 2*a) - 2*(-I*b*x*\cos(2*b*x + 2*a) + I*b*x)*\text{dilog}(\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a)) - 2*(I*b*x*\cos(2*b*x + 2*a) - I*b*x)*\text{dilog}(\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a)) + 2*(a^2 - (a^2 - 1)*\cos(2*b*x + 2*a) - 1)*\log(-1/2*\cos(2*b*x + 2*a) + 1/2*I*\sin(2*b*x + 2*a) + 1/2) + 2*(a^2 - (a^2 - 1)*\cos(2*b*x + 2*a) - 1)*\log(-1/2*\cos(2*b*x + 2*a) - 1/2*I*\sin(2*b*x + 2*a) + 1/2) + 2*(b^2*x^2 - a^2 - (b^2*x^2 - a^2)*\cos(2*$

```
b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 2*(b^2*x^2 -
a^2 - (b^2*x^2 - a^2)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x
+ 2*a) + 1) - (cos(2*b*x + 2*a) - 1)*polylog(3, cos(2*b*x + 2*a) + I*sin(2
*b*x + 2*a)) - (cos(2*b*x + 2*a) - 1)*polylog(3, cos(2*b*x + 2*a) - I*sin(2
*b*x + 2*a)))/(b^3*cos(2*b*x + 2*a) - b^3)
```

Sympy [F]

$$\int x^2 \cot^3(a + bx) dx = \int x^2 \cot^3(a + bx) dx$$

```
[In] integrate(x**2*cot(b*x+a)**3,x)
```

```
[Out] Integral(x**2*cot(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1208 vs. $2(107) = 214$.

Time = 0.45 (sec) , antiderivative size = 1208, normalized size of antiderivative = 9.59

$$\int x^2 \cot^3(a + bx) dx = \text{Too large to display}$$

```
[In] integrate(x^2*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 2*(2*(b*x + a)^3 - 6*(
b*x + a)^2*a - 6*((b*x + a)^2 - 2*(b*x + a)*a + ((b*x + a)^2 - 2*(b*x + a)*
a - 1)*cos(4*b*x + 4*a) - 2*((b*x + a)^2 - 2*(b*x + a)*a - 1)*cos(2*b*x + 2
*a) - (-I*(b*x + a)^2 + 2*I*(b*x + a)*a + I)*sin(4*b*x + 4*a) - 2*(I*(b*x +
a)^2 - 2*I*(b*x + a)*a - I)*sin(2*b*x + 2*a) - 1)*arctan2(sin(b*x + a), co
s(b*x + a) + 1) + 6*(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + I*sin(4*b*x +
4*a) - 2*I*sin(2*b*x + 2*a) + 1)*arctan2(sin(b*x + a), cos(b*x + a) - 1) +
6*((b*x + a)^2 - 2*(b*x + a)*a + ((b*x + a)^2 - 2*(b*x + a)*a)*cos(4*b*x +
4*a) - 2*((b*x + a)^2 - 2*(b*x + a)*a)*cos(2*b*x + 2*a) + (I*(b*x + a)^2 -
2*I*(b*x + a)*a)*sin(4*b*x + 4*a) + 2*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*si
n(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 2*((b*x + a)^3 -
3*(b*x + a)^2*a - 6*b*x - 6*a)*cos(4*b*x + 4*a) - 4*((b*x + a)^3 - 3*(b*x
+ a)^2*(a - I) - 3*(b*x + a)*(2*I*a + 1) - 3*a)*cos(2*b*x + 2*a) + 12*(b*x*
cos(4*b*x + 4*a) - 2*b*x*cos(2*b*x + 2*a) + I*b*x*sin(4*b*x + 4*a) - 2*I*b*
x*sin(2*b*x + 2*a) + b*x)*dilog(-e^(I*b*x + I*a)) + 12*(b*x*cos(4*b*x + 4*a
) - 2*b*x*cos(2*b*x + 2*a) + I*b*x*sin(4*b*x + 4*a) - 2*I*b*x*sin(2*b*x + 2
*a) + b*x)*dilog(e^(I*b*x + I*a)) + 3*(I*(b*x + a)^2 - 2*I*(b*x + a)*a + (I
```

```

*(b*x + a)^2 - 2*I*(b*x + a)*a - I)*cos(4*b*x + 4*a) + 2*(-I*(b*x + a)^2 +
2*I*(b*x + a)*a + I)*cos(2*b*x + 2*a) - ((b*x + a)^2 - 2*(b*x + a)*a - 1)*s
in(4*b*x + 4*a) + 2*((b*x + a)^2 - 2*(b*x + a)*a - 1)*sin(2*b*x + 2*a) - I
*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 3*(I*(b*x + a)
^2 - 2*I*(b*x + a)*a + (I*(b*x + a)^2 - 2*I*(b*x + a)*a - I)*cos(4*b*x + 4*
a) + 2*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a + I)*cos(2*b*x + 2*a) - ((b*x + a)
^2 - 2*(b*x + a)*a - 1)*sin(4*b*x + 4*a) + 2*((b*x + a)^2 - 2*(b*x + a)*a -
1)*sin(2*b*x + 2*a) - I)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x +
a) + 1) + 12*(I*cos(4*b*x + 4*a) - 2*I*cos(2*b*x + 2*a) - sin(4*b*x + 4*a)
+ 2*sin(2*b*x + 2*a) + I)*polylog(3, -e^(I*b*x + I*a)) + 12*(I*cos(4*b*x +
4*a) - 2*I*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a) + I)*p
olylog(3, e^(I*b*x + I*a)) + 2*(I*(b*x + a)^3 - 3*I*(b*x + a)^2*a - 6*I*b*x
- 6*I*a)*sin(4*b*x + 4*a) + 4*(-I*(b*x + a)^3 + 3*(b*x + a)^2*(I*a + 1) -
3*(b*x + a)*(2*a - I) + 3*I*a)*sin(2*b*x + 2*a) - 12*a)/(-6*I*cos(4*b*x + 4
*a) + 12*I*cos(2*b*x + 2*a) + 6*sin(4*b*x + 4*a) - 12*sin(2*b*x + 2*a) - 6*
I))/b^3

```

Giac [F]

$$\int x^2 \cot^3(a + bx) dx = \int x^2 \cot (bx + a)^3 dx$$

```
[In] integrate(x^2*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cot(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^3(a + bx) dx = \int x^2 \cot(a + bx)^3 dx$$

```
[In] int(x^2*cot(a + b*x)^3,x)
```

```
[Out] int(x^2*cot(a + b*x)^3, x)
```

3.13 $\int x \cot^3(a + bx) dx$

Optimal result	101
Rubi [A] (verified)	101
Mathematica [A] (verified)	103
Maple [B] (verified)	103
Fricas [B] (verification not implemented)	104
Sympy [F]	104
Maxima [B] (verification not implemented)	104
Giac [F]	105
Mupad [F(-1)]	105

Optimal result

Integrand size = 10, antiderivative size = 91

$$\int x \cot^3(a + bx) dx = -\frac{x}{2b} + \frac{ix^2}{2} - \frac{\cot(a + bx)}{2b^2} - \frac{x \cot^2(a + bx)}{2b} - \frac{x \log(1 - e^{2i(a+bx)})}{b} + \frac{i \operatorname{PolyLog}(2, e^{2i(a+bx)})}{2b^2}$$

[Out] $-1/2*x/b + 1/2*I*x^2 - 1/2*\cot(b*x+a)/b^2 - 1/2*x*\cot(b*x+a)^2/b - x*\ln(1-\exp(2*I*(b*x+a)))/b + 1/2*I*\operatorname{polylog}(2, \exp(2*I*(b*x+a)))/b^2$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3801, 3554, 8, 3798, 2221, 2317, 2438}

$$\int x \cot^3(a + bx) dx = \frac{i \operatorname{PolyLog}(2, e^{2i(a+bx)})}{2b^2} - \frac{\cot(a + bx)}{2b^2} - \frac{x \log(1 - e^{2i(a+bx)})}{b} - \frac{x \cot^2(a + bx)}{2b} - \frac{x}{2b} + \frac{ix^2}{2}$$

[In] `Int[x*Cot[a + b*x]^3,x]`

[Out] $-1/2*x/b + (I/2)*x^2 - \cot[a + b*x]/(2*b^2) - (x*\cot[a + b*x]^2)/(2*b) - (x*\log[1 - E^((2*I)*(a + b*x))])/b + ((I/2)*\operatorname{PolyLog}[2, E^((2*I)*(a + b*x))])/b^2$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2221

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3554

```
Int[((b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3798

```
Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol
] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_) + (d_)*(x_))^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symb
ol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{x \cot^2(a + bx)}{2b} + \frac{\int \cot^2(a + bx) dx}{2b} - \int x \cot(a + bx) dx \\ &= \frac{ix^2}{2} - \frac{\cot(a + bx)}{2b^2} - \frac{x \cot^2(a + bx)}{2b} + 2i \int \frac{e^{2i(a+bx)} x}{1 - e^{2i(a+bx)}} dx - \frac{\int 1 dx}{2b} \end{aligned}$$

$$\begin{aligned}
&= -\frac{x}{2b} + \frac{ix^2}{2} - \frac{\cot(a+bx)}{2b^2} - \frac{x \cot^2(a+bx)}{2b} - \frac{x \log(1 - e^{2i(a+bx)})}{b} + \frac{\int \log(1 - e^{2i(a+bx)}) dx}{b} \\
&= -\frac{x}{2b} + \frac{ix^2}{2} - \frac{\cot(a+bx)}{2b^2} - \frac{x \cot^2(a+bx)}{2b} \\
&\quad - \frac{x \log(1 - e^{2i(a+bx)})}{b} - \frac{i \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2i(a+bx)}\right)}{2b^2} \\
&= -\frac{x}{2b} + \frac{ix^2}{2} - \frac{\cot(a+bx)}{2b^2} - \frac{x \cot^2(a+bx)}{2b} - \frac{x \log(1 - e^{2i(a+bx)})}{b} + \frac{i \text{PolyLog}(2, e^{2i(a+bx)})}{2b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.62 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.97

$$\int x \cot^3(a+bx) dx = \frac{-ib\pi x - b^2 x^2 \cot(a) - bx \csc^2(a+bx) - \pi \log(1 + e^{-2ibx}) - 2bx \log(1 - e^{2i(bx + \arctan(\tan(a)))}) + \pi \log(\cos)}{2}$$

[In] Integrate[x*Cot[a + b*x]^3,x]

[Out] ((-I)*b*Pi*x - b^2*x^2*Cot[a] - b*x*Csc[a + b*x]^2 - Pi*Log[1 + E^((-2*I)*b*x)] - 2*b*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*(I*b*x - Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))]]) + Log[Sin[b*x + ArcTan[Tan[a]]]]) + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))] + b^2*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2 + Csc[a]*Csc[a + b*x]*Sin[b*x]]/(2*b^2)

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(75) = 150.

Time = 0.18 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.16

method	result
risch	$\frac{ix^2}{2} + \frac{2x e^{2i(bx+a)} b - i e^{2i(bx+a)} + i}{b^2 (e^{2i(bx+a)} - 1)^2} + \frac{2iax}{b} + \frac{ia^2}{b^2} - \frac{\ln(1 + e^{i(bx+a)})x}{b} + \frac{i \text{polylog}(2, -e^{i(bx+a)})}{b^2} - \frac{\ln(1 - e^{i(bx+a)})x}{b} - \frac{\ln(1 - e^{i(bx+a)})}{b}$

[In] int(x*cot(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/2*I*x^2+(2*x*exp(2*I*(b*x+a))*b-I*exp(2*I*(b*x+a))+I)/b^2/(exp(2*I*(b*x+a))-1)^2+2*I/b*a*x+I/b^2*a^2-1/b*ln(1+exp(I*(b*x+a)))*x+I/b^2*polylog(2,-exp(I*(b*x+a)))-1/b*ln(1-exp(I*(b*x+a)))*x-1/b^2*ln(1-exp(I*(b*x+a)))*a+I/b^2*

$\text{polylog}(2, \exp(I*(b*x+a))) - 2/b^2*a*\ln(\exp(I*(b*x+a))) + 1/b^2*a*\ln(\exp(I*(b*x+a))) - 1)$

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(72) = 144$.

Time = 0.27 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.20

$$\int x \cot^3(a + bx) dx = \frac{4bx + (i \cos(2bx + 2a) - i)\text{Li}_2(\cos(2bx + 2a) + i \sin(2bx + 2a)) + (-i \cos(2bx + 2a) + i)\text{Li}_2(\cos(2bx + 2a) - i \sin(2bx + 2a))}{b^2 \cos(2bx + 2a) - b^2}$$

[In] integrate(x*cot(b*x+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{4}*(4*b*x + (I*\cos(2*b*x + 2*a) - I)*\text{dilog}(\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a)) + (-I*\cos(2*b*x + 2*a) + I)*\text{dilog}(\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a)) + 2*(a*\cos(2*b*x + 2*a) - a)*\log(-1/2*\cos(2*b*x + 2*a) + 1/2*I*\sin(2*b*x + 2*a) + 1/2) + 2*(a*\cos(2*b*x + 2*a) - a)*\log(-1/2*\cos(2*b*x + 2*a) - 1/2*I*\sin(2*b*x + 2*a) + 1/2) + 2*(b*x - (b*x + a)*\cos(2*b*x + 2*a) + a)*\log(-\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a) + 1) + 2*(b*x - (b*x + a)*\cos(2*b*x + 2*a) + a)*\log(-\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a) + 1) + 2*\sin(2*b*x + 2*a))/(b^2*\cos(2*b*x + 2*a) - b^2)$

Sympy [F]

$$\int x \cot^3(a + bx) dx = \int x \cot^3(a + bx) dx$$

[In] integrate(x*cot(b*x+a)**3,x)

[Out] Integral(x*cot(a + b*x)**3, x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(72) = 144$.

Time = 0.39 (sec) , antiderivative size = 586, normalized size of antiderivative = 6.44

$$\int x \cot^3(a + bx) dx = \frac{b^2 x^2 \cos(4bx + 4a) + i b^2 x^2 \sin(4bx + 4a) + b^2 x^2 - 2(bx \cos(4bx + 4a) - 2bx \cos(2bx + 2a) + i bx \sin(4bx + 4a))}{b^2 \cos(2bx + 2a) - b^2}$$

[In] integrate(x*cot(b*x+a)^3,x, algorithm="maxima")

[Out] (b^2*x^2*cos(4*b*x + 4*a) + I*b^2*x^2*sin(4*b*x + 4*a) + b^2*x^2 - 2*(b*x*cos(4*b*x + 4*a) - 2*b*x*cos(2*b*x + 2*a) + I*b*x*sin(4*b*x + 4*a) - 2*I*b*x*sin(2*b*x + 2*a) + b*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(b*x*cos(4*b*x + 4*a) - 2*b*x*cos(2*b*x + 2*a) + I*b*x*sin(4*b*x + 4*a) - 2*I*b*x*sin(2*b*x + 2*a) + b*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(b^2*x^2 + 2*I*b*x + 1)*cos(2*b*x + 2*a) + 2*(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + I*sin(4*b*x + 4*a) - 2*I*sin(2*b*x + 2*a) + 1)*dilog(-e^(I*b*x + I*a)) + 2*(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + I*sin(4*b*x + 4*a) - 2*I*sin(2*b*x + 2*a) + 1)*dilog(e^(I*b*x + I*a)) - (-I*b*x*cos(4*b*x + 4*a) + 2*I*b*x*cos(2*b*x + 2*a) + b*x*sin(4*b*x + 4*a) - 2*b*x*sin(2*b*x + 2*a) - I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-I*b*x*cos(4*b*x + 4*a) + 2*I*b*x*cos(2*b*x + 2*a) + b*x*sin(4*b*x + 4*a) - 2*b*x*sin(2*b*x + 2*a) - I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 2*(-I*b^2*x^2 + 2*b*x - I)*sin(2*b*x + 2*a) + 2)/(-2*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2)

Giac [F]

$$\int x \cot^3(a + bx) dx = \int x \cot(bx + a)^3 dx$$

[In] integrate(x*cot(b*x+a)^3,x, algorithm="giac")

[Out] integrate(x*cot(b*x + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int x \cot^3(a + bx) dx = \int x \cot(a + bx)^3 dx$$

[In] int(x*cot(a + b*x)^3,x)

[Out] int(x*cot(a + b*x)^3, x)

3.14 $\int \frac{\cot^3(a+bx)}{x} dx$

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Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\cot^3(a+bx)}{x} dx = \text{Int}\left(\frac{\cot^3(a+bx)}{x}, x\right)$$

[Out] Unintegrable(cot(b*x+a)^3/x,x)

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^3(a+bx)}{x} dx = \int \frac{\cot^3(a+bx)}{x} dx$$

[In] Int[Cot[a + b*x]^3/x,x]

[Out] Defer[Int][Cot[a + b*x]^3/x, x]

Rubi steps

$$\text{integral} = \int \frac{\cot^3(a+bx)}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 9.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^3(a + bx)}{x} dx = \int \frac{\cot^3(a + bx)}{x} dx$$

[In] Integrate[Cot[a + b*x]^3/x,x]

[Out] Integrate[Cot[a + b*x]^3/x, x]

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cot^3(bx + a)}{x} dx$$

[In] int(cot(b*x+a)^3/x,x)

[Out] int(cot(b*x+a)^3/x,x)

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^3(a + bx)}{x} dx = \int \frac{\cot^3(bx + a)}{x} dx$$

[In] integrate(cot(b*x+a)^3/x,x, algorithm="fricas")

[Out] integral(cot(b*x + a)^3/x, x)

Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\cot^3(a + bx)}{x} dx = \int \frac{\cot^3(a + bx)}{x} dx$$

[In] integrate(cot(b*x+a)**3/x,x)

[Out] Integral(cot(a + b*x)**3/x, x)

Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 763, normalized size of antiderivative = 63.58

$$\int \frac{\cot^3(a + bx)}{x} dx = \int \frac{\cot (bx + a)^3}{x} dx$$

[In] integrate(cot(b*x+a)^3/x,x, algorithm="maxima")

```
[Out] -(4*b*x*cos(2*b*x + 2*a)^2 + 4*b*x*sin(2*b*x + 2*a)^2 - 2*b*x*cos(2*b*x + 2*a) - (2*b*x*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (b^2*x^2*cos(4*b*x + 4*a)^2 + 4*b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(4*b*x + 4*a)^2 - 4*b^2*x^2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^2*sin(2*b*x + 2*a)^2 - 4*b^2*x^2*cos(2*b*x + 2*a) + b^2*x^2 - 2*(2*b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*cos(4*b*x + 4*a))*integrate((b^2*x^2 - 1)*sin(b*x + a)/(b^2*x^3*cos(b*x + a)^2 + b^2*x^3*sin(b*x + a)^2 + 2*b^2*x^3*cos(b*x + a) + b^2*x^3), x) + (b^2*x^2*cos(4*b*x + 4*a)^2 + 4*b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(4*b*x + 4*a)^2 - 4*b^2*x^2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^2*sin(2*b*x + 2*a)^2 - 4*b^2*x^2*cos(2*b*x + 2*a) + b^2*x^2 - 2*(2*b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*cos(4*b*x + 4*a))*integrate((b^2*x^2 - 1)*sin(b*x + a)/(b^2*x^3*cos(b*x + a)^2 + b^2*x^3*sin(b*x + a)^2 - 2*b^2*x^3*cos(b*x + a) + b^2*x^3), x) - (2*b*x*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))/(b^2*x^2*cos(4*b*x + 4*a)^2 + 4*b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(4*b*x + 4*a)^2 - 4*b^2*x^2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^2*sin(2*b*x + 2*a)^2 - 4*b^2*x^2*cos(2*b*x + 2*a) + b^2*x^2 - 2*(2*b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*cos(4*b*x + 4*a))
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^3(a + bx)}{x} dx = \int \frac{\cot (bx + a)^3}{x} dx$$

[In] integrate(cot(b*x+a)^3/x,x, algorithm="giac")

[Out] integrate(cot(b*x + a)^3/x, x)

Mupad [N/A]

Not integrable

Time = 12.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^3(a + bx)}{x} dx = \int \frac{\cot(a + bx)^3}{x} dx$$

```
[In] int(cot(a + b*x)^3/x,x)
```

```
[Out] int(cot(a + b*x)^3/x, x)
```

3.15 $\int \frac{\cot^3(a+bx)}{x^2} dx$

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Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\cot^3(a+bx)}{x^2} dx = \text{Int}\left(\frac{\cot^3(a+bx)}{x^2}, x\right)$$

[Out] Unintegrable(cot(b*x+a)^3/x^2,x)

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^3(a+bx)}{x^2} dx = \int \frac{\cot^3(a+bx)}{x^2} dx$$

[In] Int[Cot[a + b*x]^3/x^2,x]

[Out] Defer[Int][Cot[a + b*x]^3/x^2, x]

Rubi steps

$$\text{integral} = \int \frac{\cot^3(a+bx)}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 6.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^3(a + bx)}{x^2} dx = \int \frac{\cot^3(a + bx)}{x^2} dx$$

[In] Integrate[Cot[a + b*x]^3/x^2,x]

[Out] Integrate[Cot[a + b*x]^3/x^2, x]

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cot^3(bx + a)}{x^2} dx$$

[In] int(cot(b*x+a)^3/x^2,x)

[Out] int(cot(b*x+a)^3/x^2,x)

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^3(a + bx)}{x^2} dx = \int \frac{\cot^3(bx + a)}{x^2} dx$$

[In] integrate(cot(b*x+a)^3/x^2,x, algorithm="fricas")

[Out] integral(cot(b*x + a)^3/x^2, x)

Sympy [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cot^3(a + bx)}{x^2} dx = \int \frac{\cot^3(a + bx)}{x^2} dx$$

[In] integrate(cot(b*x+a)**3/x**2,x)

[Out] Integral(cot(a + b*x)**3/x**2, x)

Maxima [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 761, normalized size of antiderivative = 63.42

$$\int \frac{\cot^3(a + bx)}{x^2} dx = \int \frac{\cot (bx + a)^3}{x^2} dx$$

```
[In] integrate(cot(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -(4*b*x*cos(2*b*x + 2*a)^2 + 4*b*x*sin(2*b*x + 2*a)^2 - 2*b*x*cos(2*b*x + 2*a) - 2*(b*x*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (b^2*x^3*cos(4*b*x + 4*a)^2 + 4*b^2*x^3*cos(2*b*x + 2*a)^2 + b^2*x^3*sin(4*b*x + 4*a)^2 - 4*b^2*x^3*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^3*sin(2*b*x + 2*a)^2 - 4*b^2*x^3*cos(2*b*x + 2*a) + b^2*x^3 - 2*(2*b^2*x^3*cos(2*b*x + 2*a) - b^2*x^3)*cos(4*b*x + 4*a))*integrate((b^2*x^2 - 3)*sin(b*x + a)/(b^2*x^4*cos(b*x + a)^2 + b^2*x^4*sin(b*x + a)^2 + 2*b^2*x^4*cos(b*x + a) + b^2*x^4), x) + (b^2*x^3*cos(4*b*x + 4*a)^2 + 4*b^2*x^3*cos(2*b*x + 2*a)^2 + b^2*x^3*sin(4*b*x + 4*a)^2 - 4*b^2*x^3*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^3*sin(2*b*x + 2*a)^2 - 4*b^2*x^3*cos(2*b*x + 2*a) + b^2*x^3 - 2*(2*b^2*x^3*cos(2*b*x + 2*a) - b^2*x^3)*cos(4*b*x + 4*a))*integrate((b^2*x^2 - 3)*sin(b*x + a)/(b^2*x^4*cos(b*x + a)^2 + b^2*x^4*sin(b*x + a)^2 - 2*b^2*x^4*cos(b*x + a) + b^2*x^4), x) - 2*(b*x*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))/(b^2*x^3*cos(4*b*x + 4*a)^2 + 4*b^2*x^3*cos(2*b*x + 2*a)^2 + b^2*x^3*sin(4*b*x + 4*a)^2 - 4*b^2*x^3*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*x^3*sin(2*b*x + 2*a)^2 - 4*b^2*x^3*cos(2*b*x + 2*a) + b^2*x^3 - 2*(2*b^2*x^3*cos(2*b*x + 2*a) - b^2*x^3)*cos(4*b*x + 4*a))
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^3(a + bx)}{x^2} dx = \int \frac{\cot (bx + a)^3}{x^2} dx$$

```
[In] integrate(cot(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^3/x^2, x)
```


Mupad [N/A]

Not integrable

Time = 12.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cot^3(a + bx)}{x^2} dx = \int \frac{\cot(a + bx)^3}{x^2} dx$$

```
[In] int(cot(a + b*x)^3/x^2,x)
```

```
[Out] int(cot(a + b*x)^3/x^2, x)
```

3.16 $\int \frac{(c+dx)^3}{a+ia \cot(e+fx)} dx$

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Optimal result

Integrand size = 23, antiderivative size = 189

$$\int \frac{(c+dx)^3}{a+ia \cot(e+fx)} dx = -\frac{3id^3x}{8af^3} - \frac{3d(c+dx)^2}{8af^2} + \frac{i(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad}$$

$$- \frac{3d^3}{8f^4(a+ia \cot(e+fx))} + \frac{3id^2(c+dx)}{4f^3(a+ia \cot(e+fx))}$$

$$+ \frac{3d(c+dx)^2}{4f^2(a+ia \cot(e+fx))} - \frac{i(c+dx)^3}{2f(a+ia \cot(e+fx))}$$

[Out] -3/8*I*d^3*x/a/f^3-3/8*d*(d*x+c)^2/a/f^2+1/4*I*(d*x+c)^3/a/f+1/8*(d*x+c)^4/a/d-3/8*d^3/f^4/(a+I*a*cot(f*x+e))+3/4*I*d^2*(d*x+c)/f^3/(a+I*a*cot(f*x+e))+3/4*d*(d*x+c)^2/f^2/(a+I*a*cot(f*x+e))-1/2*I*(d*x+c)^3/f/(a+I*a*cot(f*x+e))

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3804, 3560, 8}

$$\int \frac{(c+dx)^3}{a+ia \cot(e+fx)} dx = \frac{3id^2(c+dx)}{4f^3(a+ia \cot(e+fx))} + \frac{3d(c+dx)^2}{4f^2(a+ia \cot(e+fx))}$$

$$- \frac{i(c+dx)^3}{2f(a+ia \cot(e+fx))} - \frac{3d(c+dx)^2}{8af^2} + \frac{4af}{4af}$$

$$+ \frac{(c+dx)^4}{8ad} - \frac{3d^3}{8f^4(a+ia \cot(e+fx))} - \frac{3id^3x}{8af^3}$$

[In] Int[(c + d*x)^3/(a + I*a*Cot[e + f*x]),x]

[Out] (((-3*I)/8)*d^3*x)/(a*f^3) - (3*d*(c + d*x)^2)/(8*a*f^2) + ((I/4)*(c + d*x)^3)/(a*f) + (c + d*x)^4/(8*a*d) - (3*d^3)/(8*f^4*(a + I*a*Cot[e + f*x])) + (((3*I)/4)*d^2*(c + d*x))/(f^3*(a + I*a*Cot[e + f*x])) + (3*d*(c + d*x)^2)/(4*f^2*(a + I*a*Cot[e + f*x])) - ((I/2)*(c + d*x)^3)/(f*(a + I*a*Cot[e + f*x]))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3560

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[a*((a + b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 3804

Int[((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[a*d*(m/(2*b*f)), Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m/(2*b*f*(a + b*Tan[e + f*x]))], x)) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{(c + dx)^4}{8ad} - \frac{i(c + dx)^3}{2f(a + ia \cot(e + fx))} + \frac{(3id) \int \frac{(c+dx)^2}{a+ia \cot(e+fx)} dx}{2f} \\
 &= \frac{i(c + dx)^3}{4af} + \frac{(c + dx)^4}{8ad} + \frac{3d(c + dx)^2}{4f^2(a + ia \cot(e + fx))} \\
 &\quad - \frac{i(c + dx)^3}{2f(a + ia \cot(e + fx))} - \frac{(3d^2) \int \frac{c+dx}{a+ia \cot(e+fx)} dx}{2f^2} \\
 &= -\frac{3d(c + dx)^2}{8af^2} + \frac{i(c + dx)^3}{4af} + \frac{(c + dx)^4}{8ad} + \frac{3id^2(c + dx)}{4f^3(a + ia \cot(e + fx))} \\
 &\quad + \frac{3d(c + dx)^2}{4f^2(a + ia \cot(e + fx))} - \frac{i(c + dx)^3}{2f(a + ia \cot(e + fx))} - \frac{(3id^3) \int \frac{1}{a+ia \cot(e+fx)} dx}{4f^3} \\
 &= -\frac{3d(c + dx)^2}{8af^2} + \frac{i(c + dx)^3}{4af} + \frac{(c + dx)^4}{8ad} - \frac{3d^3}{8f^4(a + ia \cot(e + fx))} \\
 &\quad + \frac{3id^2(c + dx)}{4f^3(a + ia \cot(e + fx))} + \frac{3d(c + dx)^2}{4f^2(a + ia \cot(e + fx))} \\
 &\quad - \frac{i(c + dx)^3}{2f(a + ia \cot(e + fx))} - \frac{(3id^3) \int 1 dx}{8af^3}
 \end{aligned}$$

$$= -\frac{3id^3x}{8af^3} - \frac{3d(c+dx)^2}{8af^2} + \frac{i(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d^3}{8f^4(a+ia\cot(e+fx))}$$

$$+ \frac{3id^2(c+dx)}{4f^3(a+ia\cot(e+fx))} + \frac{3d(c+dx)^2}{4f^2(a+ia\cot(e+fx))} - \frac{i(c+dx)^3}{2f(a+ia\cot(e+fx))}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx)^3}{a+ia\cot(e+fx)} dx$$

$$= \frac{2f^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+i(4c^3f^3+6c^2df^2(i+2fx)+6cd^2f(-1+2ifx+2f^2x^2)+d^3(-3i-$$

[In] Integrate[(c + d*x)^3/(a + I*a*Cot[e + f*x]),x]

[Out] (2*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + I*(4*c^3*f^3 + 6*c^2*d*f^2*(I + 2*f*x) + 6*c*d^2*f*(-1 + (2*I)*f*x + 2*f^2*x^2) + d^3*(-3*I - 6*f*x + (6*I)*f^2*x^2 + 4*f^3*x^3))*Cos[2*f*x]*(Cos[2*e] + I*Sin[2*e]) - (4*c^3*f^3 + 6*c^2*d*f^2*(I + 2*f*x) + 6*c*d^2*f*(-1 + (2*I)*f*x + 2*f^2*x^2) + d^3*(-3*I - 6*f*x + (6*I)*f^2*x^2 + 4*f^3*x^3))*(Cos[2*e] + I*Sin[2*e])*Sin[2*f*x])/(16*a*f^4)

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.90

method	result
risch	$\frac{d^3x^4}{8a} + \frac{d^2cx^3}{2a} + \frac{3dc^2x^2}{4a} + \frac{c^3x}{2a} + \frac{c^4}{8ad} + \frac{i(4d^3x^3f^3+12cd^2f^3x^2+6id^3f^2x^2+12c^2df^3x+12icd^2f^2x+4c^3f^3+6id^3x^3)}{16af^4}$
parallelrisch	$\frac{-6f\left(\left(-\frac{1}{6}d^3x^3-\frac{2}{3}cd^2x^2-c^2dx-\frac{2}{3}c^3\right)f^3+id\left(\frac{1}{3}d^2x^2+cdx+c^2\right)f^2+\left(-\frac{1}{2}d^3x-cd^2\right)f-\frac{id^3}{2}\right)x\tan(fx+e)+4i\left(\frac{dx}{2}+c\right)\left(\frac{1}{2}d^2\right)}{8f^4a(i+\tan(fx+e))}$
derivativdivides	Expression too large to display
default	Expression too large to display

[In] int((d*x+c)^3/(a+I*a*cot(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 1/8/a*d^3*x^4+1/2/a*d^2*c*x^3+3/4/a*d*c^2*x^2+1/2/a*c^3*x+1/8/a/d*c^4+1/16*I*(4*d^3*x^3*f^3+6*I*d^3*f^2*x^2+12*c*d^2*f^3*x^2+12*I*c*d^2*f^2*x+12*c^2*d*f^3*x+6*I*c^2*d*f^2+4*c^3*f^3-6*d^3*f*x-3*I*d^3-6*c*d^2*f)/a/f^4*exp(2*I*(f*x+e))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.81

$$\int \frac{(c + dx)^3}{a + ia \cot(e + fx)} dx$$

$$= \frac{2d^3 f^4 x^4 + 8cd^2 f^4 x^3 + 12c^2 df^4 x^2 + 8c^3 f^4 x + (4id^3 f^3 x^3 + 4ic^3 f^3 - 6c^2 df^2 - 6icd^2 f + 3d^3 - 6(-2icd^2 f^2 + d^3 f^2))x^2 - 6(-2icd^2 f^2 + d^3 f^2)x + 2icd^2 f^2 + d^3 f^2}{16af^4}$$

[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="fricas")

```
[Out] 1/16*(2*d^3*f^4*x^4 + 8*c*d^2*f^4*x^3 + 12*c^2*d*f^4*x^2 + 8*c^3*f^4*x + (4
*I*d^3*f^3*x^3 + 4*I*c^3*f^3 - 6*c^2*d*f^2 - 6*I*c*d^2*f + 3*d^3 - 6*(-2*I*
c*d^2*f^2 + d^3*f^2))*x^2 - 6*(-2*I*c^2*d*f^3 + 2*c*d^2*f^2 + I*d^3*f)*x)*e^
(2*I*f*x + 2*I*e))/(a*f^4)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.66

$$\int \frac{(c + dx)^3}{a + ia \cot(e + fx)} dx$$

$$= \left\{ \frac{(4ic^3 f^3 e^{2ie} + 12ic^2 df^3 x e^{2ie} - 6c^2 df^2 e^{2ie} + 12icd^2 f^3 x^2 e^{2ie} - 12cd^2 f^2 x e^{2ie} - 6icd^2 f e^{2ie} + 4id^3 f^3 x^3 e^{2ie} - 6d^3 f^2 x^2 e^{2ie} - 6id^3 f x e^{2ie} + 3d^3 e^{2ie})e^{2ifx}}{16af^4} \right.$$

$$\left. - \frac{c^3 x e^{2ie}}{2a} - \frac{3c^2 dx^2 e^{2ie}}{4a} - \frac{cd^2 x^3 e^{2ie}}{2a} - \frac{d^3 x^4 e^{2ie}}{8a} \right.$$

$$\left. + \frac{c^3 x}{2a} + \frac{3c^2 dx^2}{4a} + \frac{cd^2 x^3}{2a} + \frac{d^3 x^4}{8a} \right.$$

[In] integrate((d*x+c)**3/(a+I*a*cot(f*x+e)),x)

```
[Out] Piecewise(((4*I*c**3*f**3*exp(2*I*e) + 12*I*c**2*d*f**3*x*exp(2*I*e) - 6*c*
*2*d*f**2*exp(2*I*e) + 12*I*c*d**2*f**3*x**2*exp(2*I*e) - 12*c*d**2*f**2*x*
exp(2*I*e) - 6*I*c*d**2*f*exp(2*I*e) + 4*I*d**3*f**3*x**3*exp(2*I*e) - 6*d*
*3*f**2*x**2*exp(2*I*e) - 6*I*d**3*f*x*exp(2*I*e) + 3*d**3*exp(2*I*e))*exp(
2*I*f*x)/(16*a*f**4), Ne(a*f**4, 0)), (-c**3*x*exp(2*I*e)/(2*a) - 3*c**2*d*
x**2*exp(2*I*e)/(4*a) - c*d**2*x**3*exp(2*I*e)/(2*a) - d**3*x**4*exp(2*I*e)
/(8*a), True)) + c**3*x/(2*a) + 3*c**2*d*x**2/(4*a) + c*d**2*x**3/(2*a) + d
**3*x**4/(8*a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + dx)^3}{a + ia \cot(e + fx)} dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.23

$$\int \frac{(c + dx)^3}{a + ia \cot(e + fx)} dx$$

$$= \frac{2d^3 f^4 x^4 + 8cd^2 f^4 x^3 + 4id^3 f^3 x^3 e^{(2ifx+2ie)} + 12c^2 df^4 x^2 + 12icd^2 f^3 x^2 e^{(2ifx+2ie)} + 8c^3 f^4 x + 12ic^2 df^3 x e^{(2ifx+2ie)}}{a^2}$$

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/16*(2*d^3*f^4*x^4 + 8*c*d^2*f^4*x^3 + 4*I*d^3*f^3*x^3*e^(2*I*f*x + 2*I*e)
+ 12*c^2*d*f^4*x^2 + 12*I*c*d^2*f^3*x^2*e^(2*I*f*x + 2*I*e) + 8*c^3*f^4*x
+ 12*I*c^2*d*f^3*x*e^(2*I*f*x + 2*I*e) - 6*d^3*f^2*x^2*e^(2*I*f*x + 2*I*e)
+ 4*I*c^3*f^3*e^(2*I*f*x + 2*I*e) - 12*c*d^2*f^2*x*e^(2*I*f*x + 2*I*e) - 6*
c^2*d*f^2*e^(2*I*f*x + 2*I*e) - 6*I*d^3*f*x*e^(2*I*f*x + 2*I*e) - 6*I*c*d^2*
*f*e^(2*I*f*x + 2*I*e) + 3*d^3*e^(2*I*f*x + 2*I*e))/(a*f^4)
```

Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.24

$$\int \frac{(c + dx)^3}{a + ia \cot(e + fx)} dx$$

$$= \frac{3d^3 \cos(2e + 2fx) + 8c^3 f^4 x - 4c^3 f^3 \sin(2e + 2fx) + 2d^3 f^4 x^4 - 6c^2 d f^2 \cos(2e + 2fx) + 12c^2 d f^3 x \sin(2e + 2fx)}{a^2}$$

```
[In] int((c + d*x)^3/(a + a*cot(e + f*x)*1i),x)
```

```
[Out] (3*d^3*cos(2*e + 2*f*x) + d^3*sin(2*e + 2*f*x)*3i + 8*c^3*f^4*x + c^3*f^3*cos(2*e + 2*f*x)*4i - 4*c^3*f^3*sin(2*e + 2*f*x) + 2*d^3*f^4*x^4 - 6*c^2*d*f
```

$$\begin{aligned}
&^2 \cos(2e + 2f*x) - c^2 d f^2 \sin(2e + 2f*x) * 6i + 12 c^2 d f^4 x^2 + 8 * \\
&c d^2 f^4 x^3 - 6 d^3 f^2 x^2 \cos(2e + 2f*x) + d^3 f^3 x^3 \cos(2e + 2f * \\
&x) * 4i - d^3 f^2 x^2 \sin(2e + 2f*x) * 6i - 4 d^3 f^3 x^3 \sin(2e + 2f*x) - \\
&c d^2 f \cos(2e + 2f*x) * 6i + 6 c d^2 f \sin(2e + 2f*x) - d^3 f x \cos(2e \\
&+ 2f*x) * 6i + 6 d^3 f x \sin(2e + 2f*x) - 12 c d^2 f^2 x \cos(2e + 2f*x) \\
&+ c^2 d f^3 x \cos(2e + 2f*x) * 12i - c d^2 f^2 x \sin(2e + 2f*x) * 12i - 12 * \\
&c^2 d f^3 x \sin(2e + 2f*x) + c d^2 f^3 x^2 \cos(2e + 2f*x) * 12i - 12 c d^ \\
&2 f^3 x^2 \sin(2e + 2f*x)) / (16 a f^4)
\end{aligned}$$

3.17 $\int \frac{(c+dx)^2}{a+ia \cot(e+fx)} dx$

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Rubi [A] (verified)	120
Mathematica [A] (verified)	122
Maple [A] (verified)	122
Fricas [A] (verification not implemented)	122
Sympy [A] (verification not implemented)	123
Maxima [F(-2)]	123
Giac [A] (verification not implemented)	123
Mupad [B] (verification not implemented)	124

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{(c+dx)^2}{a+ia \cot(e+fx)} dx = -\frac{d^2x}{4af^2} + \frac{i(c+dx)^2}{4af} + \frac{(c+dx)^3}{6ad} + \frac{id^2}{4f^3(a+ia \cot(e+fx))} + \frac{d(c+dx)}{2f^2(a+ia \cot(e+fx))} - \frac{i(c+dx)^2}{2f(a+ia \cot(e+fx))}$$

[Out] $-1/4*d^2*x/a/f^2+1/4*I*(d*x+c)^2/a/f+1/6*(d*x+c)^3/a/d+1/4*I*d^2/f^3/(a+I*a*\cot(f*x+e))+1/2*d*(d*x+c)/f^2/(a+I*a*\cot(f*x+e))-1/2*I*(d*x+c)^2/f/(a+I*a*\cot(f*x+e))$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3804, 3560, 8}

$$\int \frac{(c+dx)^2}{a+ia \cot(e+fx)} dx = \frac{d(c+dx)}{2f^2(a+ia \cot(e+fx))} - \frac{i(c+dx)^2}{2f(a+ia \cot(e+fx))} + \frac{i(c+dx)^2}{4af} + \frac{(c+dx)^3}{6ad} + \frac{id^2}{4f^3(a+ia \cot(e+fx))} - \frac{d^2x}{4af^2}$$

[In] $\text{Int}[(c+d*x)^2/(a+I*a*\text{Cot}[e+f*x]),x]$

[Out] $-1/4*(d^2*x)/(a*f^2) + ((I/4)*(c+d*x)^2)/(a*f) + (c+d*x)^3/(6*a*d) + ((I/4)*d^2)/(f^3*(a+I*a*\text{Cot}[e+f*x])) + (d*(c+d*x))/(2*f^2*(a+I*a*\text{Cot}[e+f*x])) - (((I/2)*(c+d*x)^2)/(f*(a+I*a*\text{Cot}[e+f*x])))$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3560

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[a*((a + b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 3804

Int[((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[a*d*(m/(2*b*f)), Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m/(2*b*f*(a + b*Tan[e + f*x]))], x)) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{(c + dx)^3}{6ad} - \frac{i(c + dx)^2}{2f(a + ia \cot(e + fx))} + \frac{(id) \int \frac{c+dx}{a+ia \cot(e+fx)} dx}{f} \\
 &= \frac{i(c + dx)^2}{4af} + \frac{(c + dx)^3}{6ad} + \frac{d(c + dx)}{2f^2(a + ia \cot(e + fx))} \\
 &\quad - \frac{i(c + dx)^2}{2f(a + ia \cot(e + fx))} - \frac{d^2 \int \frac{1}{a+ia \cot(e+fx)} dx}{2f^2} \\
 &= \frac{i(c + dx)^2}{4af} + \frac{(c + dx)^3}{6ad} + \frac{id^2}{4f^3(a + ia \cot(e + fx))} \\
 &\quad + \frac{d(c + dx)}{2f^2(a + ia \cot(e + fx))} - \frac{i(c + dx)^2}{2f(a + ia \cot(e + fx))} - \frac{d^2 \int 1 dx}{4af^2} \\
 &= -\frac{d^2 x}{4af^2} + \frac{i(c + dx)^2}{4af} + \frac{(c + dx)^3}{6ad} + \frac{id^2}{4f^3(a + ia \cot(e + fx))} \\
 &\quad + \frac{d(c + dx)}{2f^2(a + ia \cot(e + fx))} - \frac{i(c + dx)^2}{2f(a + ia \cot(e + fx))}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\int \frac{(c + dx)^2}{a + ia \cot(e + fx)} dx$$

$$= \frac{4f^3x(3c^2 + 3cdx + d^2x^2) + 3((1 + i)cf + d(-1 + (1 + i)fx))((1 + i)cf + d(i + (1 + i)fx)) \cos(2fx)(\cos(2e) + i \sin(2e))}{24af^3}$$

[In] Integrate[(c + d*x)^2/(a + I*a*Cot[e + f*x]),x]

[Out] (4*f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 3*((1 + I)*c*f + d*(-1 + (1 + I)*f*x))*((1 + I)*c*f + d*(I + (1 + I)*f*x))*Cos[2*f*x]*(Cos[2*e] + I*Sin[2*e]) + (3*I)*((1 + I)*c*f + d*(-1 + (1 + I)*f*x))*((1 + I)*c*f + d*(I + (1 + I)*f*x))*((Cos[2*e] + I*Sin[2*e])*Sin[2*f*x])/(24*a*f^3)

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

method	result
risch	$\frac{d^2x^3}{6a} + \frac{dcx^2}{2a} + \frac{c^2x}{2a} + \frac{c^3}{6ad} + \frac{i(2d^2x^2f^2 + 4cdf^2x + 2id^2fx + 2c^2f^2 + 2icdf - d^2)e^{2i(fx+e)}}{8af^3}$
parallelrisc	$\frac{-2\left(-\frac{1}{3}d^2x^2 - cdx - c^2\right)f^2 + i\left(\frac{dx}{2} + c\right)df - \frac{d^2}{2}}{4f^3a(i + \tan(fx+e))}fx \tan(fx+e) + 2i\left(\frac{1}{3}d^2x^2 + cdx + c^2\right)x f^3 + (-d^2x^2 - 2cdx - 2c^2)f^2 - 2i\left(\frac{dx}{2} + c\right)df + \frac{d^2}{2}}$

[In] int((d*x+c)^2/(a+I*a*cot(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 1/6/a*d^2*x^3+1/2/a*d*c*x^2+1/2/a*c^2*x+1/6/a/d*c^3+1/8*I*(2*d^2*x^2*f^2+2*I*d^2*f*x+4*c*d*f^2*x+2*I*c*d*f+2*c^2*f^2-d^2)/a/f^3*exp(2*I*(f*x+e))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$\int \frac{(c + dx)^2}{a + ia \cot(e + fx)} dx$$

$$= \frac{4d^2f^3x^3 + 12cdf^3x^2 + 12c^2f^3x - 3(-2id^2f^2x^2 - 2ic^2f^2 + 2cdf + id^2 + 2(-2icdf^2 + d^2f)x)e^{(2ifx+2ie)}}{24af^3}$$

[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="fricas")

[Out] 1/24*(4*d^2*f^3*x^3 + 12*c*d*f^3*x^2 + 12*c^2*f^3*x - 3*(-2*I*d^2*f^2*x^2 - 2*I*c^2*f^2 + 2*c*d*f + I*d^2 + 2*(-2*I*c*d*f^2 + d^2*f)*x)*e^(2*I*f*x + 2*I*e))/(a*f^3)

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.42

$$\int \frac{(c + dx)^2}{a + ia \cot(e + fx)} dx$$

$$= \begin{cases} \frac{(2ic^2 f^2 e^{2ie} + 4icdf^2 x e^{2ie} - 2cdf e^{2ie} + 2id^2 f^2 x^2 e^{2ie} - 2d^2 f x e^{2ie} - id^2 e^{2ie}) e^{2ifx}}{8af^3} & \text{for } af^3 \neq 0 \\ -\frac{c^2 x e^{2ie}}{2a} - \frac{cdx^2 e^{2ie}}{2a} - \frac{d^2 x^3 e^{2ie}}{6a} & \text{otherwise} \end{cases}$$

$$+ \frac{c^2 x}{2a} + \frac{cdx^2}{2a} + \frac{d^2 x^3}{6a}$$

[In] integrate((d*x+c)**2/(a+I*a*cot(f*x+e)),x)

[Out] Piecewise(((2*I*c**2*f**2*exp(2*I*e) + 4*I*c*d*f**2*x*exp(2*I*e) - 2*c*d*f*exp(2*I*e) + 2*I*d**2*f**2*x**2*exp(2*I*e) - 2*d**2*f*x*exp(2*I*e) - I*d**2*exp(2*I*e))*exp(2*I*f*x)/(8*a*f**3), Ne(a*f**3, 0)), (-c**2*x*exp(2*I*e)/(2*a) - c*d*x**2*exp(2*I*e)/(2*a) - d**2*x**3*exp(2*I*e)/(6*a), True)) + c**2*x/(2*a) + c*d*x**2/(2*a) + d**2*x**3/(6*a)

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + dx)^2}{a + ia \cot(e + fx)} dx = \text{Exception raised: RuntimeError}$$

[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00

$$\int \frac{(c + dx)^2}{a + ia \cot(e + fx)} dx$$

$$= \frac{4d^2 f^3 x^3 + 12cdf^3 x^2 + 6id^2 f^2 x^2 e^{(2ifx+2ie)} + 12c^2 f^3 x + 12icdf^2 x e^{(2ifx+2ie)} + 6ic^2 f^2 e^{(2ifx+2ie)} - 6d^2 f x}{24af^3}$$

[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="giac")

[Out] 1/24*(4*d^2*f^3*x^3 + 12*c*d*f^3*x^2 + 6*I*d^2*f^2*x^2*e^(2*I*f*x + 2*I*e) + 12*c^2*f^3*x + 12*I*c*d*f^2*x*e^(2*I*f*x + 2*I*e) + 6*I*c^2*f^2*e^(2*I*f*x + 2*I*e) - 6*d^2*f*x*e^(2*I*f*x + 2*I*e) - 6*c*d*f*e^(2*I*f*x + 2*I*e) - 3*I*d^2*e^(2*I*f*x + 2*I*e))/(a*f^3)

Mupad [B] (verification not implemented)

Time = 12.78 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.76

$$\int \frac{(c + dx)^2}{a + ia \cot(e + fx)} dx = \frac{6c^2 f^2 \sin(2e + 2fx) - 12c^2 f^3 x - 3d^2 \sin(2e + 2fx) - 4d^2 f^3 x^3 + 6cdf \cos(2e + 2fx) + 6d^2 f^2 x^2 \cos(2e + 2fx) + 6d^2 f^2 x^2 \sin(2e + 2fx)}{24a^2 f^3}$$

[In] int((c + d*x)^2/(a + a*cot(e + f*x)*1i),x)

[Out] -(d^2*cos(2*e + 2*f*x)*3i - 3*d^2*sin(2*e + 2*f*x) - 12*c^2*f^3*x - c^2*f^2*cos(2*e + 2*f*x)*6i + 6*c^2*f^2*sin(2*e + 2*f*x) - 4*d^2*f^3*x^3 + 6*c*d*f*cos(2*e + 2*f*x) + c*d*f*sin(2*e + 2*f*x)*6i - d^2*f^2*x^2*cos(2*e + 2*f*x)*6i + 6*d^2*f^2*x^2*sin(2*e + 2*f*x) - 12*c*d*f^3*x^2 + 6*d^2*f*x*cos(2*e + 2*f*x) + d^2*f*x*sin(2*e + 2*f*x)*6i - c*d*f^2*x*cos(2*e + 2*f*x)*12i + 12*c*d*f^2*x*sin(2*e + 2*f*x))/(24*a*f^3)

3.18 $\int \frac{c+dx}{a+ia \cot(e+fx)} dx$

Optimal result	125
Rubi [A] (verified)	125
Mathematica [A] (verified)	126
Maple [A] (verified)	127
Fricas [A] (verification not implemented)	127
Sympy [A] (verification not implemented)	127
Maxima [F(-2)]	128
Giac [A] (verification not implemented)	128
Mupad [B] (verification not implemented)	128

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{c+dx}{a+ia \cot(e+fx)} dx = \frac{idx}{4af} + \frac{(c+dx)^2}{4ad} + \frac{d}{4f^2(a+ia \cot(e+fx))} - \frac{i(c+dx)}{2f(a+ia \cot(e+fx))}$$

[Out] $1/4*I*d*x/a/f+1/4*(d*x+c)^2/a/d+1/4*d/f^2/(a+I*a*\cot(f*x+e))-1/2*I*(d*x+c)/f/(a+I*a*\cot(f*x+e))$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3804, 3560, 8}

$$\int \frac{c+dx}{a+ia \cot(e+fx)} dx = -\frac{i(c+dx)}{2f(a+ia \cot(e+fx))} + \frac{(c+dx)^2}{4ad} + \frac{d}{4f^2(a+ia \cot(e+fx))} + \frac{idx}{4af}$$

[In] Int[(c + d*x)/(a + I*a*Cot[e + f*x]),x]

[Out] ((I/4)*d*x)/(a*f) + (c + d*x)^2/(4*a*d) + d/(4*f^2*(a + I*a*Cot[e + f*x])) - ((I/2)*(c + d*x))/(f*(a + I*a*Cot[e + f*x]))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3560

```
Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3804

```
Int[((c_) + (d_)*(x_)^(m_))/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Sy
mbol] := Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[a*d*(m/(2*b*f))
, Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m/
(2*b*f*(a + b*Tan[e + f*x]))], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 + b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(c + dx)^2}{4ad} - \frac{i(c + dx)}{2f(a + ia \cot(e + fx))} + \frac{(id) \int \frac{1}{a + ia \cot(e + fx)} dx}{2f} \\ &= \frac{(c + dx)^2}{4ad} + \frac{d}{4f^2(a + ia \cot(e + fx))} - \frac{i(c + dx)}{2f(a + ia \cot(e + fx))} + \frac{(id) \int 1 dx}{4af} \\ &= \frac{id x}{4af} + \frac{(c + dx)^2}{4ad} + \frac{d}{4f^2(a + ia \cot(e + fx))} - \frac{i(c + dx)}{2f(a + ia \cot(e + fx))} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \frac{c + dx}{a + ia \cot(e + fx)} dx = \frac{(\cos(e + fx) + i \sin(e + fx)) ((2cf(i + 2fx) + d(-1 + 2ifx + 2f^2x^2)) \cos(e + fx) - i(2cf(-i + 2fx) + a))}{8af^2}$$

```
[In] Integrate[(c + d*x)/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] ((Cos[e + f*x] + I*Sin[e + f*x])*((2*c*f*(I + 2*f*x) + d*(-1 + (2*I)*f*x +
2*f^2*x^2))*Cos[e + f*x] - I*(2*c*f*(-I + 2*f*x) + d*(1 - (2*I)*f*x + 2*f^2
*x^2))*Sin[e + f*x]))/(8*a*f^2)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.60

method	result	size
risch	$\frac{dx^2}{4a} + \frac{xc}{2a} + \frac{i(2dfx+2cf+id)e^{2i(fx+e)}}{8af^2}$	50
parallelrisch	$\frac{-f((-dx-2c)f+id)x \tan(fx+e)+2i\left(\frac{dx}{2}+c\right)xf^2+(-dx-2c)f-id}{4f^2a(i+\tan(fx+e))}$	73
norman	$\frac{\frac{dx^2}{4a} - \frac{-2icf+d}{4af^2} + \frac{dx^2 \tan(fx+e)^2}{4a} - \frac{(2cf+id) \tan(fx+e)}{4f^2a} + \frac{(2cf+id)x}{4af} - \frac{dx \tan(fx+e)}{2af} + \frac{(2cf-id)x \tan(fx+e)^2}{4af}}{1+\tan(fx+e)^2}$	139

```
[In] int((d*x+c)/(a+I*a*cot(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*d*x^2+1/2/a*x*c+1/8*I*(2*d*f*x+I*d+2*c*f)/a/f^2*exp(2*I*(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.57

$$\int \frac{c + dx}{a + ia \cot(e + fx)} dx = \frac{2df^2x^2 + 4cfx + (2idfx + 2icf - d)e^{(2ifx+2ie)}}{8af^2}$$

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*d*f^2*x^2 + 4*c*f^2*x + (2*I*d*f*x + 2*I*c*f - d)*e^(2*I*f*x + 2*I*e))/(a*f^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int \frac{c + dx}{a + ia \cot(e + fx)} dx = \begin{cases} \frac{(2icfe^{2ie}+2idfxe^{2ie}-de^{2ie})e^{2ifx}}{8af^2} & \text{for } af^2 \neq 0 \\ -\frac{cxe^{2ie}}{2a} - \frac{dx^2e^{2ie}}{4a} & \text{otherwise} \end{cases} + \frac{cx}{2a} + \frac{dx^2}{4a}$$

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e)),x)
```

```
[Out] Piecewise(((2*I*c*f*exp(2*I*e) + 2*I*d*f*x*exp(2*I*e) - d*exp(2*I*e))*exp(2*I*f*x)/(8*a*f**2), Ne(a*f**2, 0)), (-c*x*exp(2*I*e)/(2*a) - d*x**2*exp(2*I*e)/(4*a), True)) + c*x/(2*a) + d*x**2/(4*a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{c + dx}{a + ia \cot(e + fx)} dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \frac{c + dx}{a + ia \cot(e + fx)} dx = \frac{2df^2x^2 + 4cf^2x + 2idfxe^{(2ifx+2ie)} + 2icfe^{(2ifx+2ie)} - de^{(2ifx+2ie)}}{8af^2}$$

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/8*(2*d*f^2*x^2 + 4*c*f^2*x + 2*I*d*f*x*e^(2*I*f*x + 2*I*e) + 2*I*c*f*e^(2*I*f*x + 2*I*e) - d*e^(2*I*f*x + 2*I*e))/(a*f^2)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \frac{c + dx}{a + ia \cot(e + fx)} dx = \frac{d \cos(2e + 2fx) - 2df^2x^2 + 2cf \sin(2e + 2fx) - 4cf^2x + 2dfx \sin(2e + 2fx) + d \sin(2e + 2fx)}{8af^2}$$

```
[In] int((c + d*x)/(a + a*cot(e + f*x)*1i),x)
```

```
[Out] -(d*cos(2*e + 2*f*x) + d*sin(2*e + 2*f*x)*1i - 2*d*f^2*x^2 - c*f*cos(2*e + 2*f*x)*2i + 2*c*f*sin(2*e + 2*f*x) - 4*c*f^2*x - d*f*x*cos(2*e + 2*f*x)*2i + 2*d*f*x*sin(2*e + 2*f*x))/(8*a*f^2)
```


$$3.19 \quad \int \frac{1}{(c+dx)(a+ia \cot(e+fx))} dx$$

Optimal result	129
Rubi [A] (verified)	129
Mathematica [A] (verified)	131
Maple [A] (verified)	131
Fricas [A] (verification not implemented)	131
Sympy [F]	132
Maxima [A] (verification not implemented)	132
Giac [B] (verification not implemented)	132
Mupad [F(-1)]	133

Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))} dx = -\frac{\cos(2e - \frac{2cf}{d}) \operatorname{CosIntegral}(\frac{2cf}{d} + 2fx)}{2ad} + \frac{\log(c+dx)}{2ad} \\ - \frac{i \operatorname{CosIntegral}(\frac{2cf}{d} + 2fx) \sin(2e - \frac{2cf}{d})}{2ad} \\ - \frac{i \cos(2e - \frac{2cf}{d}) \operatorname{Si}(\frac{2cf}{d} + 2fx)}{2ad} \\ + \frac{\sin(2e - \frac{2cf}{d}) \operatorname{Si}(\frac{2cf}{d} + 2fx)}{2ad}$$

[Out] $-1/2*\operatorname{Ci}(2*c*f/d+2*f*x)*\cos(-2*e+2*c*f/d)/a/d+1/2*\ln(d*x+c)/a/d-1/2*I*\cos(-2$
 $*e+2*c*f/d)*\operatorname{Si}(2*c*f/d+2*f*x)/a/d+1/2*I*\operatorname{Ci}(2*c*f/d+2*f*x)*\sin(-2*e+2*c*f/d)$
 $/a/d-1/2*\operatorname{Si}(2*c*f/d+2*f*x)*\sin(-2*e+2*c*f/d)/a/d$

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00,
 number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used
 = {3807, 3384, 3380, 3383}

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))} dx = -\frac{i \operatorname{CosIntegral}(2xf + \frac{2cf}{d}) \sin(2e - \frac{2cf}{d})}{2ad} \\ - \frac{\operatorname{CosIntegral}(2xf + \frac{2cf}{d}) \cos(2e - \frac{2cf}{d})}{2ad} \\ + \frac{\sin(2e - \frac{2cf}{d}) \operatorname{Si}(2xf + \frac{2cf}{d})}{2ad} \\ - \frac{i \cos(2e - \frac{2cf}{d}) \operatorname{Si}(2xf + \frac{2cf}{d})}{2ad} + \frac{\log(c+dx)}{2ad}$$

[In] Int[1/((c + d*x)*(a + I*a*Cot[e + f*x])),x]
 [Out] -1/2*(Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(a*d) + Log[c + d*x]/(2*a*d) - ((I/2)*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a*d) - ((I/2)*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a*d) + (Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*a*d)

Rule 3380

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3383

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3384

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3807

Int[1/(((c_.) + (d_.)*(x_.))*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])), x_Symbol] := Simp[Log[c + d*x]/(2*a*d), x] + (Dist[1/(2*a), Int[Cos[2*e + 2*f*x]/(c + d*x), x], x] + Dist[1/(2*b), Int[Sin[2*e + 2*f*x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\log(c + dx)}{2ad} + \frac{i \int \frac{\sin(2(e + \frac{\pi}{2}) + 2fx)}{c + dx} dx}{2a} + \frac{\int \frac{\cos(2(e + \frac{\pi}{2}) + 2fx)}{c + dx} dx}{2a} \\
 &= \frac{\log(c + dx)}{2ad} - \frac{(i \cos(2e - \frac{2cf}{d})) \int \frac{\sin(\frac{2cf}{d} + 2fx)}{c + dx} dx}{2a} - \frac{\cos(2e - \frac{2cf}{d}) \int \frac{\cos(\frac{2cf}{d} + 2fx)}{c + dx} dx}{2a} \\
 &\quad - \frac{(i \sin(2e - \frac{2cf}{d})) \int \frac{\cos(\frac{2cf}{d} + 2fx)}{c + dx} dx}{2a} + \frac{\sin(2e - \frac{2cf}{d}) \int \frac{\sin(\frac{2cf}{d} + 2fx)}{c + dx} dx}{2a} \\
 &= -\frac{\cos(2e - \frac{2cf}{d}) \text{CosIntegral}(\frac{2cf}{d} + 2fx)}{2ad} + \frac{\log(c + dx)}{2ad} \\
 &\quad - \frac{i \text{CosIntegral}(\frac{2cf}{d} + 2fx) \sin(2e - \frac{2cf}{d})}{2ad} \\
 &\quad - \frac{i \cos(2e - \frac{2cf}{d}) \text{Si}(\frac{2cf}{d} + 2fx)}{2ad} + \frac{\sin(2e - \frac{2cf}{d}) \text{Si}(\frac{2cf}{d} + 2fx)}{2ad}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.48

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))} dx = \frac{\log(c+dx) - \left(\cos\left(2e - \frac{2cf}{d}\right) + i \sin\left(2e - \frac{2cf}{d}\right)\right) \left(\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right) + i \text{Si}\left(\frac{2f(c+dx)}{d}\right)\right)}{2ad}$$

[In] Integrate[1/((c + d*x)*(a + I*a*Cot[e + f*x])),x]

[Out] (Log[c + d*x] - (Cos[2*e - (2*c*f)/d] + I*Sin[2*e - (2*c*f)/d])*(CosIntegral[1[(2*f*(c + d*x))/d] + I*SinIntegral[(2*f*(c + d*x))/d]])/(2*a*d)

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.42

method	result
risch	$\frac{\ln(dx+c)}{2ad} + \frac{e^{-\frac{2i(cf-de)}{d}} \text{Ei}_1\left(-2ifx-2ie-\frac{2(icf-ide)}{d}\right)}{2ad}$
derivativedivides	$-\frac{i\left(\frac{2 \text{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right) \cos\left(\frac{2cf-2de}{d}\right) - 2 \text{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right) \sin\left(\frac{2cf-2de}{d}\right)}{4}\right) + \frac{\ln(cf-de+d(fx+e))}{a} - \frac{\text{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)}{a}}{a}$
default	$-\frac{i\left(\frac{2 \text{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right) \cos\left(\frac{2cf-2de}{d}\right) - 2 \text{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right) \sin\left(\frac{2cf-2de}{d}\right)}{4}\right) + \frac{\ln(cf-de+d(fx+e))}{2d} - \frac{\text{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)}{a}}{a}$

[In] int(1/(d*x+c)/(a+I*a*cot(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 1/2*ln(d*x+c)/a/d+1/2/a/d*exp(-2*I*(c*f-d*e)/d)*Ei(1,-2*I*f*x-2*I*e-2*(I*c*f-I*d*e)/d)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.33

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))} dx = -\frac{\text{Ei}\left(-\frac{2(-idf_x-icf)}{d}\right) e^{\left(-\frac{2(-ide+icf)}{d}\right)} - \log\left(\frac{dx+c}{d}\right)}{2ad}$$

[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="fricas")

[Out] -1/2*(Ei(-2*(-I*d*f*x - I*c*f)/d)*e^(-2*(-I*d*e + I*c*f)/d) - log((d*x + c)/d))/(a*d)

Sympy [F]

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))} dx = -\frac{i \int \frac{1}{c \cot(e+fx) - ic + dx \cot(e+fx) - idx} dx}{a}$$

[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e)),x)

[Out] -I*Integral(1/(c*cot(e + f*x) - I*c + d*x*cot(e + f*x) - I*d*x), x)/a

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.69

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))} dx = \frac{f \cos\left(-\frac{2(de-cf)}{d}\right) E_1\left(\frac{2(-i(fx+e)d+ide-icf)}{d}\right) - i f E_1\left(\frac{2(-i(fx+e)d+ide-icf)}{d}\right) \sin\left(-\frac{2(de-cf)}{d}\right) + f \log((fx+e))}{2adf}$$

[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="maxima")

[Out] 1/2*(f*cos(-2*(d*e - c*f)/d)*exp_integral_e(1, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) - I*f*exp_integral_e(1, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d)*sin(-2*(d*e - c*f)/d) + f*log((f*x + e)*d - d*e + c*f))/(a*d*f)

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(147) = 294.

Time = 0.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.18

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))} dx = \frac{\cos(e)^2 \cos\left(\frac{2cf}{d}\right) \text{Ci}\left(\frac{2(dfx+cf)}{d}\right) + 2i \cos(e) \cos\left(\frac{2cf}{d}\right) \text{Ci}\left(\frac{2(dfx+cf)}{d}\right) \sin(e) - \cos\left(\frac{2cf}{d}\right) \text{Ci}\left(\frac{2(dfx+cf)}{d}\right) \sin(e)}{2adf}$$

[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e)),x, algorithm="giac")

[Out] -1/2*(cos(e)^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d) + 2*I*cos(e)*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) - cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2 - I*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) + 2*cos(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)*sin(2*c*f/d)

) + I*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2*sin(2*c*f/d) + I*cos(e)^2*cos(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - 2*cos(e)*cos(2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*f)/d) - I*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) + cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 2*I*cos(e)*sin(e)*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - sin(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - log(d*x + c)/(a*d)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c + dx)(a + ia \cot(e + fx))} dx = \int \frac{1}{(a + a \cot(e + fx) \text{li}) (c + dx)} dx$$

[In] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)),x)

[Out] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)), x)

3.20 $\int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))} dx$

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Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))} dx = -\frac{if \cos\left(2e - \frac{2cf}{d}\right) \text{CosIntegral}\left(\frac{2cf}{d} + 2fx\right)}{ad^2} - \frac{1}{d(c+dx)(a+ia \cot(e+fx))} + \frac{f \text{CosIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(2e - \frac{2cf}{d}\right)}{ad^2} + \frac{f \cos\left(2e - \frac{2cf}{d}\right) \text{Si}\left(\frac{2cf}{d} + 2fx\right)}{ad^2} + \frac{if \sin\left(2e - \frac{2cf}{d}\right) \text{Si}\left(\frac{2cf}{d} + 2fx\right)}{ad^2}$$

[Out] $-I*f*Ci(2*c*f/d+2*f*x)*\cos(-2*e+2*c*f/d)/a/d^2-1/d/(d*x+c)/(a+I*a*\cot(f*x+e)) + f*\cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a/d^2-f*Ci(2*c*f/d+2*f*x)*\sin(-2*e+2*c*f/d)/a/d^2-I*f*Si(2*c*f/d+2*f*x)*\sin(-2*e+2*c*f/d)/a/d^2$

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used

= {3805, 3384, 3380, 3383}

$$\int \frac{1}{(c + dx)^2(a + ia \cot(e + fx))} dx = \frac{f \operatorname{CosIntegral}\left(2xf + \frac{2cf}{d}\right) \sin\left(2e - \frac{2cf}{d}\right)}{ad^2} - \frac{if \operatorname{CosIntegral}\left(2xf + \frac{2cf}{d}\right) \cos\left(2e - \frac{2cf}{d}\right)}{ad^2} + \frac{if \sin\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(2xf + \frac{2cf}{d}\right)}{ad^2} + \frac{f \cos\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(2xf + \frac{2cf}{d}\right)}{ad^2} - \frac{1}{d(c + dx)(a + ia \cot(e + fx))}$$

[In] Int[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])),x]

[Out] ((-I)*f*cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(a*d^2) - 1/(d*(c + d*x)*(a + I*a*Cot[e + f*x])) + (f*cosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a*d^2) + (f*cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a*d^2) + (I*f*sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a*d^2)

Rule 3380

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3383

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3384

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3805

Int[1/(((c_.) + (d_.)*(x_.))^2*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])), x_Symbol] := -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Dist[f/(a*d), Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Dist[f/(b*d), Int[Cos[2*e + 2*f*x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{1}{d(c+dx)(a+ia\cot(e+fx))} + \frac{(if) \int \frac{\cos(2(e+\frac{\pi}{2})+2fx)}{c+dx} dx}{ad} - \frac{f \int \frac{\sin(2(e+\frac{\pi}{2})+2fx)}{c+dx} dx}{ad} \\
 &= -\frac{1}{d(c+dx)(a+ia\cot(e+fx))} - \frac{(if \cos(2e - \frac{2cf}{d})) \int \frac{\cos(\frac{2cf}{d}+2fx)}{c+dx} dx}{ad} \\
 &\quad + \frac{(f \cos(2e - \frac{2cf}{d})) \int \frac{\sin(\frac{2cf}{d}+2fx)}{c+dx} dx}{ad} + \frac{(if \sin(2e - \frac{2cf}{d})) \int \frac{\sin(\frac{2cf}{d}+2fx)}{c+dx} dx}{ad} \\
 &\quad + \frac{(f \sin(2e - \frac{2cf}{d})) \int \frac{\cos(\frac{2cf}{d}+2fx)}{c+dx} dx}{ad} \\
 &= -\frac{if \cos(2e - \frac{2cf}{d}) \text{CosIntegral}(\frac{2cf}{d} + 2fx)}{ad^2} - \frac{1}{d(c+dx)(a+ia\cot(e+fx))} \\
 &\quad + \frac{f \text{CosIntegral}(\frac{2cf}{d} + 2fx) \sin(2e - \frac{2cf}{d})}{ad^2} \\
 &\quad + \frac{f \cos(2e - \frac{2cf}{d}) \text{Si}(\frac{2cf}{d} + 2fx)}{ad^2} + \frac{if \sin(2e - \frac{2cf}{d}) \text{Si}(\frac{2cf}{d} + 2fx)}{ad^2}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.30

$$\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))} dx$$

$$(\cos(e+f(-\frac{c}{d}+x)) + i \sin(e+f(-\frac{c}{d}+x))) (d(-\cos(e+f(-\frac{c}{d}+x)) + \cos(e+f(\frac{c}{d}+x))) + i(\sin($$

[In] Integrate[1/((c+d*x)^2*(a+I*a*Cot[e+f*x])),x]

[Out] ((Cos[e+f*(-(c/d)+x)] + I*Sin[e+f*(-(c/d)+x)])*(d*(-Cos[e+f*(-(c/d)+x)] + Cos[e+f*(c/d+x)] + I*(Sin[e+f*(-(c/d)+x)] + Sin[e+f*(c/d+x)])) + 2*f*(c+d*x)*CosIntegral[(2*f*(c+d*x))/d]*((-I)*Cos[e-(f*(c+d*x))/d] + Sin[e-(f*(c+d*x))/d]) + 2*f*(c+d*x)*(Cos[e-(f*(c+d*x))/d] + I*Sin[e-(f*(c+d*x))/d])*SinIntegral[(2*f*(c+d*x))/d]))/(2*a*d^2*(c+d*x))

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{1}{2d(dx+c)a} + \frac{if e^{2i(fx+e)}}{2ad^2\left(ifx + \frac{icf}{d} \right)} + \frac{if e^{-\frac{2i(cf-de)}{d}} \text{Ei}_1\left(-2ifx-2ie-\frac{2(icf-ide)}{d}\right)}{ad^2}$
derivativedivides	$f \left(\frac{i \left(-\frac{2 \sin(2fx+2e)}{(cf-de+d(fx+e))d} + \frac{4 \text{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right) \sin\left(\frac{2cf-2de}{d}\right)}{d} + \frac{4 \text{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right) \cos\left(\frac{2cf-2de}{d}\right)}{d} \right)}{4} \right) - \frac{a}{2(cf-de+}$
default	$f \left(\frac{i \left(-\frac{2 \sin(2fx+2e)}{(cf-de+d(fx+e))d} + \frac{4 \text{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right) \sin\left(\frac{2cf-2de}{d}\right)}{d} + \frac{4 \text{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right) \cos\left(\frac{2cf-2de}{d}\right)}{d} \right)}{4} \right) - \frac{a}{2(cf-de+}$

[In] `int(1/(d*x+c)^2/(a+I*a*cot(f*x+e)),x,method=_RETURNVERBOSE)`

[Out]
$$-1/2/d/(d*x+c)/a+1/2*I/a*f/d^2*\exp(2*I*(f*x+e))/(I*f*x+I/d*c*f)+I/a*f/d^2*\exp(-2*I*(c*f-d*e)/d)*\text{Ei}\left(1,-2*I*f*x-2*I*e-2*(I*c*f-I*d*e)/d\right)$$

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.45

$$\int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))} dx$$

$$= -\frac{2(i dfx + icf) \text{Ei}\left(-\frac{2(-i dfx - icf)}{d}\right) e^{\left(-\frac{2(-ide+icf)}{d}\right)} - de^{(2ifx+2ie)} + d}{2(ad^3x + acd^2)}$$

[In] `integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="fricas")`

[Out]
$$-1/2*(2*(I*d*f*x + I*c*f)*\text{Ei}\left(-2*(-I*d*f*x - I*c*f)/d\right)*e^{(-2*(-I*d*e + I*c*f)/d)} - d*e^{(2*I*f*x + 2*I*e)} + d)/(a*d^3*x + a*c*d^2)$$

Sympy [F]

$$\int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))} dx$$

$$= -\frac{i \int \frac{1}{c^2 \cot(e+fx) - ic^2 + 2cdx \cot(e+fx) - 2icdx + d^2 x^2 \cot(e+fx) - id^2 x^2} dx}{a}$$

[In] integrate(1/(d*x+c)**2/(a+I*a*cot(f*x+e)),x)

[Out] -I*Integral(1/(c**2*cot(e + f*x) - I*c**2 + 2*c*d*x*cot(e + f*x) - 2*I*c*d*x + d**2*x**2*cot(e + f*x) - I*d**2*x**2), x)/a

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.73

$$\int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))} dx$$

$$= \frac{f^2 \cos\left(-\frac{2(de-cf)}{d}\right) E_2\left(\frac{2(-i(fx+e)d+ide-icf)}{d}\right) - i f^2 E_2\left(\frac{2(-i(fx+e)d+ide-icf)}{d}\right) \sin\left(-\frac{2(de-cf)}{d}\right) - f^2}{2((fx+e)ad^2 - ad^2e + acdf)f}$$

[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="maxima")

[Out] 1/2*(f^2*cos(-2*(d*e - c*f)/d)*exp_integral_e(2, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) - I*f^2*exp_integral_e(2, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) *sin(-2*(d*e - c*f)/d) - f^2)/(((f*x + e)*a*d^2 - a*d^2*e + a*c*d*f)*f)

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(161) = 322.

Time = 1.08 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.15

$$\int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))} dx =$$

$$\frac{i \left(-2i(dx+c) \left(\frac{ide}{dx+c} - \frac{icf}{dx+c} + if \right) f^2 \text{Ei} \left(\frac{2 \left((dx+c) \left(\frac{ide}{dx+c} - \frac{icf}{dx+c} + if \right) - ide + icf \right)}{d} \right) e^{\left(-\frac{2(-ide+icf)}{d} \right)} - 2def^2 \text{Ei} \left(\frac{2(-ide+icf)}{d} \right) \right)}{2(dx+c)ad}$$

[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e)),x, algorithm="giac")

[Out]
$$-1/2*I*(-2*I*(d*x + c)*(I*d*e/(d*x + c) - I*c*f/(d*x + c) + I*f)*f^2*Ei(2*(d*x + c)*(I*d*e/(d*x + c) - I*c*f/(d*x + c) + I*f) - I*d*e + I*c*f)/d)*e^{(-2*(-I*d*e + I*c*f)/d) - 2*d*e*f^2*Ei(2*((d*x + c)*(I*d*e/(d*x + c) - I*c*f/(d*x + c) + I*f) - I*d*e + I*c*f)/d)*e^{(-2*(-I*d*e + I*c*f)/d) + 2*c*f^3*Ei(2*((d*x + c)*(I*d*e/(d*x + c) - I*c*f/(d*x + c) + I*f) - I*d*e + I*c*f)/d)*e^{(-2*(-I*d*e + I*c*f)/d) + I*d*f^2*e^{(-2*(d*x + c)*(-I*d*e/(d*x + c) + I*c*f/(d*x + c) - I*f)/d)}*d^2/((-I*(d*x + c)*d^4*(I*d*e/(d*x + c) - I*c*f/(d*x + c) + I*f) - d^5*e + c*d^4*f)*a*f) - 1/2/((d*x + c)*a*d)$$

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c + dx)^2(a + ia \cot(e + fx))} dx = \int \frac{1}{(a + a \cot(e + fx) \operatorname{li})(c + dx)^2} dx$$

[In] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)^2),x)

[Out] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)^2), x)

$$3.21 \quad \int \frac{1}{(c+dx)^3(a+ia \cot(e+fx))} dx$$

Optimal result	140
Rubi [A] (verified)	141
Mathematica [A] (verified)	143
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Optimal result

Integrand size = 23, antiderivative size = 227

$$\int \frac{1}{(c+dx)^3(a+ia \cot(e+fx))} dx = \frac{if}{2ad^2(c+dx)} + \frac{f^2 \cos(2e - \frac{2cf}{d}) \operatorname{CosIntegral}(\frac{2cf}{d} + 2fx)}{ad^3} - \frac{1}{2d(c+dx)^2(a+ia \cot(e+fx))} - \frac{if}{d^2(c+dx)(a+ia \cot(e+fx))} + \frac{if^2 \operatorname{CosIntegral}(\frac{2cf}{d} + 2fx) \sin(2e - \frac{2cf}{d})}{ad^3} + \frac{if^2 \cos(2e - \frac{2cf}{d}) \operatorname{Si}(\frac{2cf}{d} + 2fx)}{ad^3} - \frac{f^2 \sin(2e - \frac{2cf}{d}) \operatorname{Si}(\frac{2cf}{d} + 2fx)}{ad^3}$$

```
[Out] 1/2*I*f/a/d^2/(d*x+c)+f^2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a/d^3-1/2/d/(d*x+c)^2/(a+I*a*cot(f*x+e))-I*f/d^2/(d*x+c)/(a+I*a*cot(f*x+e))+I*f^2*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a/d^3-I*f^2*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a/d^3+f^2*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a/d^3
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3806, 3805, 3384, 3380, 3383}

$$\int \frac{1}{(c+dx)^3(a+ia \cot(e+fx))} dx = \frac{if^2 \operatorname{CosIntegral}\left(2xf + \frac{2cf}{d}\right) \sin\left(2e - \frac{2cf}{d}\right)}{ad^3} + \frac{f^2 \operatorname{CosIntegral}\left(2xf + \frac{2cf}{d}\right) \cos\left(2e - \frac{2cf}{d}\right)}{ad^3} - \frac{f^2 \sin\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(2xf + \frac{2cf}{d}\right)}{ad^3} + \frac{if^2 \cos\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(2xf + \frac{2cf}{d}\right)}{ad^3} - \frac{if}{d^2(c+dx)(a+ia \cot(e+fx))} + \frac{if}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+ia \cot(e+fx))}$$

[In] Int[1/((c + d*x)^3*(a + I*a*Cot[e + f*x])),x]

[Out] ((I/2)*f)/(a*d^2*(c + d*x)) + (f^2*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(a*d^3) - 1/(2*d*(c + d*x)^2*(a + I*a*Cot[e + f*x])) - (I*f)/(d^2*(c + d*x)*(a + I*a*Cot[e + f*x])) + (I*f^2*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a*d^3) + (I*f^2*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a*d^3) - (f^2*Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a*d^3)

Rule 3380

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3383

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3384

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3805

```
Int[1/(((c_.) + (d_.)*(x_))2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] := -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))(-1), x] + (-Dist[f/(a*d)
, Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Dist[f/(b*d), Int[Cos[2*e + 2*f*
x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a2 + b2, 0]
```

Rule 3806

```
Int[((c_.) + (d_.)*(x_))(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] := Simp[f*((c + d*x)(m + 2)/(b*d2*(m + 1)*(m + 2))], x] + (Dist[2*b*
(f/(a*d*(m + 1))), Int[(c + d*x)(m + 1)/(a + b*Tan[e + f*x]), x], x] + Sim
p[(c + d*x)(m + 1)/(d*(m + 1)*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[a2 + b2, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{if}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+ia\cot(e+fx))} + \frac{(if) \int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))} dx}{d} \\
&= \frac{if}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+ia\cot(e+fx))} - \frac{if}{d^2(c+dx)(a+ia\cot(e+fx))} \\
&\quad - \frac{(if^2) \int \frac{\sin(2(e+\frac{\pi}{2})+2fx)}{c+dx} dx}{ad^2} - \frac{f^2 \int \frac{\cos(2(e+\frac{\pi}{2})+2fx)}{c+dx} dx}{ad^2} \\
&= \frac{if}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+ia\cot(e+fx))} - \frac{if}{d^2(c+dx)(a+ia\cot(e+fx))} \\
&\quad + \frac{(if^2 \cos(2e - \frac{2cf}{d})) \int \frac{\sin(\frac{2cf}{d}+2fx)}{c+dx} dx}{ad^2} + \frac{(f^2 \cos(2e - \frac{2cf}{d})) \int \frac{\cos(\frac{2cf}{d}+2fx)}{c+dx} dx}{ad^2} \\
&\quad + \frac{(if^2 \sin(2e - \frac{2cf}{d})) \int \frac{\cos(\frac{2cf}{d}+2fx)}{c+dx} dx}{ad^2} - \frac{(f^2 \sin(2e - \frac{2cf}{d})) \int \frac{\sin(\frac{2cf}{d}+2fx)}{c+dx} dx}{ad^2} \\
&= \frac{if}{2ad^2(c+dx)} + \frac{f^2 \cos(2e - \frac{2cf}{d}) \text{CosIntegral}(\frac{2cf}{d} + 2fx)}{ad^3} \\
&\quad - \frac{1}{2d(c+dx)^2(a+ia\cot(e+fx))} - \frac{if}{d^2(c+dx)(a+ia\cot(e+fx))} \\
&\quad + \frac{if^2 \text{CosIntegral}(\frac{2cf}{d} + 2fx) \sin(2e - \frac{2cf}{d})}{ad^3} \\
&\quad + \frac{if^2 \cos(2e - \frac{2cf}{d}) \text{Si}(\frac{2cf}{d} + 2fx)}{ad^3} - \frac{f^2 \sin(2e - \frac{2cf}{d}) \text{Si}(\frac{2cf}{d} + 2fx)}{ad^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.25

$$\int \frac{1}{(c+dx)^3(a+ia \cot(e+fx))} dx$$

$$= \frac{(\cos(e+f(-\frac{c}{d}+x)) + i \sin(e+f(-\frac{c}{d}+x))) \left(4f^2(c+dx)^2 \operatorname{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)\right) \left(\cos\left(e - \frac{f(c+dx)}{d}\right)\right)}{}$$

[In] Integrate[1/((c + d*x)^3*(a + I*a*Cot[e + f*x])),x]

[Out] ((Cos[e + f*(-(c/d) + x)] + I*Sin[e + f*(-(c/d) + x)])*(4*f^2*(c + d*x)^2*CosIntegral[(2*f*(c + d*x))/d]*(Cos[e - (f*(c + d*x))/d] + I*Sin[e - (f*(c + d*x))/d]) + I*(d*(I*d*Cos[e + f*(-(c/d) + x)] + ((-I)*d + 2*c*f + 2*d*f*x)*Cos[e + f*(c/d + x)] + d*Sin[e + f*(-(c/d) + x)] + d*Sin[e + f*(c/d + x)] + (2*I)*c*f*Sin[e + f*(c/d + x)] + (2*I)*d*f*x*Sin[e + f*(c/d + x)]) + 4*f^2*(c + d*x)^2*(Cos[e - (f*(c + d*x))/d] + I*Sin[e - (f*(c + d*x))/d])*SinIntegral[(2*f*(c + d*x))/d]))/(4*a*d^3*(c + d*x)^2)

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{1}{4d(dx+c)^2a} - \frac{f^2 e^{2i(fx+e)}}{4a d^3 \left(i f x + \frac{icf}{d} \right)^2} - \frac{f^2 e^{2i(fx+e)}}{2a d^3 \left(i f x + \frac{icf}{d} \right)} - \frac{f^2 e^{-\frac{2i(cf-de)}{d}} \operatorname{Ei}_1 \left(-2i f x - 2ie - \frac{2(icf-ide)}{d} \right)}{a d^3}$ $f^2 \left(i \left(-\frac{\sin(2fx+2e)}{(cf-de+d(fx+e))^2 d} + \frac{2 \cos(2fx+2e)}{(cf-de+d(fx+e))d} - \frac{2 \left(\frac{2 \operatorname{Si} \left(2fx+2e + \frac{2cf-2de}{d} \right) \cos \left(\frac{2cf-2de}{d} \right) - 2 \operatorname{Ci} \left(2fx+2e + \frac{2cf-2de}{d} \right) \sin \left(\frac{2cf-2de}{d} \right)}{d} \right)}{d} \right) \right)$
derivativedivides	$f^2 \left(i \left(-\frac{\sin(2fx+2e)}{(cf-de+d(fx+e))^2 d} + \frac{2 \cos(2fx+2e)}{(cf-de+d(fx+e))d} - \frac{2 \left(\frac{2 \operatorname{Si} \left(2fx+2e + \frac{2cf-2de}{d} \right) \cos \left(\frac{2cf-2de}{d} \right) - 2 \operatorname{Ci} \left(2fx+2e + \frac{2cf-2de}{d} \right) \sin \left(\frac{2cf-2de}{d} \right)}{d} \right)}{d} \right) \right)$
default	$f^2 \left(i \left(-\frac{\sin(2fx+2e)}{(cf-de+d(fx+e))^2 d} + \frac{2 \cos(2fx+2e)}{(cf-de+d(fx+e))d} - \frac{2 \left(\frac{2 \operatorname{Si} \left(2fx+2e + \frac{2cf-2de}{d} \right) \cos \left(\frac{2cf-2de}{d} \right) - 2 \operatorname{Ci} \left(2fx+2e + \frac{2cf-2de}{d} \right) \sin \left(\frac{2cf-2de}{d} \right)}{d} \right)}{d} \right) \right)$

[In] `int(1/(d*x+c)^3/(a+I*a*cot(f*x+e)),x,method=_RETURNVERBOSE)`

[Out] $-\frac{1}{4} \frac{1}{d} \frac{1}{(d*x+c)^2} \frac{1}{a} - \frac{1}{4} \frac{f^2}{a} \frac{1}{d^3} \exp(2*I*(f*x+e)) / (I*f*x + I/d*c*f)^2 - \frac{1}{2} \frac{f^2}{a} \frac{1}{d^3} \exp(2*I*(f*x+e)) / (I*f*x + I/d*c*f) - f^2 \frac{1}{a} \frac{1}{d^3} \exp(-2*I*(c*f-d*e)/d) * \operatorname{Ei} \left(1, -2*I*f*x - 2*I*e - 2*(I*c*f - I*d*e)/d \right)$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.53

$$\int \frac{1}{(c+dx)^3(a+ia \cot(e+fx))} dx$$

$$= \frac{4(d^2 f^2 x^2 + 2cdf^2 x + c^2 f^2) \operatorname{Ei} \left(-\frac{2(-idf x - icf)}{d} \right) e^{\left(-\frac{2(-ide+icf)}{d} \right)} - d^2 + (2i d^2 f x + 2i cdf + d^2) e^{(2i f x + 2i e)}}{4(ad^5 x^2 + 2acd^4 x + ac^2 d^3)}$$

[In] `integrate(1/(d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="fricas")`

[Out] $\frac{1}{4} * (4 * (d^2 * f^2 * x^2 + 2 * c * d * f^2 * x + c^2 * f^2) * \operatorname{Ei}(-2 * (-I * d * f * x - I * c * f) / d) * e^{(-2 * (-I * d * e + I * c * f) / d)} - d^2 + (2 * I * d^2 * f * x + 2 * I * c * d * f + d^2) * e^{(2 * I * f * x + 2 * I * e)}) / (a * d^5 * x^2 + 2 * a * c * d^4 * x + a * c^2 * d^3)$

SymPy [F]

$$\int \frac{1}{(c+dx)^3(a+ia \cot(e+fx))} dx$$

$$= \frac{i \int \frac{1}{c^3 \cot(e+fx) - ic^3 + 3c^2 dx \cot(e+fx) - 3ic^2 dx + 3cd^2 x^2 \cot(e+fx) - 3icd^2 x^2 + d^3 x^3 \cot(e+fx) - id^3 x^3} dx}{a}$$

[In] integrate(1/(d*x+c)**3/(a+I*a*cot(f*x+e)),x)

[Out] -I*Integral(1/(c**3*cot(e + f*x) - I*c**3 + 3*c**2*d*x*cot(e + f*x) - 3*I*c**2*d*x + 3*c*d**2*x**2*cot(e + f*x) - 3*I*c*d**2*x**2 + d**3*x**3*cot(e + f*x) - I*d**3*x**3), x)/a

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.70

$$\int \frac{1}{(c+dx)^3(a+ia \cot(e+fx))} dx$$

$$= \frac{2 f^3 \cos\left(-\frac{2(de-cf)}{d}\right) E_3\left(\frac{2(-i(fx+e)d+i de-icf)}{d}\right) - 2i f^3 E_3\left(\frac{2(-i(fx+e)d+i de-icf)}{d}\right) \sin\left(-\frac{2(de-cf)}{d}\right) - f^3}{4((fx+e)^2 ad^3 + ad^3 e^2 - 2acd^2 ef + ac^2 df^2 - 2(ad^3 e - acd^2 f)(fx+e))f}$$

[In] integrate(1/(d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="maxima")

[Out] 1/4*(2*f^3*cos(-2*(d*e - c*f)/d)*exp_integral_e(3, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) - 2*I*f^3*exp_integral_e(3, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d)*sin(-2*(d*e - c*f)/d) - f^3/(((f*x + e)^2*a*d^3 + a*d^3*e^2 - 2*a*c*d^2*e*f + a*c^2*d*f^2 - 2*(a*d^3*e - a*c*d^2*f)*(f*x + e))*f)

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1558 vs. 2(210) = 420.

Time = 0.30 (sec) , antiderivative size = 1558, normalized size of antiderivative = 6.86

$$\int \frac{1}{(c+dx)^3(a+ia \cot(e+fx))} dx = \text{Too large to display}$$

[In] integrate(1/(d*x+c)^3/(a+I*a*cot(f*x+e)),x, algorithm="giac")

[Out] 1/4*(4*d^2*f^2*x^2*cos(e)^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d) + 8*I*d^2*f^2*x^2*cos(e)*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)

$$\begin{aligned}
& - 4*d^2*f^2*x^2*\cos(2*c*f/d)*\cos_integral(2*(d*f*x + c*f)/d)*\sin(e)^2 - 4*I \\
& *d^2*f^2*x^2*\cos(e)^2*\cos_integral(2*(d*f*x + c*f)/d)*\sin(2*c*f/d) + 8*d^2* \\
& f^2*x^2*\cos(e)*\cos_integral(2*(d*f*x + c*f)/d)*\sin(e)*\sin(2*c*f/d) + 4*I*d^ \\
& 2*f^2*x^2*\cos_integral(2*(d*f*x + c*f)/d)*\sin(e)^2*\sin(2*c*f/d) + 4*I*d^2*f \\
& ^2*x^2*\cos(e)^2*\cos(2*c*f/d)*\sin_integral(2*(d*f*x + c*f)/d) - 8*d^2*f^2*x^ \\
& 2*\cos(e)*\cos(2*c*f/d)*\sin(e)*\sin_integral(2*(d*f*x + c*f)/d) - 4*I*d^2*f^2* \\
& x^2*\cos(2*c*f/d)*\sin(e)^2*\sin_integral(2*(d*f*x + c*f)/d) + 4*d^2*f^2*x^2*c \\
& os(e)^2*\sin(2*c*f/d)*\sin_integral(2*(d*f*x + c*f)/d) + 8*I*d^2*f^2*x^2*\cos(\\
& e)*\sin(e)*\sin(2*c*f/d)*\sin_integral(2*(d*f*x + c*f)/d) - 4*d^2*f^2*x^2*\sin(\\
& e)^2*\sin(2*c*f/d)*\sin_integral(2*(d*f*x + c*f)/d) + 8*c*d*f^2*x*\cos(e)^2*c \\
& os(2*c*f/d)*\cos_integral(2*(d*f*x + c*f)/d) + 16*I*c*d*f^2*x*\cos(e)*\cos(2*c* \\
& f/d)*\cos_integral(2*(d*f*x + c*f)/d)*\sin(e) - 8*c*d*f^2*x*\cos(2*c*f/d)*\cos_ \\
& integral(2*(d*f*x + c*f)/d)*\sin(e)^2 - 8*I*c*d*f^2*x*\cos(e)^2*\cos_integral(\\
& 2*(d*f*x + c*f)/d)*\sin(2*c*f/d) + 16*c*d*f^2*x*\cos(e)*\cos_integral(2*(d*f*x \\
& + c*f)/d)*\sin(e)*\sin(2*c*f/d) + 8*I*c*d*f^2*x*\cos_integral(2*(d*f*x + c*f) \\
& /d)*\sin(e)^2*\sin(2*c*f/d) + 8*I*c*d*f^2*x*\cos(e)^2*\cos(2*c*f/d)*\sin_integra \\
& l(2*(d*f*x + c*f)/d) - 16*c*d*f^2*x*\cos(e)*\cos(2*c*f/d)*\sin(e)*\sin_integral \\
& (2*(d*f*x + c*f)/d) - 8*I*c*d*f^2*x*\cos(2*c*f/d)*\sin(e)^2*\sin_integral(2*(d \\
& *f*x + c*f)/d) + 8*c*d*f^2*x*\cos(e)^2*\sin(2*c*f/d)*\sin_integral(2*(d*f*x + \\
& c*f)/d) + 16*I*c*d*f^2*x*\cos(e)*\sin(e)*\sin(2*c*f/d)*\sin_integral(2*(d*f*x + \\
& c*f)/d) - 8*c*d*f^2*x*\sin(e)^2*\sin(2*c*f/d)*\sin_integral(2*(d*f*x + c*f)/d \\
&) + 4*c^2*f^2*\cos(e)^2*\cos(2*c*f/d)*\cos_integral(2*(d*f*x + c*f)/d) + 8*I*c \\
& ^2*f^2*\cos(e)*\cos(2*c*f/d)*\cos_integral(2*(d*f*x + c*f)/d)*\sin(e) - 4*c^2*f \\
& ^2*\cos(2*c*f/d)*\cos_integral(2*(d*f*x + c*f)/d)*\sin(e)^2 - 4*I*c^2*f^2*\cos(\\
& e)^2*\cos_integral(2*(d*f*x + c*f)/d)*\sin(2*c*f/d) + 8*c^2*f^2*\cos(e)*\cos_in \\
& tegral(2*(d*f*x + c*f)/d)*\sin(e)*\sin(2*c*f/d) + 4*I*c^2*f^2*\cos_integral(2* \\
& (d*f*x + c*f)/d)*\sin(e)^2*\sin(2*c*f/d) + 4*I*c^2*f^2*\cos(e)^2*\cos(2*c*f/d)* \\
& \sin_integral(2*(d*f*x + c*f)/d) - 8*c^2*f^2*\cos(e)*\cos(2*c*f/d)*\sin(e)*\sin_ \\
& integral(2*(d*f*x + c*f)/d) - 4*I*c^2*f^2*\cos(2*c*f/d)*\sin(e)^2*\sin_integra \\
& l(2*(d*f*x + c*f)/d) + 4*c^2*f^2*\cos(e)^2*\sin(2*c*f/d)*\sin_integral(2*(d*f* \\
& x + c*f)/d) + 8*I*c^2*f^2*\cos(e)*\sin(e)*\sin(2*c*f/d)*\sin_integral(2*(d*f*x \\
& + c*f)/d) - 4*c^2*f^2*\sin(e)^2*\sin(2*c*f/d)*\sin_integral(2*(d*f*x + c*f)/d) \\
& + 2*I*d^2*f*x*\cos(2*f*x)*\cos(e)^2 - 2*d^2*f*x*\cos(e)^2*\sin(2*f*x) - 4*d^2* \\
& f*x*\cos(2*f*x)*\cos(e)*\sin(e) - 4*I*d^2*f*x*\cos(e)*\sin(2*f*x)*\sin(e) - 2*I*d \\
& ^2*f*x*\cos(2*f*x)*\sin(e)^2 + 2*d^2*f*x*\sin(2*f*x)*\sin(e)^2 + 2*I*c*d*f*\cos(\\
& 2*f*x)*\cos(e)^2 - 2*c*d*f*\cos(e)^2*\sin(2*f*x) - 4*c*d*f*\cos(2*f*x)*\cos(e)*s \\
& in(e) - 4*I*c*d*f*\cos(e)*\sin(2*f*x)*\sin(e) - 2*I*c*d*f*\cos(2*f*x)*\sin(e)^2 \\
& + 2*c*d*f*\sin(2*f*x)*\sin(e)^2 + d^2*\cos(2*f*x)*\cos(e)^2 + I*d^2*\cos(e)^2*si \\
& n(2*f*x) + 2*I*d^2*\cos(2*f*x)*\cos(e)*\sin(e) - 2*d^2*\cos(e)*\sin(2*f*x)*\sin(e \\
&) - d^2*\cos(2*f*x)*\sin(e)^2 - I*d^2*\sin(2*f*x)*\sin(e)^2 - d^2)/(a*d^5*x^2 + \\
& 2*a*c*d^4*x + a*c^2*d^3)
\end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c + dx)^3(a + ia \cot(e + fx))} dx = \int \frac{1}{(a + a \cot(e + fx) 1i) (c + dx)^3} dx$$

```
[In] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)^3), x)
```

```
[Out] int(1/((a + a*cot(e + f*x)*1i)*(c + d*x)^3), x)
```

3.22 $\int \frac{(c+dx)^3}{(a+ia \cot(e+fx))^2} dx$

Optimal result	148
Rubi [A] (verified)	148
Mathematica [A] (verified)	150
Maple [A] (verified)	151
Fricas [A] (verification not implemented)	151
Sympy [A] (verification not implemented)	152
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Giac [B] (verification not implemented)	153
Mupad [B] (verification not implemented)	153

Optimal result

Integrand size = 23, antiderivative size = 270

$$\int \frac{(c+dx)^3}{(a+ia \cot(e+fx))^2} dx = \frac{3d^3 e^{2ie+2ifx}}{16a^2 f^4} - \frac{3d^3 e^{4ie+4ifx}}{512a^2 f^4} - \frac{3id^2 e^{2ie+2ifx}(c+dx)}{8a^2 f^3} + \frac{3id^2 e^{4ie+4ifx}(c+dx)}{128a^2 f^3} - \frac{3de^{2ie+2ifx}(c+dx)^2}{8a^2 f^2} + \frac{3de^{4ie+4ifx}(c+dx)^2}{64a^2 f^2} + \frac{ie^{2ie+2ifx}(c+dx)^3}{4a^2 f} - \frac{ie^{4ie+4ifx}(c+dx)^3}{16a^2 f} + \frac{(c+dx)^4}{16a^2 d}$$

[Out] 3/16*d^3*exp(2*I*e+2*I*f*x)/a^2/f^4-3/512*d^3*exp(4*I*e+4*I*f*x)/a^2/f^4-3/8*I*d^2*exp(2*I*e+2*I*f*x)*(d*x+c)/a^2/f^3+3/128*I*d^2*exp(4*I*e+4*I*f*x)*(d*x+c)/a^2/f^3-3/8*d*exp(2*I*e+2*I*f*x)*(d*x+c)^2/a^2/f^2+3/64*d*exp(4*I*e+4*I*f*x)*(d*x+c)^2/a^2/f^2+1/4*I*exp(2*I*e+2*I*f*x)*(d*x+c)^3/a^2/f-1/16*I*exp(4*I*e+4*I*f*x)*(d*x+c)^3/a^2/f+1/16*(d*x+c)^4/a^2/d

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used

= {3810, 2207, 2225}

$$\int \frac{(c+dx)^3}{(a+ia \cot(e+fx))^2} dx = -\frac{3id^2(c+dx)e^{2ie+2ifx}}{8a^2f^3} + \frac{3id^2(c+dx)e^{4ie+4ifx}}{128a^2f^3} - \frac{3d(c+dx)^2e^{2ie+2ifx}}{8a^2f^2} + \frac{3d(c+dx)^2e^{4ie+4ifx}}{64a^2f^2} + \frac{i(c+dx)^3e^{2ie+2ifx}}{4a^2f} - \frac{i(c+dx)^3e^{4ie+4ifx}}{16a^2f} + \frac{(c+dx)^4}{16a^2d} + \frac{3d^3e^{2ie+2ifx}}{16a^2f^4} - \frac{3d^3e^{4ie+4ifx}}{512a^2f^4}$$

[In] Int[(c + d*x)^3/(a + I*a*Cot[e + f*x])^2,x]

[Out] (3*d^3*E^((2*I)*e + (2*I)*f*x))/(16*a^2*f^4) - (3*d^3*E^((4*I)*e + (4*I)*f*x))/(512*a^2*f^4) - (((3*I)/8)*d^2*E^((2*I)*e + (2*I)*f*x)*(c + d*x))/(a^2*f^3) + (((3*I)/128)*d^2*E^((4*I)*e + (4*I)*f*x)*(c + d*x))/(a^2*f^3) - (3*d*E^((2*I)*e + (2*I)*f*x)*(c + d*x)^2)/(8*a^2*f^2) + (3*d*E^((4*I)*e + (4*I)*f*x)*(c + d*x)^2)/(64*a^2*f^2) + ((I/4)*E^((2*I)*e + (2*I)*f*x)*(c + d*x)^3)/(a^2*f) - ((I/16)*E^((4*I)*e + (4*I)*f*x)*(c + d*x)^3)/(a^2*f) + (c + d*x)^4/(16*a^2*d)

Rule 2207

Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[\$UseGamma]

Rule 2225

Int[(F)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 3810

Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*x)))/(2*a)]^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, 0]

Rubi steps

$$\text{integral} = \int \left(\frac{(c+dx)^3}{4a^2} - \frac{e^{2ie+2ifx}(c+dx)^3}{2a^2} + \frac{e^{4ie+4ifx}(c+dx)^3}{4a^2} \right) dx$$

$$\begin{aligned}
&= \frac{(c+dx)^4}{16a^2d} + \frac{\int e^{4ie+4ifx}(c+dx)^3 dx}{4a^2} - \frac{\int e^{2ie+2ifx}(c+dx)^3 dx}{2a^2} \\
&= \frac{ie^{2ie+2ifx}(c+dx)^3}{4a^2f} - \frac{ie^{4ie+4ifx}(c+dx)^3}{16a^2f} + \frac{(c+dx)^4}{16a^2d} \\
&\quad + \frac{(3id) \int e^{4ie+4ifx}(c+dx)^2 dx}{16a^2f} - \frac{(3id) \int e^{2ie+2ifx}(c+dx)^2 dx}{4a^2f} \\
&= -\frac{3de^{2ie+2ifx}(c+dx)^2}{8a^2f^2} + \frac{3de^{4ie+4ifx}(c+dx)^2}{64a^2f^2} \\
&\quad + \frac{ie^{2ie+2ifx}(c+dx)^3}{4a^2f} - \frac{ie^{4ie+4ifx}(c+dx)^3}{16a^2f} + \frac{(c+dx)^4}{16a^2d} \\
&\quad - \frac{(3d^2) \int e^{4ie+4ifx}(c+dx) dx}{32a^2f^2} + \frac{(3d^2) \int e^{2ie+2ifx}(c+dx) dx}{4a^2f^2} \\
&= -\frac{3id^2e^{2ie+2ifx}(c+dx)}{8a^2f^3} + \frac{3id^2e^{4ie+4ifx}(c+dx)}{128a^2f^3} - \frac{3de^{2ie+2ifx}(c+dx)^2}{8a^2f^2} \\
&\quad + \frac{3de^{4ie+4ifx}(c+dx)^2}{64a^2f^2} + \frac{ie^{2ie+2ifx}(c+dx)^3}{4a^2f} - \frac{ie^{4ie+4ifx}(c+dx)^3}{16a^2f} \\
&\quad + \frac{(c+dx)^4}{16a^2d} - \frac{(3id^3) \int e^{4ie+4ifx} dx}{128a^2f^3} + \frac{(3id^3) \int e^{2ie+2ifx} dx}{8a^2f^3} \\
&= \frac{3d^3e^{2ie+2ifx}}{16a^2f^4} - \frac{3d^3e^{4ie+4ifx}}{512a^2f^4} - \frac{3id^2e^{2ie+2ifx}(c+dx)}{8a^2f^3} \\
&\quad + \frac{3id^2e^{4ie+4ifx}(c+dx)}{128a^2f^3} - \frac{3de^{2ie+2ifx}(c+dx)^2}{8a^2f^2} + \frac{3de^{4ie+4ifx}(c+dx)^2}{64a^2f^2} \\
&\quad + \frac{ie^{2ie+2ifx}(c+dx)^3}{4a^2f} - \frac{ie^{4ie+4ifx}(c+dx)^3}{16a^2f} + \frac{(c+dx)^4}{16a^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.17 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx)^3}{(a+ia \cot(e+fx))^2} dx$$

$$= \frac{(\cos(2(e+fx)) + i \sin(2(e+fx))) ((32c^3f^3(-i+4fx) + 24c^2df^2(1-4ifx+8f^2x^2) + 4cd^2f(3i+12fx -$$

[In] Integrate[(c + d*x)^3/(a + I*a*Cot[e + f*x])^2,x]

[Out] ((Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*((32*c^3*f^3*(-I + 4*f*x) + 24*c^2*d*f^2*(1 - (4*I)*f*x + 8*f^2*x^2) + 4*c*d^2*f*(3*I + 12*f*x - (24*I)*f^2*x^2 + 32*f^3*x^3) + d^3*(-3 + (12*I)*f*x + 24*f^2*x^2 - (32*I)*f^3*x^3 + 32*f^4*x^4))*Cos[2*(e + f*x)] - I*(-32*(4*c^3*f^3 + 6*c^2*d*f^2*(I + 2*f*x) + 6*c*d^2*f*(-1 + (2*I)*f*x + 2*f^2*x^2) + d^3*(-3*I - 6*f*x + (6*I)*f^2*x^2

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(204) = 408$.

Time = 0.32 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.53

$$\int \frac{(c + dx)^3}{(a + ia \cot(e + fx))^2} dx$$

$$= \frac{32 d^3 f^4 x^4 + 128 cd^2 f^4 x^3 - 32i d^3 f^3 x^3 e^{(4i fx + 4i e)} + 128i d^3 f^3 x^3 e^{(2i fx + 2i e)} + 192 c^2 d f^4 x^2 - 96i cd^2 f^3 x^2 e^{(4i e)}}{}$$

[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")

[Out] $\frac{1}{512} (32 d^3 f^4 x^4 + 128 c d^2 f^4 x^3 - 32 I d^3 f^3 x^3 e^{(4 I f x + 4 I e)} + 128 I d^3 f^3 x^3 e^{(2 I f x + 2 I e)} + 192 c^2 d f^4 x^2 - 96 I c d^2 f^3 x^2 e^{(4 I f x + 4 I e)} + 384 I c d^2 f^3 x^2 e^{(2 I f x + 2 I e)} + 128 c^3 f^4 x - 96 I c^2 d f^3 x e^{(4 I f x + 4 I e)} + 24 d^3 f^2 x^2 e^{(4 I f x + 4 I e)} + 384 I c^2 d f^3 x e^{(2 I f x + 2 I e)} - 192 d^3 f^2 x^2 e^{(2 I f x + 2 I e)} - 32 I c^3 f^3 e^{(4 I f x + 4 I e)} + 48 c d^2 f^2 x e^{(4 I f x + 4 I e)} + 128 I c^3 f^3 e^{(2 I f x + 2 I e)} - 384 c d^2 f^2 x e^{(2 I f x + 2 I e)} + 24 c^2 d f^2 e^{(4 I f x + 4 I e)} + 12 I d^3 f x e^{(4 I f x + 4 I e)} - 192 c^2 d f^2 e^{(2 I f x + 2 I e)} - 192 I d^3 f x e^{(2 I f x + 2 I e)} + 12 I c d^2 f e^{(4 I f x + 4 I e)} - 192 I c d^2 f e^{(2 I f x + 2 I e)} - 3 d^3 e^{(4 I f x + 4 I e)} + 96 d^3 e^{(2 I f x + 2 I e)}) / (a^2 f^4)$

Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.09

$$\int \frac{(c + dx)^3}{(a + ia \cot(e + fx))^2} dx = e^{e^{2i+fx} 2i} \left(-\frac{(-4c^3 f^3 - c^2 d f^2 6i + 6cd^2 f + d^3 3i) li}{16a^2 f^4} + \frac{d^3 x^3 li}{4a^2 f} + \frac{dx(2c^2 f^2 + cdf 2i - d^2) 3i}{8a^2 f^3} + \frac{d^2 x^2 (2cf + d li) 3i}{8a^2 f^2} \right) - e^{e^{4i+fx} 4i} \left(-\frac{(-32c^3 f^3 - c^2 d f^2 24i + 12cd^2 f + d^3 3i) li}{512a^2 f^4} + \frac{d^3 x^3 li}{16a^2 f} + \frac{dx(8c^2 f^2 + cdf 4i - d^2) 3i}{128a^2 f^3} + \frac{d^2 x^2 (4cf + d li) 3i}{64a^2 f^2} \right) + \frac{c^3 x}{4a^2} + \frac{d^3 x^4}{16a^2} + \frac{3c^2 dx^2}{8a^2} + \frac{cd^2 x^3}{4a^2}$$

[In] int((c + d*x)^3/(a + a*cot(e + f*x)*1i)^2,x)

```
[Out] exp(e*2i + f*x*2i)*((d^3*x^3*1i)/(4*a^2*f) - ((d^3*3i - 4*c^3*f^3 - c^2*d*f
^2*6i + 6*c*d^2*f)*1i)/(16*a^2*f^4) + (d*x*(2*c^2*f^2 - d^2 + c*d*f*2i)*3i)
/(8*a^2*f^3) + (d^2*x^2*(d*1i + 2*c*f)*3i)/(8*a^2*f^2)) - exp(e*4i + f*x*4i
)*((d^3*x^3*1i)/(16*a^2*f) - ((d^3*3i - 32*c^3*f^3 - c^2*d*f^2*24i + 12*c*d
^2*f)*1i)/(512*a^2*f^4) + (d*x*(8*c^2*f^2 - d^2 + c*d*f*4i)*3i)/(128*a^2*f^
3) + (d^2*x^2*(d*1i + 4*c*f)*3i)/(64*a^2*f^2)) + (c^3*x)/(4*a^2) + (d^3*x^4
)/(16*a^2) + (3*c^2*d*x^2)/(8*a^2) + (c*d^2*x^3)/(4*a^2)
```

3.23 $\int \frac{(c+dx)^2}{(a+ia \cot(e+fx))^2} dx$

Optimal result	155
Rubi [A] (verified)	155
Mathematica [A] (verified)	157
Maple [A] (verified)	157
Fricas [A] (verification not implemented)	158
Sympy [A] (verification not implemented)	158
Maxima [F(-2)]	159
Giac [A] (verification not implemented)	159
Mupad [B] (verification not implemented)	159

Optimal result

Integrand size = 23, antiderivative size = 202

$$\int \frac{(c+dx)^2}{(a+ia \cot(e+fx))^2} dx = -\frac{id^2e^{2ie+2ifx}}{8a^2f^3} + \frac{id^2e^{4ie+4ifx}}{128a^2f^3} - \frac{de^{2ie+2ifx}(c+dx)}{4a^2f^2} + \frac{de^{4ie+4ifx}(c+dx)}{32a^2f^2} + \frac{ie^{2ie+2ifx}(c+dx)^2}{4a^2f} - \frac{ie^{4ie+4ifx}(c+dx)^2}{16a^2f} + \frac{(c+dx)^3}{12a^2d}$$

[Out] $-1/8*I*d^2*\exp(2*I*e+2*I*f*x)/a^2/f^3+1/128*I*d^2*\exp(4*I*e+4*I*f*x)/a^2/f^3-1/4*d*\exp(2*I*e+2*I*f*x)*(d*x+c)/a^2/f^2+1/32*d*\exp(4*I*e+4*I*f*x)*(d*x+c)/a^2/f^2+1/4*I*\exp(2*I*e+2*I*f*x)*(d*x+c)^2/a^2/f-1/16*I*\exp(4*I*e+4*I*f*x)*(d*x+c)^2/a^2/f+1/12*(d*x+c)^3/a^2/d$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3810, 2207, 2225}

$$\int \frac{(c+dx)^2}{(a+ia \cot(e+fx))^2} dx = -\frac{d(c+dx)e^{2ie+2ifx}}{4a^2f^2} + \frac{d(c+dx)e^{4ie+4ifx}}{32a^2f^2} + \frac{i(c+dx)^2e^{2ie+2ifx}}{4a^2f} - \frac{i(c+dx)^2e^{4ie+4ifx}}{16a^2f} + \frac{(c+dx)^3}{12a^2d} - \frac{id^2e^{2ie+2ifx}}{8a^2f^3} + \frac{id^2e^{4ie+4ifx}}{128a^2f^3}$$

[In] $\text{Int}[(c+d*x)^2/(a+I*a*\text{Cot}[e+f*x])^2,x]$

[Out] $((-1/8*I)*d^2*E^((2*I)*e+(2*I)*f*x))/(a^2*f^3)+((I/128)*d^2*E^((4*I)*e+(4*I)*f*x))/(a^2*f^3)-(d*E^((2*I)*e+(2*I)*f*x)*(c+d*x))/(4*a^2*f^2)$

$$+ (d * E^{(4 * I) * e + (4 * I) * f * x} * (c + d * x)) / (32 * a^2 * f^2) + ((I / 4) * E^{(2 * I) * e + (2 * I) * f * x} * (c + d * x)^2) / (a^2 * f) - ((I / 16) * E^{(4 * I) * e + (4 * I) * f * x} * (c + d * x)^2) / (a^2 * f) + (c + d * x)^3 / (12 * a^2 * d)$$

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x]
/; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol]
:> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3810

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Symbol]
:> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*x)))/(2*a))^(n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{(c + dx)^2}{4a^2} - \frac{e^{2ie+2ifx}(c + dx)^2}{2a^2} + \frac{e^{4ie+4ifx}(c + dx)^2}{4a^2} \right) dx \\ &= \frac{(c + dx)^3}{12a^2d} + \frac{\int e^{4ie+4ifx}(c + dx)^2 dx}{4a^2} - \frac{\int e^{2ie+2ifx}(c + dx)^2 dx}{2a^2} \\ &= \frac{ie^{2ie+2ifx}(c + dx)^2}{4a^2f} - \frac{ie^{4ie+4ifx}(c + dx)^2}{16a^2f} + \frac{(c + dx)^3}{12a^2d} \\ &\quad + \frac{(id) \int e^{4ie+4ifx}(c + dx) dx}{8a^2f} - \frac{(id) \int e^{2ie+2ifx}(c + dx) dx}{2a^2f} \\ &= -\frac{de^{2ie+2ifx}(c + dx)}{4a^2f^2} + \frac{de^{4ie+4ifx}(c + dx)}{32a^2f^2} + \frac{ie^{2ie+2ifx}(c + dx)^2}{4a^2f} \\ &\quad - \frac{ie^{4ie+4ifx}(c + dx)^2}{16a^2f} + \frac{(c + dx)^3}{12a^2d} - \frac{d^2 \int e^{4ie+4ifx} dx}{32a^2f^2} + \frac{d^2 \int e^{2ie+2ifx} dx}{4a^2f^2} \\ &= -\frac{id^2e^{2ie+2ifx}}{8a^2f^3} + \frac{id^2e^{4ie+4ifx}}{128a^2f^3} - \frac{de^{2ie+2ifx}(c + dx)}{4a^2f^2} + \frac{de^{4ie+4ifx}(c + dx)}{32a^2f^2} \\ &\quad + \frac{ie^{2ie+2ifx}(c + dx)^2}{4a^2f} - \frac{ie^{4ie+4ifx}(c + dx)^2}{16a^2f} + \frac{(c + dx)^3}{12a^2d} \end{aligned}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.26

$$\int \frac{(c + dx)^2}{(a + ia \cot(e + fx))^2} dx$$

$$= \frac{32f^3x(3c^2 + 3cdx + d^2x^2) + 48((1+i)cf + d(-1 + (1+i)fx))((1+i)cf + d(i + (1+i)fx)) \cos(2fx)(c + dx) + \dots}{(384a^2f^3)}$$

[In] Integrate[(c + d*x)^2/(a + I*a*Cot[e + f*x])^2,x]

```
[Out] (32*f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 48*((1 + I)*c*f + d*(-1 + (1 + I)*f*x))*((1 + I)*c*f + d*(I + (1 + I)*f*x))*Cos[2*f*x]*(Cos[2*e] + I*Sin[2*e]) - 3*((2 + 2*I)*c*f + d*(-1 + (2 + 2*I)*f*x))*((2 + 2*I)*c*f + d*(I + (2 + 2*I)*f*x))*Cos[4*f*x]*(Cos[4*e] + I*Sin[4*e]) + (48*I)*((1 + I)*c*f + d*(-1 + (1 + I)*f*x))*((1 + I)*c*f + d*(I + (1 + I)*f*x))*Cos[2*e] + I*Sin[2*e])*Sin[2*f*x] - 3*(d - (2 + 2*I)*c*f - (2 + 2*I)*d*f*x)*(d + (2 - 2*I)*c*f + (2 - 2*I)*d*f*x)*(Cos[4*e] + I*Sin[4*e])*Sin[4*f*x])/(384*a^2*f^3)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.86

method	result
risch	$\frac{d^2x^3}{12a^2} + \frac{dcx^2}{4a^2} + \frac{c^2x}{4a^2} + \frac{c^3}{12a^2d} - \frac{i(8d^2x^2f^2 + 16cdf^2x + 4id^2fx + 8c^2f^2 + 4icdf - d^2)e^{4i(fx+e)}}{128a^2f^3} + \frac{i(2d^2x^2f^2 + 4cdf^2x + 2c^2f^2)}{96f^3a^2}(-1 + 2i \tan(fx+e))$
parallelrisch	$\frac{-60\left(-\frac{2}{15}d^2x^2 - \frac{2}{5}cdx - \frac{2}{5}c^2\right)f^2 + i\left(\frac{dx}{2} + c\right)df - \frac{9d^2}{20}fx \tan(fx+e)^2 + \left(48i\left(\frac{1}{3}d^2x^2 + cdx + c^2\right)xf^3 + (-12d^2x^2 - 24cdx - 72c^2)f^2\right)}{96f^3a^2(-1 + 2i \tan(fx+e))}$

[In] int((d*x+c)^2/(a+I*a*cot(f*x+e))^2,x,method=_RETURNVERBOSE)

```
[Out] 1/12/a^2*d^2*x^3+1/4/a^2*d*c*x^2+1/4/a^2*c^2*x+1/12/a^2/d*c^3-1/128*I*(8*d^2*x^2*f^2+4*I*d^2*f*x+16*c*d*f^2*x+4*I*c*d*f+8*c^2*f^2-d^2)/a^2/f^3*exp(4*I*(f*x+e))+1/8*I*(2*d^2*x^2*f^2+2*I*d^2*f*x+4*c*d*f^2*x+2*I*c*d*f+2*c^2*f^2-d^2)/a^2/f^3*exp(2*I*(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.76

$$\int \frac{(c + dx)^2}{(a + ia \cot(e + fx))^2} dx$$

$$= \frac{32 d^2 f^3 x^3 + 96 c d f^3 x^2 + 96 c^2 f^3 x - 3(8i d^2 f^2 x^2 + 8i c^2 f^2 - 4 c d f - i d^2 + 4(4i c d f^2 - d^2 f)x) e^{(4i f x + 4i e)} - 384 a^2 f^3}{384 a^2 f^3}$$

[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")

```
[Out] 1/384*(32*d^2*f^3*x^3 + 96*c*d*f^3*x^2 + 96*c^2*f^3*x - 3*(8*I*d^2*f^2*x^2 + 8*I*c^2*f^2 - 4*c*d*f - I*d^2 + 4*(4*I*c*d*f^2 - d^2*f)*x)*e^(4*I*f*x + 4*I*e) - 48*(-2*I*d^2*f^2*x^2 - 2*I*c^2*f^2 + 2*c*d*f + I*d^2 + 2*(-2*I*c*d*f^2 + d^2*f)*x)*e^(2*I*f*x + 2*I*e))/(a^2*f^3)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.00

$$\int \frac{(c + dx)^2}{(a + ia \cot(e + fx))^2} dx$$

$$= \left\{ \frac{(256ia^2c^2f^5e^{2ie} + 512ia^2cdf^5xe^{2ie} - 256a^2cdf^4e^{2ie} + 256ia^2d^2f^5x^2e^{2ie} - 256a^2d^2f^4xe^{2ie} - 128ia^2d^2f^3e^{2ie})e^{2ifx} + (-64ia^2c^2f^5e^{4ie} - 128ia^2cdf^5e^{4ie} + 32a^2c^2f^5e^{4ie} + 32a^2d^2f^5x^2e^{4ie} - 64a^2cdf^4xe^{4ie} - 128a^2d^2f^3e^{4ie})e^{2ifx}}{1024a^4f^6} \right.$$

$$\left. + \frac{x^3(d^2e^{4ie} - 2d^2e^{2ie})}{12a^2} + \frac{x^2(cde^{4ie} - 2cde^{2ie})}{4a^2} + \frac{x(c^2e^{4ie} - 2c^2e^{2ie})}{4a^2} + \frac{c^2x}{4a^2} + \frac{cdx^2}{4a^2} + \frac{d^2x^3}{12a^2} \right.$$

[In] integrate((d*x+c)**2/(a+I*a*cot(f*x+e))**2,x)

```
[Out] Piecewise((((256*I*a**2*c**2*f**5*exp(2*I*e) + 512*I*a**2*c*d*f**5*x*exp(2*I*e) - 256*a**2*c*d*f**4*exp(2*I*e) + 256*I*a**2*d**2*f**5*x**2*exp(2*I*e) - 256*a**2*d**2*f**4*x*exp(2*I*e) - 128*I*a**2*d**2*f**3*exp(2*I*e))*exp(2*I*f*x) + (-64*I*a**2*c**2*f**5*exp(4*I*e) - 128*I*a**2*c*d*f**5*x*exp(4*I*e) + 32*a**2*c*d*f**4*exp(4*I*e) - 64*I*a**2*d**2*f**5*x**2*exp(4*I*e) + 32*a**2*d**2*f**4*x*exp(4*I*e) + 8*I*a**2*d**2*f**3*exp(4*I*e))*exp(4*I*f*x))/(1024*a**4*f**6), Ne(a**4*f**6, 0)), (x**3*(d**2*exp(4*I*e) - 2*d**2*exp(2*I*e))/(12*a**2) + x**2*(c*d*exp(4*I*e) - 2*c*d*exp(2*I*e))/(4*a**2) + x*(c**2*exp(4*I*e) - 2*c**2*exp(2*I*e))/(4*a**2), True)) + c**2*x/(4*a**2) + c*d*x**2/(4*a**2) + d**2*x**3/(12*a**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + dx)^2}{(a + ia \cot(e + fx))^2} dx = \text{Exception raised: RuntimeError}$$

[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.16

$$\int \frac{(c + dx)^2}{(a + ia \cot(e + fx))^2} dx = \frac{32 d^2 f^3 x^3 + 96 c d f^3 x^2 - 24 i d^2 f^2 x^2 e^{(4i f x + 4i e)} + 96 i d^2 f^2 x^2 e^{(2i f x + 2i e)} + 96 c^2 f^3 x - 48 i c d f^2 x e^{(4i f x + 4i e)} + \dots}{\dots}$$

[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")

[Out] 1/384*(32*d^2*f^3*x^3 + 96*c*d*f^3*x^2 - 24*I*d^2*f^2*x^2*e^(4*I*f*x + 4*I*e) + 96*I*d^2*f^2*x^2*e^(2*I*f*x + 2*I*e) + 96*c^2*f^3*x - 48*I*c*d*f^2*x*e^(4*I*f*x + 4*I*e) + 192*I*c*d*f^2*x*e^(2*I*f*x + 2*I*e) - 24*I*c^2*f^2*e^(4*I*f*x + 4*I*e) + 12*d^2*f*x*e^(4*I*f*x + 4*I*e) + 96*I*c^2*f^2*e^(2*I*f*x + 2*I*e) - 96*d^2*f*x*e^(2*I*f*x + 2*I*e) + 12*c*d*f*e^(4*I*f*x + 4*I*e) - 96*c*d*f*e^(2*I*f*x + 2*I*e) + 3*I*d^2*e^(4*I*f*x + 4*I*e) - 48*I*d^2*e^(2*I*f*x + 2*I*e))/(a^2*f^3)

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.92

$$\int \frac{(c + dx)^2}{(a + ia \cot(e + fx))^2} dx = e^{e^{2i+fx} 2i} \left(\frac{(2c^2 f^2 + c d f 2i - d^2) li}{8 a^2 f^3} + \frac{d^2 x^2 li}{4 a^2 f} + \frac{d x (2 c f + d li) li}{4 a^2 f^2} \right) - e^{e^{4i+fx} 4i} \left(\frac{(8 c^2 f^2 + c d f 4i - d^2) li}{128 a^2 f^3} + \frac{d^2 x^2 li}{16 a^2 f} + \frac{d x (4 c f + d li) li}{32 a^2 f^2} \right) + \frac{c^2 x}{4 a^2} + \frac{d^2 x^3}{12 a^2} + \frac{c d x^2}{4 a^2}$$

[In] int((c + d*x)^2/(a + a*cot(e + f*x)*1i)^2,x)

```
[Out] exp(e*2i + f*x*2i)*(((2*c^2*f^2 - d^2 + c*d*f*2i)*1i)/(8*a^2*f^3) + (d^2*x^
2*1i)/(4*a^2*f) + (d*x*(d*1i + 2*c*f)*1i)/(4*a^2*f^2)) - exp(e*4i + f*x*4i)
*(((8*c^2*f^2 - d^2 + c*d*f*4i)*1i)/(128*a^2*f^3) + (d^2*x^2*1i)/(16*a^2*f)
+ (d*x*(d*1i + 4*c*f)*1i)/(32*a^2*f^2)) + (c^2*x)/(4*a^2) + (d^2*x^3)/(12*
a^2) + (c*d*x^2)/(4*a^2)
```


3.24 $\int \frac{c+dx}{(a+ia \cot(e+fx))^2} dx$

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Optimal result

Integrand size = 21, antiderivative size = 151

$$\int \frac{c+dx}{(a+ia \cot(e+fx))^2} dx = \frac{3idx}{16a^2f} - \frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} + \frac{d}{16f^2(a+ia \cot(e+fx))^2}$$

$$- \frac{i(c+dx)}{4f(a+ia \cot(e+fx))^2} + \frac{3d}{16f^2(a^2+ia^2 \cot(e+fx))}$$

$$- \frac{i(c+dx)}{4f(a^2+ia^2 \cot(e+fx))}$$

[Out] 3/16*I*d*x/a^2/f-1/8*d*x^2/a^2+1/4*x*(d*x+c)/a^2+1/16*d/f^2/(a+I*a*cot(f*x+e))^2-1/4*I*(d*x+c)/f/(a+I*a*cot(f*x+e))^2+3/16*d/f^2/(a^2+I*a^2*cot(f*x+e))-1/4*I*(d*x+c)/f/(a^2+I*a^2*cot(f*x+e))

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3560, 8, 3811}

$$\int \frac{c+dx}{(a+ia \cot(e+fx))^2} dx = -\frac{i(c+dx)}{4f(a^2+ia^2 \cot(e+fx))} + \frac{x(c+dx)}{4a^2}$$

$$+ \frac{3d}{16f^2(a^2+ia^2 \cot(e+fx))} + \frac{3idx}{16a^2f} - \frac{dx^2}{8a^2}$$

$$- \frac{i(c+dx)}{4f(a+ia \cot(e+fx))^2} + \frac{d}{16f^2(a+ia \cot(e+fx))^2}$$

[In] Int[(c + d*x)/(a + I*a*Cot[e + f*x])^2,x]

[Out] $((3I/16)d*x)/(a^2*f) - (d*x^2)/(8*a^2) + (x*(c + d*x))/(4*a^2) + d/(16*f^2*(a + I*a*Cot[e + f*x])^2) - ((I/4)*(c + d*x))/(f*(a + I*a*Cot[e + f*x])^2) + (3*d)/(16*f^2*(a^2 + I*a^2*Cot[e + f*x])) - ((I/4)*(c + d*x))/(f*(a^2 + I*a^2*Cot[e + f*x]))$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 3560

$\text{Int}[(a_) + (b_)*\tan[(c_) + (d_)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[a*((a + b*\tan[c + d*x])^n/(2*b*d*n)), x] + \text{Dist}[1/(2*a), \text{Int}[(a + b*\tan[c + d*x])^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[n, 0]$

Rule 3811

$\text{Int}[(c_) + (d_)*(x_)^{(m_)}*((a_) + (b_)*\tan[(e_) + (f_)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[(a + b*\tan[e + f*x])^n, x]\}, \text{Dist}[(c + d*x)^m, u, x] - \text{Dist}[d*m, \text{Int}[\text{Dist}[(c + d*x)^{(m - 1)}, u, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{ILtQ}[n, -1] \ \&\& \ \text{GtQ}[m, 0]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{x(c + dx)}{4a^2} - \frac{i(c + dx)}{4f(a + ia \cot(e + fx))^2} - \frac{i(c + dx)}{4f(a^2 + ia^2 \cot(e + fx))} \\
 &\quad - d \int \left(\frac{x}{4a^2} - \frac{i}{4f(a + ia \cot(e + fx))^2} - \frac{i}{4f(a^2 + ia^2 \cot(e + fx))} \right) dx \\
 &= -\frac{dx^2}{8a^2} + \frac{x(c + dx)}{4a^2} - \frac{i(c + dx)}{4f(a + ia \cot(e + fx))^2} - \frac{i(c + dx)}{4f(a^2 + ia^2 \cot(e + fx))} \\
 &\quad + \frac{(id) \int \frac{1}{(a + ia \cot(e + fx))^2} dx}{4f} + \frac{(id) \int \frac{1}{a^2 + ia^2 \cot(e + fx)} dx}{4f} \\
 &= -\frac{dx^2}{8a^2} + \frac{x(c + dx)}{4a^2} + \frac{d}{16f^2(a + ia \cot(e + fx))^2} \\
 &\quad - \frac{i(c + dx)}{4f(a + ia \cot(e + fx))^2} + \frac{d}{8f^2(a^2 + ia^2 \cot(e + fx))} \\
 &\quad - \frac{i(c + dx)}{4f(a^2 + ia^2 \cot(e + fx))} + \frac{(id) \int 1 dx}{8a^2 f} + \frac{(id) \int \frac{1}{a + ia \cot(e + fx)} dx}{8af} \\
 &= \frac{id x}{8a^2 f} - \frac{dx^2}{8a^2} + \frac{x(c + dx)}{4a^2} + \frac{d}{16f^2(a + ia \cot(e + fx))^2} - \frac{i(c + dx)}{4f(a + ia \cot(e + fx))^2} \\
 &\quad + \frac{3d}{16f^2(a^2 + ia^2 \cot(e + fx))} - \frac{i(c + dx)}{4f(a^2 + ia^2 \cot(e + fx))} + \frac{(id) \int 1 dx}{16a^2 f}
 \end{aligned}$$

$$= \frac{3idx}{16a^2f} - \frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} + \frac{d}{16f^2(a+ia\cot(e+fx))^2} - \frac{i(c+dx)}{4f(a+ia\cot(e+fx))^2}$$

$$+ \frac{3d}{16f^2(a^2+ia^2\cot(e+fx))} - \frac{i(c+dx)}{4f(a^2+ia^2\cot(e+fx))}$$

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.09

$$\int \frac{c+dx}{(a+ia\cot(e+fx))^2} dx$$

$$= \frac{-8de^2 + 16cef + 16cf^2x + 8df^2x^2 + 8i(2cf + d(i + 2fx))\cos(2(e+fx)) + (d - 4icf - 4idf)\cos(4(e+fx))}{64a^2f^2}$$

[In] Integrate[(c + d*x)/(a + I*a*Cot[e + f*x])^2,x]

[Out] $(-8*d*e^2 + 16*c*e*f + 16*c*f^2*x + 8*d*f^2*x^2 + (8*I)*(2*c*f + d*(I + 2*f*x))*\text{Cos}[2*(e + f*x)] + (d - (4*I)*c*f - (4*I)*d*f*x)*\text{Cos}[4*(e + f*x)] - (8*I)*d*\text{Sin}[2*(e + f*x)] - 16*c*f*\text{Sin}[2*(e + f*x)] - 16*d*f*x*\text{Sin}[2*(e + f*x)] + I*d*\text{Sin}[4*(e + f*x)] + 4*c*f*\text{Sin}[4*(e + f*x)] + 4*d*f*x*\text{Sin}[4*(e + f*x)])/(64*a^2*f^2)$

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.54

method	result
risch	$\frac{dx^2}{8a^2} + \frac{xc}{4a^2} - \frac{i(4dfx+4cf+id)e^{4i(fx+e)}}{64a^2f^2} + \frac{i(2dfx+2cf+id)e^{2i(fx+e)}}{8a^2f^2}$
parallelrisch	$\frac{((2dx^2+4xc)f^2-5idf)\tan(fx+e)^2 + (8i(\frac{dx}{2}+c)x f^2 + (-2dx-12c)f-5id)\tan(fx+e) + (-2dx^2-4xc)f^2 + (-3idx-8ic)f}{16f^2a^2(-1+2i\tan(fx+e)+\tan(fx+e)^2)}$

[In] int((d*x+c)/(a+I*a*cot(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] $1/8*d*x^2/a^2+1/4/a^2*x*c-1/64*I*(4*d*f*x+I*d+4*c*f)/a^2/f^2*\exp(4*I*(f*x+e))+1/8*I*(2*d*f*x+I*d+2*c*f)/a^2/f^2*\exp(2*I*(f*x+e))$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.45

$$\int \frac{c + dx}{(a + ia \cot(e + fx))^2} dx$$

$$= \frac{8df^2x^2 + 16cf^2x + (-4idfx - 4icf + d)e^{(4ifx+4ie)} - 8(-2idfx - 2icf + d)e^{(2ifx+2ie)}}{64a^2f^2}$$

[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")

[Out] 1/64*(8*d*f^2*x^2 + 16*c*f^2*x + (-4*I*d*f*x - 4*I*c*f + d)*e^(4*I*f*x + 4*I*e) - 8*(-2*I*d*f*x - 2*I*c*f + d)*e^(2*I*f*x + 2*I*e))/(a^2*f^2)

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.40

$$\int \frac{c + dx}{(a + ia \cot(e + fx))^2} dx$$

$$= \begin{cases} \frac{(128ia^2cf^3e^{2ie} + 128ia^2df^3xe^{2ie} - 64a^2df^2e^{2ie})e^{2ifx} + (-32ia^2cf^3e^{4ie} - 32ia^2df^3xe^{4ie} + 8a^2df^2e^{4ie})e^{4ifx}}{512a^4f^4} & \text{for } a^4f^4 \neq 0 \\ \frac{x^2(de^{4ie} - 2de^{2ie})}{8a^2} + \frac{x(ce^{4ie} - 2ce^{2ie})}{4a^2} & \text{otherwise} \\ + \frac{cx}{4a^2} + \frac{dx^2}{8a^2} \end{cases}$$

[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))**2,x)

[Out] Piecewise((((128*I*a**2*c*f**3*exp(2*I*e) + 128*I*a**2*d*f**3*x*exp(2*I*e) - 64*a**2*d*f**2*exp(2*I*e))*exp(2*I*f*x) + (-32*I*a**2*c*f**3*exp(4*I*e) - 32*I*a**2*d*f**3*x*exp(4*I*e) + 8*a**2*d*f**2*exp(4*I*e))*exp(4*I*f*x))/(512*a**4*f**4), Ne(a**4*f**4, 0)), (x**2*(d*exp(4*I*e) - 2*d*exp(2*I*e))/(8*a**2) + x*(c*exp(4*I*e) - 2*c*exp(2*I*e))/(4*a**2), True)) + c*x/(4*a**2) + d*x**2/(8*a**2)

Maxima [F(-2)]

Exception generated.

$$\int \frac{c + dx}{(a + ia \cot(e + fx))^2} dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.68

$$\int \frac{c + dx}{(a + ia \cot(e + fx))^2} dx = \frac{8df^2x^2 + 16cf^2x - 4idfxe^{(4ifx+4ie)} + 16idfxe^{(2ifx+2ie)} - 4icfe^{(4ifx+4ie)} + 16icfe^{(2ifx+2ie)} + de^{(4ifx+4ie)}}{64a^2f^2}$$

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/64*(8*d*f^2*x^2 + 16*c*f^2*x - 4*I*d*f*x*e^(4*I*f*x + 4*I*e) + 16*I*d*f*x*e^(2*I*f*x + 2*I*e) - 4*I*c*f*e^(4*I*f*x + 4*I*e) + 16*I*c*f*e^(2*I*f*x + 2*I*e) + d*e^(4*I*f*x + 4*I*e) - 8*d*e^(2*I*f*x + 2*I*e))/(a^2*f^2)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.68

$$\int \frac{c + dx}{(a + ia \cot(e + fx))^2} dx = e^{e^{2i+fx}2i} \left(\frac{(2cf + dli) li}{8a^2 f^2} + \frac{dx li}{4a^2 f} \right) - e^{e^{4i+fx}4i} \left(\frac{(4cf + dli) li}{64a^2 f^2} + \frac{dx li}{16a^2 f} \right) + \frac{dx^2}{8a^2} + \frac{cx}{4a^2}$$

```
[In] int((c + d*x)/(a + a*cot(e + f*x)*1i)^2,x)
```

```
[Out] exp(e*2i + f*x*2i)*(((d*1i + 2*c*f)*1i)/(8*a^2*f^2) + (d*x*1i)/(4*a^2*f)) - exp(e*4i + f*x*4i)*(((d*1i + 4*c*f)*1i)/(64*a^2*f^2) + (d*x*1i)/(16*a^2*f)) + (d*x^2)/(8*a^2) + (c*x)/(4*a^2)
```

3.25 $\int \frac{1}{(c+dx)(a+ia \cot(e+fx))^2} dx$

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Optimal result

Integrand size = 23, antiderivative size = 305

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))^2} dx = -\frac{\cos\left(2e - \frac{2cf}{d}\right) \operatorname{CosIntegral}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} + \frac{\cos\left(4e - \frac{4cf}{d}\right) \operatorname{CosIntegral}\left(\frac{4cf}{d} + 4fx\right)}{4a^2d} + \frac{\log(c+dx)}{4a^2d} + \frac{i \operatorname{CosIntegral}\left(\frac{4cf}{d} + 4fx\right) \sin\left(4e - \frac{4cf}{d}\right)}{4a^2d} - \frac{i \operatorname{CosIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(2e - \frac{2cf}{d}\right)}{2a^2d} - \frac{i \cos\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} + \frac{\sin\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} + \frac{i \cos\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(\frac{4cf}{d} + 4fx\right)}{4a^2d} - \frac{\sin\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(\frac{4cf}{d} + 4fx\right)}{4a^2d}$$

[Out] 1/4*Ci(4*c*f/d+4*f*x)*cos(-4*e+4*c*f/d)/a^2/d-1/2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a^2/d+1/4*ln(d*x+c)/a^2/d-1/2*I*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a^2/d+1/4*I*cos(-4*e+4*c*f/d)*Si(4*c*f/d+4*f*x)/a^2/d-1/4*I*Ci(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^2/d+1/4*Si(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^2/d+1/2*I*Ci(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a^2/d-1/2*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/a^2/d

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3809, 3384, 3380, 3383, 3393}

$$\int \frac{1}{(c + dx)(a + ia \cot(e + fx))^2} dx = -\frac{i \operatorname{CosIntegral}\left(2xf + \frac{2cf}{d}\right) \sin\left(2e - \frac{2cf}{d}\right)}{2a^2d} + \frac{i \operatorname{CosIntegral}\left(4xf + \frac{4cf}{d}\right) \sin\left(4e - \frac{4cf}{d}\right)}{4a^2d} - \frac{\operatorname{CosIntegral}\left(2xf + \frac{2cf}{d}\right) \cos\left(2e - \frac{2cf}{d}\right)}{2a^2d} + \frac{\operatorname{CosIntegral}\left(4xf + \frac{4cf}{d}\right) \cos\left(4e - \frac{4cf}{d}\right)}{4a^2d} + \frac{\sin\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(2xf + \frac{2cf}{d}\right)}{2a^2d} - \frac{\sin\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(4xf + \frac{4cf}{d}\right)}{4a^2d} - \frac{i \cos\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(2xf + \frac{2cf}{d}\right)}{2a^2d} + \frac{i \cos\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(4xf + \frac{4cf}{d}\right)}{4a^2d} + \frac{\log(c + dx)}{4a^2d}$$

[In] Int[1/((c + d*x)*(a + I*a*Cot[e + f*x])^2),x]

[Out] -1/2*(Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(a^2*d) + (Cos[4*e - (4*c*f)/d]*CosIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d) + Log[c + d*x]/(4*a^2*d) + ((I/4)*CosIntegral[(4*c*f)/d + 4*f*x]*Sin[4*e - (4*c*f)/d])/(a^2*d) - ((I/2)*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/(a^2*d) - ((I/2)*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(a^2*d) + (Sin[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) + ((I/4)*Cos[4*e - (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(a^2*d) - (Sin[4*e - (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d)

Rule 3380

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3383

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3809

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_),
x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{4a^2(c+dx)} - \frac{\cos(2e+2fx)}{2a^2(c+dx)} + \frac{\cos^2(2e+2fx)}{4a^2(c+dx)} - \frac{i \sin(2e+2fx)}{2a^2(c+dx)} \right. \\
&\quad \left. - \frac{\sin^2(2e+2fx)}{4a^2(c+dx)} + \frac{i \sin(4e+4fx)}{4a^2(c+dx)} \right) dx \\
&= \frac{\log(c+dx)}{4a^2d} + \frac{i \int \frac{\sin(4e+4fx)}{c+dx} dx}{4a^2} - \frac{i \int \frac{\sin(2e+2fx)}{c+dx} dx}{2a^2} \\
&\quad + \frac{\int \frac{\cos^2(2e+2fx)}{c+dx} dx}{4a^2} - \frac{\int \frac{\sin^2(2e+2fx)}{c+dx} dx}{4a^2} - \frac{\int \frac{\cos(2e+2fx)}{c+dx} dx}{2a^2} \\
&= \frac{\log(c+dx)}{4a^2d} - \frac{\int \left(\frac{1}{2(c+dx)} - \frac{\cos(4e+4fx)}{2(c+dx)} \right) dx}{4a^2} + \frac{\int \left(\frac{1}{2(c+dx)} + \frac{\cos(4e+4fx)}{2(c+dx)} \right) dx}{4a^2} \\
&\quad + \frac{(i \cos(4e - \frac{4cf}{d})) \int \frac{\sin(\frac{4cf}{d} + 4fx)}{c+dx} dx}{4a^2} - \frac{(i \cos(2e - \frac{2cf}{d})) \int \frac{\sin(\frac{2cf}{d} + 2fx)}{c+dx} dx}{2a^2} \\
&\quad - \frac{\cos(2e - \frac{2cf}{d}) \int \frac{\cos(\frac{2cf}{d} + 2fx)}{c+dx} dx}{2a^2} + \frac{(i \sin(4e - \frac{4cf}{d})) \int \frac{\cos(\frac{4cf}{d} + 4fx)}{c+dx} dx}{4a^2} \\
&\quad - \frac{(i \sin(2e - \frac{2cf}{d})) \int \frac{\cos(\frac{2cf}{d} + 2fx)}{c+dx} dx}{2a^2} + \frac{\sin(2e - \frac{2cf}{d}) \int \frac{\sin(\frac{2cf}{d} + 2fx)}{c+dx} dx}{2a^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\cos\left(2e - \frac{2cf}{d}\right) \operatorname{CosIntegral}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} + \frac{\log(c + dx)}{4a^2d} \\
&\quad + \frac{i \operatorname{CosIntegral}\left(\frac{4cf}{d} + 4fx\right) \sin\left(4e - \frac{4cf}{d}\right)}{4a^2d} \\
&\quad - \frac{i \operatorname{CosIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(2e - \frac{2cf}{d}\right)}{2a^2d} - \frac{i \cos\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} \\
&\quad + \frac{\sin\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} + \frac{i \cos\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(\frac{4cf}{d} + 4fx\right)}{4a^2d} + 2 \frac{\int \frac{\cos(4e+4fx)}{c+dx} dx}{8a^2} \\
&= -\frac{\cos\left(2e - \frac{2cf}{d}\right) \operatorname{CosIntegral}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} + \frac{\log(c + dx)}{4a^2d} \\
&\quad + \frac{i \operatorname{CosIntegral}\left(\frac{4cf}{d} + 4fx\right) \sin\left(4e - \frac{4cf}{d}\right)}{4a^2d} \\
&\quad - \frac{i \operatorname{CosIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(2e - \frac{2cf}{d}\right)}{2a^2d} - \frac{i \cos\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} \\
&\quad + \frac{\sin\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} + \frac{i \cos\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(\frac{4cf}{d} + 4fx\right)}{4a^2d} \\
&\quad + 2 \left(\frac{\cos\left(4e - \frac{4cf}{d}\right) \int \frac{\cos\left(\frac{4cf}{d} + 4fx\right)}{c+dx} dx}{8a^2} - \frac{\sin\left(4e - \frac{4cf}{d}\right) \int \frac{\sin\left(\frac{4cf}{d} + 4fx\right)}{c+dx} dx}{8a^2} \right) \\
&= -\frac{\cos\left(2e - \frac{2cf}{d}\right) \operatorname{CosIntegral}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} + \frac{\log(c + dx)}{4a^2d} \\
&\quad + \frac{i \operatorname{CosIntegral}\left(\frac{4cf}{d} + 4fx\right) \sin\left(4e - \frac{4cf}{d}\right)}{4a^2d} \\
&\quad - \frac{i \operatorname{CosIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(2e - \frac{2cf}{d}\right)}{2a^2d} - \frac{i \cos\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} \\
&\quad + \frac{\sin\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} + \frac{i \cos\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(\frac{4cf}{d} + 4fx\right)}{4a^2d} \\
&\quad + 2 \left(\frac{\cos\left(4e - \frac{4cf}{d}\right) \operatorname{CosIntegral}\left(\frac{4cf}{d} + 4fx\right)}{8a^2d} - \frac{\sin\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(\frac{4cf}{d} + 4fx\right)}{8a^2d} \right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.45

$$\begin{aligned}
&\int \frac{1}{(c + dx)(a + ia \cot(e + fx))^2} dx \\
&= \frac{\log(c + dx) - 2\left(\cos\left(2e - \frac{2cf}{d}\right) + i \sin\left(2e - \frac{2cf}{d}\right)\right) \left(\operatorname{CosIntegral}\left(\frac{2f(c+dx)}{d}\right) + i \operatorname{Si}\left(\frac{2f(c+dx)}{d}\right)\right) + (\cos(4e - \dots)}{4a^2d}
\end{aligned}$$

[In] Integrate[1/((c + d*x)*(a + I*a*Cot[e + f*x])^2),x]

[Out] $(\text{Log}[c + d*x] - 2*(\text{Cos}[2*e - (2*c*f)/d] + I*\text{Sin}[2*e - (2*c*f)/d])*(\text{CosIntegral}[(2*f*(c + d*x))/d] + I*\text{SinIntegral}[(2*f*(c + d*x))/d]) + (\text{Cos}[4*e - (4*c*f)/d] + I*\text{Sin}[4*e - (4*c*f)/d])*(\text{CosIntegral}[(4*f*(c + d*x))/d] + I*\text{SinIntegral}[(4*f*(c + d*x))/d]))/(4*a^2*d)$

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{\ln(dx+c)}{4a^2d} + \frac{e^{-\frac{2i(cf-de)}{d}} \text{Ei}_1\left(-2ifx-2ie-\frac{2(icf-ide)}{d}\right)}{2a^2d} - \frac{e^{-\frac{4i(cf-de)}{d}} \text{Ei}_1\left(-4ifx-4ie-\frac{4(icf-ide)}{d}\right)}{4a^2d}$	118

[In] `int(1/(d*x+c)/(a+I*a*cot(f*x+e))^2,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4}*\ln(d*x+c)/a^2/d + 1/2/a^2/d*\exp(-2*I*(c*f-d*e)/d)*\text{Ei}(1,-2*I*f*x-2*I*e-2*(I*c*f-I*d*e)/d) - 1/4/a^2/d*\exp(-4*I*(c*f-d*e)/d)*\text{Ei}(1,-4*I*f*x-4*I*e-4*(I*c*f-I*d*e)/d)$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.29

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))^2} dx$$

$$= -\frac{2 \text{Ei}\left(-\frac{2(-i dfx-icf)}{d}\right) e^{\left(-\frac{2(-ide+icf)}{d}\right)} - \text{Ei}\left(-\frac{4(-i dfx-icf)}{d}\right) e^{\left(-\frac{4(-ide+icf)}{d}\right)} - \log\left(\frac{dx+c}{d}\right)}{4a^2d}$$

[In] `integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")`

[Out] $-1/4*(2*\text{Ei}(-2*(-I*d*f*x - I*c*f)/d)*e^{(-2*(-I*d*e + I*c*f)/d)} - \text{Ei}(-4*(-I*d*f*x - I*c*f)/d)*e^{(-4*(-I*d*e + I*c*f)/d)} - \log((d*x + c)/d))/(a^2*d)$

Sympy [F]

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))^2} dx$$

$$= -\frac{\int \frac{1}{c \cot^2(e+fx) - 2ic \cot(e+fx) - c + dx \cot^2(e+fx) - 2idx \cot(e+fx) - dx} dx}{a^2}$$

[In] `integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))**2,x)`

[Out] $-\text{Integral}(1/(c*\cot(e + f*x)**2 - 2*I*c*\cot(e + f*x) - c + d*x*\cot(e + f*x))*2 - 2*I*d*x*\cot(e + f*x) - d*x), x)/a**2$

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.64

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))^2} dx = \frac{f \cos\left(-\frac{4(de-cf)}{d}\right) E_1\left(\frac{4(-i(fx+e)d+ide-icf)}{d}\right) - 2f \cos\left(-\frac{2(de-cf)}{d}\right) E_1\left(\frac{2(-i(fx+e)d+ide-icf)}{d}\right) + 2if E_1\left(\frac{2(-i(fx+e)d+ide-icf)}{d}\right)}{a^2 d f}$$

[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")

```
[Out] -1/4*(f*cos(-4*(d*e - c*f)/d)*exp_integral_e(1, 4*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) - 2*f*cos(-2*(d*e - c*f)/d)*exp_integral_e(1, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) + 2*I*f*exp_integral_e(1, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d)*sin(-2*(d*e - c*f)/d) - I*f*exp_integral_e(1, 4*(-I*(f*x + e)*d + I*d*e - I*c*f)/d)*sin(-4*(d*e - c*f)/d) - f*log((f*x + e)*d - d*e + c*f))/(a^2*d*f)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 939 vs. 2(279) = 558.

Time = 0.30 (sec) , antiderivative size = 939, normalized size of antiderivative = 3.08

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))^2} dx = \text{Too large to display}$$

[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")

```
[Out] 1/4*(cos(e)^4*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d) + 4*I*cos(e)^3*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e) - 6*cos(e)^2*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2 - 4*I*cos(e)*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^3 + cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4 - I*cos(e)^4*cos_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d) + 4*cos(e)^3*cos_integral(4*(d*f*x + c*f)/d)*sin(e)*sin(4*c*f/d) + 6*I*cos(e)^2*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2*sin(4*c*f/d) - 4*cos(e)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^3*sin(4*c*f/d) - I*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4*sin(4*c*f/d) + I*cos(e)^4*cos(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 4*cos(e)^3*cos(4*c*f/d)*sin(e)*sin_integral(4*(d*f*x + c*f)/d) - 6*I*cos(e)^2*cos(4*c*f/d)*sin(e)^2*sin_integral(4*(d*f*x + c*f)/d) + 4*cos(e)*cos(4*c*f/d)*sin(e)^3*sin_integral(4*(d*f*x + c*f)/d) + I*cos(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*f)/d) + cos(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 4*I*cos(e)^3*sin(e)*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d)
```

```

4*(d*f*x + c*f)/d) - 6*cos(e)^2*sin(e)^2*sin(4*c*f/d)*sin_integral(4*(d*f*x
+ c*f)/d) - 4*I*cos(e)*sin(e)^3*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/
d) + sin(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 2*cos(e)^2*cos
(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d) - 4*I*cos(e)*cos(2*c*f/d)*cos_int
egral(2*(d*f*x + c*f)/d)*sin(e) + 2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*
f)/d)*sin(e)^2 + 2*I*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d)
- 4*cos(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)*sin(2*c*f/d) - 2*I*cos_in
tegral(2*(d*f*x + c*f)/d)*sin(e)^2*sin(2*c*f/d) - 2*I*cos(e)^2*cos(2*c*f/d)
*sin_integral(2*(d*f*x + c*f)/d) + 4*cos(e)*cos(2*c*f/d)*sin(e)*sin_integra
l(2*(d*f*x + c*f)/d) + 2*I*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*
f)/d) - 2*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - 4*I*cos(e
)*sin(e)*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 2*sin(e)^2*sin(2*c*
f/d)*sin_integral(2*(d*f*x + c*f)/d) + log(d*x + c))/(a^2*d)

```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c + dx)(a + ia \cot(e + fx))^2} dx = \int \frac{1}{(a + a \cot(e + fx) li)^2 (c + dx)} dx$$

```
[In] int(1/((a + a*cot(e + f*x)*1i)^2*(c + d*x)),x)
```

```
[Out] int(1/((a + a*cot(e + f*x)*1i)^2*(c + d*x)), x)
```

$$3.26 \quad \int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))^2} dx$$

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Optimal result

Integrand size = 23, antiderivative size = 434

$$\begin{aligned} \int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))^2} dx = & -\frac{1}{4a^2d(c+dx)} + \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} \\ & - \frac{if \cos(2e - \frac{2cf}{d}) \operatorname{CosIntegral}(\frac{2cf}{d} + 2fx)}{a^2d^2} \\ & + \frac{if \cos(4e - \frac{4cf}{d}) \operatorname{CosIntegral}(\frac{4cf}{d} + 4fx)}{a^2d^2} \\ & - \frac{f \operatorname{CosIntegral}(\frac{4cf}{d} + 4fx) \sin(4e - \frac{4cf}{d})}{a^2d^2} \\ & + \frac{f \operatorname{CosIntegral}(\frac{2cf}{d} + 2fx) \sin(2e - \frac{2cf}{d})}{a^2d^2} \\ & + \frac{i \sin(2e+2fx)}{2a^2d(c+dx)} + \frac{\sin^2(2e+2fx)}{4a^2d(c+dx)} \\ & - \frac{i \sin(4e+4fx)}{4a^2d(c+dx)} + \frac{f \cos(2e - \frac{2cf}{d}) \operatorname{Si}(\frac{2cf}{d} + 2fx)}{a^2d^2} \\ & + \frac{if \sin(2e - \frac{2cf}{d}) \operatorname{Si}(\frac{2cf}{d} + 2fx)}{a^2d^2} \\ & - \frac{f \cos(4e - \frac{4cf}{d}) \operatorname{Si}(\frac{4cf}{d} + 4fx)}{a^2d^2} \\ & - \frac{if \sin(4e - \frac{4cf}{d}) \operatorname{Si}(\frac{4cf}{d} + 4fx)}{a^2d^2} \end{aligned}$$

```
[Out] -1/4/a^2/d/(d*x+c)+I*f*Ci(4*c*f/d+4*f*x)*cos(-4*e+4*c*f/d)/a^2/d^2-I*f*Ci(2
*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/a^2/d^2+1/2*cos(2*f*x+2*e)/a^2/d/(d*x+c)-1/
4*cos(2*f*x+2*e)^2/a^2/d/(d*x+c)+f*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a^2/
d^2-f*cos(-4*e+4*c*f/d)*Si(4*c*f/d+4*f*x)/a^2/d^2+f*Ci(4*c*f/d+4*f*x)*sin(-
4*e+4*c*f/d)/a^2/d^2+I*f*Si(4*c*f/d+4*f*x)*sin(-4*e+4*c*f/d)/a^2/d^2-f*Ci(2
```

$*c*f/d+2*f*x)*\sin(-2*e+2*c*f/d)/a^2/d^2-I*f*Si(2*c*f/d+2*f*x)*\sin(-2*e+2*c*f/d)/a^2/d^2+1/2*I*\sin(2*f*x+2*e)/a^2/d/(d*x+c)+1/4*\sin(2*f*x+2*e)^2/a^2/d/(d*x+c)-1/4*I*\sin(4*f*x+4*e)/a^2/d/(d*x+c)$

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3809, 3378, 3384, 3380, 3383, 3394, 12}

$$\int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))^2} dx = -\frac{f \operatorname{CosIntegral}\left(4xf + \frac{4cf}{d}\right) \sin\left(4e - \frac{4cf}{d}\right)}{a^2 d^2} + \frac{f \operatorname{CosIntegral}\left(2xf + \frac{2cf}{d}\right) \sin\left(2e - \frac{2cf}{d}\right)}{a^2 d^2} - \frac{if \operatorname{CosIntegral}\left(2xf + \frac{2cf}{d}\right) \cos\left(2e - \frac{2cf}{d}\right)}{a^2 d^2} + \frac{if \operatorname{CosIntegral}\left(4xf + \frac{4cf}{d}\right) \cos\left(4e - \frac{4cf}{d}\right)}{a^2 d^2} + \frac{if \sin\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(2xf + \frac{2cf}{d}\right)}{a^2 d^2} - \frac{if \sin\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(4xf + \frac{4cf}{d}\right)}{a^2 d^2} + \frac{f \cos\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(2xf + \frac{2cf}{d}\right)}{a^2 d^2} - \frac{f \cos\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(4xf + \frac{4cf}{d}\right)}{a^2 d^2} + \frac{\sin^2(2e+2fx)}{4a^2 d(c+dx)} + \frac{i \sin(2e+2fx)}{2a^2 d(c+dx)} - \frac{i \sin(4e+4fx)}{4a^2 d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2 d(c+dx)} + \frac{\cos(2e+2fx)}{2a^2 d(c+dx)} - \frac{1}{4a^2 d(c+dx)}$$

[In] Int[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])^2),x]

[Out] $-1/4*1/(a^2*d*(c + d*x)) + \operatorname{Cos}[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - \operatorname{Cos}[2*e + 2*f*x]^2/(4*a^2*d*(c + d*x)) - (I*f*\operatorname{Cos}[2*e - (2*c*f)/d]*\operatorname{CosIntegral}[(2*c*f)/d + 2*f*x])/(a^2*d^2) + (I*f*\operatorname{Cos}[4*e - (4*c*f)/d]*\operatorname{CosIntegral}[(4*c*f)/d + 4*f*x])/(a^2*d^2) - (f*\operatorname{CosIntegral}[(4*c*f)/d + 4*f*x]*\operatorname{Sin}[4*e - (4*c*f)/d])/(a^2*d^2) + (f*\operatorname{CosIntegral}[(2*c*f)/d + 2*f*x]*\operatorname{Sin}[2*e - (2*c*f)/d])/(a^2*d^2) + ((I/2)*\operatorname{Sin}[2*e + 2*f*x])/(a^2*d*(c + d*x)) + \operatorname{Sin}[2*e + 2*f*x]^2/(4*a^2*d*(c + d*x)) - ((I/4)*\operatorname{Sin}[4*e + 4*f*x])/(a^2*d*(c + d*x)) + (f*\operatorname{Cos}[2*e - (2*c*f)/d]*\operatorname{SinIntegral}[(2*c*f)/d + 2*f*x])/(a^2*d^2) + (I*f*\operatorname{Sin}[2*e - (2*c*f)/d]*\operatorname{SinIntegral}[(2*c*f)/d + 2*f*x])/(a^2*d^2) - (f*\operatorname{Cos}[4*e - (4*c*f)/d]*\operatorname{SinIntegral}[(4*c*f)/d + 4*f*x])/(a^2*d^2) - (I*f*\operatorname{Sin}[4*e - (4*c*f)/d]*\operatorname{SinIntegral}[(4*c*f)/d + 4*f*x])/(a^2*d^2)$

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(c + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n - 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]
```

Rule 3809

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(2*a) + Sin[2*e + 2*f*x]/(2*b))^(n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{4a^2(c+dx)^2} - \frac{\cos(2e+2fx)}{2a^2(c+dx)^2} + \frac{\cos^2(2e+2fx)}{4a^2(c+dx)^2} - \frac{i \sin(2e+2fx)}{2a^2(c+dx)^2} \right. \\
&\quad \left. - \frac{\sin^2(2e+2fx)}{4a^2(c+dx)^2} + \frac{i \sin(4e+4fx)}{4a^2(c+dx)^2} \right) dx \\
&= -\frac{1}{4a^2d(c+dx)} + \frac{i \int \frac{\sin(4e+4fx)}{(c+dx)^2} dx}{4a^2} - \frac{i \int \frac{\sin(2e+2fx)}{(c+dx)^2} dx}{2a^2} \\
&\quad + \frac{\int \frac{\cos^2(2e+2fx)}{(c+dx)^2} dx}{4a^2} - \frac{\int \frac{\sin^2(2e+2fx)}{(c+dx)^2} dx}{4a^2} - \frac{\int \frac{\cos(2e+2fx)}{(c+dx)^2} dx}{2a^2} \\
&= -\frac{1}{4a^2d(c+dx)} + \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} + \frac{i \sin(2e+2fx)}{2a^2d(c+dx)} \\
&\quad + \frac{\sin^2(2e+2fx)}{4a^2d(c+dx)} - \frac{i \sin(4e+4fx)}{4a^2d(c+dx)} - \frac{(if) \int \frac{\cos(2e+2fx)}{c+dx} dx}{a^2d} + \frac{(if) \int \frac{\cos(4e+4fx)}{c+dx} dx}{a^2d} \\
&\quad + \frac{f \int \frac{\sin(2e+2fx)}{c+dx} dx}{a^2d} + \frac{f \int -\frac{\sin(4e+4fx)}{2(c+dx)} dx}{a^2d} - \frac{f \int \frac{\sin(4e+4fx)}{2(c+dx)} dx}{a^2d} \\
&= -\frac{1}{4a^2d(c+dx)} + \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} + \frac{i \sin(2e+2fx)}{2a^2d(c+dx)} \\
&\quad + \frac{\sin^2(2e+2fx)}{4a^2d(c+dx)} - \frac{i \sin(4e+4fx)}{4a^2d(c+dx)} - \frac{2f \int \frac{\sin(4e+4fx)}{c+dx} dx}{2a^2d} \\
&\quad + \frac{(if \cos(4e - \frac{4cf}{d})) \int \frac{\cos(\frac{4cf}{d} + 4fx)}{c+dx} dx}{a^2d} - \frac{(if \cos(2e - \frac{2cf}{d})) \int \frac{\cos(\frac{2cf}{d} + 2fx)}{c+dx} dx}{a^2d} \\
&\quad + \frac{(f \cos(2e - \frac{2cf}{d})) \int \frac{\sin(\frac{2cf}{d} + 2fx)}{c+dx} dx}{a^2d} - \frac{(if \sin(4e - \frac{4cf}{d})) \int \frac{\sin(\frac{4cf}{d} + 4fx)}{c+dx} dx}{a^2d} \\
&\quad + \frac{(if \sin(2e - \frac{2cf}{d})) \int \frac{\sin(\frac{2cf}{d} + 2fx)}{c+dx} dx}{a^2d} + \frac{(f \sin(2e - \frac{2cf}{d})) \int \frac{\cos(\frac{2cf}{d} + 2fx)}{c+dx} dx}{a^2d}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{4a^2d(c+dx)} + \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} \\
&\quad - \frac{if \cos\left(2e - \frac{2cf}{d}\right) \text{CosIntegral}\left(\frac{2cf}{d} + 2fx\right)}{a^2d^2} \\
&\quad + \frac{if \cos\left(4e - \frac{4cf}{d}\right) \text{CosIntegral}\left(\frac{4cf}{d} + 4fx\right)}{a^2d^2} \\
&\quad + \frac{f \text{CosIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(2e - \frac{2cf}{d}\right)}{a^2d^2} + \frac{i \sin(2e+2fx)}{2a^2d(c+dx)} \\
&\quad + \frac{\sin^2(2e+2fx)}{4a^2d(c+dx)} - \frac{i \sin(4e+4fx)}{4a^2d(c+dx)} + \frac{f \cos\left(2e - \frac{2cf}{d}\right) \text{Si}\left(\frac{2cf}{d} + 2fx\right)}{a^2d^2} \\
&\quad + \frac{if \sin\left(2e - \frac{2cf}{d}\right) \text{Si}\left(\frac{2cf}{d} + 2fx\right)}{a^2d^2} - \frac{if \sin\left(4e - \frac{4cf}{d}\right) \text{Si}\left(\frac{4cf}{d} + 4fx\right)}{a^2d^2} \\
&\quad - 2 \left(\frac{\left(f \cos\left(4e - \frac{4cf}{d}\right)\right) \int \frac{\sin\left(\frac{4cf}{d} + 4fx\right)}{c+dx} dx}{2a^2d} + \frac{\left(f \sin\left(4e - \frac{4cf}{d}\right)\right) \int \frac{\cos\left(\frac{4cf}{d} + 4fx\right)}{c+dx} dx}{2a^2d} \right) \\
&= -\frac{1}{4a^2d(c+dx)} + \frac{\cos(2e+2fx)}{2a^2d(c+dx)} - \frac{\cos^2(2e+2fx)}{4a^2d(c+dx)} \\
&\quad - \frac{if \cos\left(2e - \frac{2cf}{d}\right) \text{CosIntegral}\left(\frac{2cf}{d} + 2fx\right)}{a^2d^2} \\
&\quad + \frac{if \cos\left(4e - \frac{4cf}{d}\right) \text{CosIntegral}\left(\frac{4cf}{d} + 4fx\right)}{a^2d^2} \\
&\quad + \frac{f \text{CosIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(2e - \frac{2cf}{d}\right)}{a^2d^2} + \frac{i \sin(2e+2fx)}{2a^2d(c+dx)} \\
&\quad + \frac{\sin^2(2e+2fx)}{4a^2d(c+dx)} - \frac{i \sin(4e+4fx)}{4a^2d(c+dx)} + \frac{f \cos\left(2e - \frac{2cf}{d}\right) \text{Si}\left(\frac{2cf}{d} + 2fx\right)}{a^2d^2} \\
&\quad + \frac{if \sin\left(2e - \frac{2cf}{d}\right) \text{Si}\left(\frac{2cf}{d} + 2fx\right)}{a^2d^2} - \frac{if \sin\left(4e - \frac{4cf}{d}\right) \text{Si}\left(\frac{4cf}{d} + 4fx\right)}{a^2d^2} \\
&\quad - 2 \left(\frac{f \text{CosIntegral}\left(\frac{4cf}{d} + 4fx\right) \sin\left(4e - \frac{4cf}{d}\right)}{2a^2d^2} + \frac{f \cos\left(4e - \frac{4cf}{d}\right) \text{Si}\left(\frac{4cf}{d} + 4fx\right)}{2a^2d^2} \right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.47

$$\begin{aligned}
&\int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))^2} dx \\
&= \frac{-d + 2d(\cos(2(e+fx)) + i \sin(2(e+fx))) - d(\cos(4(e+fx)) + i \sin(4(e+fx))) + 4f(c+dx)(-i \cos(2(e+fx)) + \sin(2(e+fx)))}{(c+dx)^3(a+ia \cot(e+fx))^2}
\end{aligned}$$

[In] Integrate[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])^2),x]

```
[Out] (-d + 2*d*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]) - d*(Cos[4*(e + f*x)] + I*Sin[4*(e + f*x)]) + 4*f*(c + d*x)*((-I)*Cos[2*e - (2*c*f)/d] + Sin[2*e - (2*c*f)/d])*(CosIntegral[(2*f*(c + d*x))/d] + I*SinIntegral[(2*f*(c + d*x))/d]) + (c + d*x)*((4*I)*f*cos[4*e - (4*c*f)/d] - 4*f*Sin[4*e - (4*c*f)/d])*(CosIntegral[(4*f*(c + d*x))/d] + I*SinIntegral[(4*f*(c + d*x))/d])/(4*a^2*d^2*(c + d*x))
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{1}{4a^2d(dx+c)} + \frac{if e^{2i(fx+e)}}{2a^2d^2\left(ifx+\frac{icf}{d}\right)} + \frac{if e^{-\frac{2i(cf-de)}{d}} \operatorname{Ei}_1\left(-2ifx-2ie-\frac{2(icf-ide)}{d}\right)}{a^2d^2} - \frac{if e^{4i(fx+e)}}{4a^2d^2\left(ifx+\frac{icf}{d}\right)} - \frac{if e^{-\frac{4i(cf-de)}{d}} \operatorname{Ei}_1\left(-4ifx-4ie-\frac{4(icf-ide)}{d}\right)}{4a^2d^2}$

```
[In] int(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^2/d/(d*x+c)+1/2*I/a^2*f/d^2*exp(2*I*(f*x+e))/(I*f*x+I/d*c*f)+I/a^2*f/d^2*exp(-2*I*(c*f-d*e)/d)*Ei(1,-2*I*f*x-2*I*e-2*(I*c*f-I*d*e)/d)-1/4*I/a^2*f/d^2*exp(4*I*(f*x+e))/(I*f*x+I/d*c*f)-I/a^2*f/d^2*exp(-4*I*(c*f-d*e)/d)*Ei(1,-4*I*f*x-4*I*e-4*(I*c*f-I*d*e)/d)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.30

$$\int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))^2} dx = \frac{4(idfx+icf)\operatorname{Ei}\left(-\frac{2(-idfx-icf)}{d}\right)e^{\left(-\frac{2(-ide+icf)}{d}\right)} + 4(-idfx-icf)\operatorname{Ei}\left(-\frac{4(-idfx-icf)}{d}\right)e^{\left(-\frac{4(-ide+icf)}{d}\right)} + \dots}{4(a^2d^3x+a^2cd^2)}$$

```
[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(I*d*f*x + I*c*f)*Ei(-2*(-I*d*f*x - I*c*f)/d)*e^(-2*(-I*d*e + I*c*f)/d) + 4*(-I*d*f*x - I*c*f)*Ei(-4*(-I*d*f*x - I*c*f)/d)*e^(-4*(-I*d*e + I*c*f)/d) + d*e^(4*I*f*x + 4*I*e) - 2*d*e^(2*I*f*x + 2*I*e) + d)/(a^2*d^3*x + a^2*c*d^2)
```

Sympy [F]

$$\int \frac{1}{(c + dx)^2(a + ia \cot(e + fx))^2} dx = \frac{\int \frac{1}{c^2 \cot^2(e+fx) - 2ic^2 \cot(e+fx) - c^2 + 2cdx \cot^2(e+fx) - 4icdx \cot(e+fx) - 2cdx + d^2 x^2 \cot^2(e+fx) - 2id^2 x^2 \cot(e+fx) - d^2 x^2} dx}{a^2}$$

[In] integrate(1/(d*x+c)**2/(a+I*a*cot(f*x+e))**2,x)

[Out] -Integral(1/(c**2*cot(e + f*x)**2 - 2*I*c**2*cot(e + f*x) - c**2 + 2*c*d*x*cot(e + f*x)**2 - 4*I*c*d*x*cot(e + f*x) - 2*c*d*x + d**2*x**2*cot(e + f*x)**2 - 2*I*d**2*x**2*cot(e + f*x) - d**2*x**2), x)/a**2

Maxima [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.49

$$\int \frac{1}{(c + dx)^2(a + ia \cot(e + fx))^2} dx = \frac{f^2 \cos\left(-\frac{4(de-cf)}{d}\right) E_2\left(\frac{4(-i(fx+e)d+ide-icf)}{d}\right) - 2f^2 \cos\left(-\frac{2(de-cf)}{d}\right) E_2\left(\frac{2(-i(fx+e)d+ide-icf)}{d}\right) + 2if^2 E_2\left(\frac{2(-i(fx+e)d+ide-icf)}{d}\right)}{4((fx+e)a^2d^2 - a^2d^2)}$$

[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")

[Out] -1/4*(f^2*cos(-4*(d*e - c*f)/d)*exp_integral_e(2, 4*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) - 2*f^2*cos(-2*(d*e - c*f)/d)*exp_integral_e(2, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) + 2*I*f^2*exp_integral_e(2, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d)*sin(-2*(d*e - c*f)/d) - I*f^2*exp_integral_e(2, 4*(-I*(f*x + e)*d + I*d*e - I*c*f)/d)*sin(-4*(d*e - c*f)/d) + f^2)/(((f*x + e)*a^2*d^2 - a^2*d^2*e + a^2*c*d*f)*f)

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2249 vs. 2(410) = 820.

Time = 1.44 (sec) , antiderivative size = 2249, normalized size of antiderivative = 5.18

$$\int \frac{1}{(c + dx)^2(a + ia \cot(e + fx))^2} dx = \text{Too large to display}$$

[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")

```
[Out] 1/4*(4*I*d*f*x*cos(e)^4*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d) - 16*d
*f*x*cos(e)^3*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e) - 24*I*d*
f*x*cos(e)^2*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2 + 16*d*f
*x*cos(e)*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^3 + 4*I*d*f*x
*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4 + 4*d*f*x*cos(e)^4*c
os_integral(4*(d*f*x + c*f)/d)*sin(4*c*f/d) + 16*I*d*f*x*cos(e)^3*cos_integ
ral(4*(d*f*x + c*f)/d)*sin(e)*sin(4*c*f/d) - 24*d*f*x*cos(e)^2*cos_integral
(4*(d*f*x + c*f)/d)*sin(e)^2*sin(4*c*f/d) - 16*I*d*f*x*cos(e)*cos_integral(
4*(d*f*x + c*f)/d)*sin(e)^3*sin(4*c*f/d) + 4*d*f*x*cos_integral(4*(d*f*x +
c*f)/d)*sin(e)^4*sin(4*c*f/d) - 4*d*f*x*cos(e)^4*cos(4*c*f/d)*sin_integral(
4*(d*f*x + c*f)/d) - 16*I*d*f*x*cos(e)^3*cos(4*c*f/d)*sin(e)*sin_integral(4
*(d*f*x + c*f)/d) + 24*d*f*x*cos(e)^2*cos(4*c*f/d)*sin(e)^2*sin_integral(4*
(d*f*x + c*f)/d) + 16*I*d*f*x*cos(e)*cos(4*c*f/d)*sin(e)^3*sin_integral(4*(
d*f*x + c*f)/d) - 4*d*f*x*cos(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*f
)/d) + 4*I*d*f*x*cos(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 16
*d*f*x*cos(e)^3*sin(e)*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 24*I*
d*f*x*cos(e)^2*sin(e)^2*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 16*d
*f*x*cos(e)*sin(e)^3*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 4*I*d*f
*x*sin(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 4*I*c*f*cos(e)^4
*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d) - 16*c*f*cos(e)^3*cos(4*c*f/d
)*cos_integral(4*(d*f*x + c*f)/d)*sin(e) - 24*I*c*f*cos(e)^2*cos(4*c*f/d)*c
os_integral(4*(d*f*x + c*f)/d)*sin(e)^2 + 16*c*f*cos(e)*cos(4*c*f/d)*cos_in
tegral(4*(d*f*x + c*f)/d)*sin(e)^3 + 4*I*c*f*cos(4*c*f/d)*cos_integral(4*(d
*f*x + c*f)/d)*sin(e)^4 + 4*c*f*cos(e)^4*cos_integral(4*(d*f*x + c*f)/d)*si
n(4*c*f/d) + 16*I*c*f*cos(e)^3*cos_integral(4*(d*f*x + c*f)/d)*sin(e)*sin(4
*c*f/d) - 24*c*f*cos(e)^2*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2*sin(4*c*
f/d) - 16*I*c*f*cos(e)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^3*sin(4*c*f/d
) + 4*c*f*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4*sin(4*c*f/d) - 4*c*f*cos
(e)^4*cos(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 16*I*c*f*cos(e)^3*cos(
4*c*f/d)*sin(e)*sin_integral(4*(d*f*x + c*f)/d) + 24*c*f*cos(e)^2*cos(4*c*f
/d)*sin(e)^2*sin_integral(4*(d*f*x + c*f)/d) + 16*I*c*f*cos(e)*cos(4*c*f/d
)*sin(e)^3*sin_integral(4*(d*f*x + c*f)/d) - 4*c*f*cos(4*c*f/d)*sin(e)^4*sin
_integral(4*(d*f*x + c*f)/d) + 4*I*c*f*cos(e)^4*sin(4*c*f/d)*sin_integral(4
*(d*f*x + c*f)/d) - 16*c*f*cos(e)^3*sin(e)*sin(4*c*f/d)*sin_integral(4*(d*f
*x + c*f)/d) - 24*I*c*f*cos(e)^2*sin(e)^2*sin(4*c*f/d)*sin_integral(4*(d*f*
x + c*f)/d) + 16*c*f*cos(e)*sin(e)^3*sin(4*c*f/d)*sin_integral(4*(d*f*x + c
*f)/d) + 4*I*c*f*sin(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 4*
I*d*f*x*cos(e)^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d) + 8*d*f*x*cos
(e)*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) + 4*I*d*f*x*cos(2*c
*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2 - 4*d*f*x*cos(e)^2*cos_integ
ral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) - 8*I*d*f*x*cos(e)*cos_integral(2*(d*f*
x + c*f)/d)*sin(e)*sin(2*c*f/d) + 4*d*f*x*cos_integral(2*(d*f*x + c*f)/d)*s
in(e)^2*sin(2*c*f/d) + 4*d*f*x*cos(e)^2*cos(2*c*f/d)*sin_integral(2*(d*f*x
+ c*f)/d) + 8*I*d*f*x*cos(e)*cos(2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*
f)/d) - 4*d*f*x*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) - 4*I
```

```

*d*f*x*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 8*d*f*x*cos(
e)*sin(e)*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 4*I*d*f*x*sin(e)^2
*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - d*cos(4*f*x)*cos(e)^4 - 4*I
*c*f*cos(e)^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d) - I*d*cos(e)^4*s
in(4*f*x) - 4*I*d*cos(4*f*x)*cos(e)^3*sin(e) + 8*c*f*cos(e)*cos(2*c*f/d)*co
s_integral(2*(d*f*x + c*f)/d)*sin(e) + 4*d*cos(e)^3*sin(4*f*x)*sin(e) + 6*d
*cos(4*f*x)*cos(e)^2*sin(e)^2 + 4*I*c*f*cos(2*c*f/d)*cos_integral(2*(d*f*x
+ c*f)/d)*sin(e)^2 + 6*I*d*cos(e)^2*sin(4*f*x)*sin(e)^2 + 4*I*d*cos(4*f*x)*
cos(e)*sin(e)^3 - 4*d*cos(e)*sin(4*f*x)*sin(e)^3 - d*cos(4*f*x)*sin(e)^4 -
I*d*sin(4*f*x)*sin(e)^4 - 4*c*f*cos(e)^2*cos_integral(2*(d*f*x + c*f)/d)*si
n(2*c*f/d) - 8*I*c*f*cos(e)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)*sin(2*c*
f/d) + 4*c*f*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2*sin(2*c*f/d) + 4*c*f*
cos(e)^2*cos(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 8*I*c*f*cos(e)*cos(
2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*f)/d) - 4*c*f*cos(2*c*f/d)*sin(e)
^2*sin_integral(2*(d*f*x + c*f)/d) - 4*I*c*f*cos(e)^2*sin(2*c*f/d)*sin_inte
gral(2*(d*f*x + c*f)/d) + 8*c*f*cos(e)*sin(e)*sin(2*c*f/d)*sin_integral(2*(
d*f*x + c*f)/d) + 4*I*c*f*sin(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f
)/d) + 2*d*cos(2*f*x)*cos(e)^2 + 2*I*d*cos(e)^2*sin(2*f*x) + 4*I*d*cos(2*f*
x)*cos(e)*sin(e) - 4*d*cos(e)*sin(2*f*x)*sin(e) - 2*d*cos(2*f*x)*sin(e)^2 -
2*I*d*sin(2*f*x)*sin(e)^2 - d)/(a^2*d^3*x + a^2*c*d^2)

```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c + dx)^2 (a + ia \cot(e + fx))^2} dx = \int \frac{1}{(a + a \cot(e + fx) li)^2 (c + dx)^2} dx$$

[In] int(1/((a + a*cot(e + f*x)*1i)^2*(c + d*x)^2), x)

[Out] int(1/((a + a*cot(e + f*x)*1i)^2*(c + d*x)^2), x)

3.27 $\int \frac{(c+dx)^3}{(a+ia \cot(e+fx))^3} dx$

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Optimal result

Integrand size = 23, antiderivative size = 396

$$\int \frac{(c+dx)^3}{(a+ia \cot(e+fx))^3} dx = \frac{9d^3 e^{2ie+2ifx}}{64a^3 f^4} - \frac{9d^3 e^{4ie+4ifx}}{1024a^3 f^4} + \frac{d^3 e^{6ie+6ifx}}{1728a^3 f^4} - \frac{9id^2 e^{2ie+2ifx}(c+dx)}{32a^3 f^3}$$

$$+ \frac{9id^2 e^{4ie+4ifx}(c+dx)}{256a^3 f^3} - \frac{id^2 e^{6ie+6ifx}(c+dx)}{288a^3 f^3}$$

$$- \frac{9de^{2ie+2ifx}(c+dx)^2}{32a^3 f^2} + \frac{9de^{4ie+4ifx}(c+dx)^2}{128a^3 f^2}$$

$$- \frac{de^{6ie+6ifx}(c+dx)^2}{96a^3 f^2} + \frac{3ie^{2ie+2ifx}(c+dx)^3}{16a^3 f}$$

$$- \frac{3ie^{4ie+4ifx}(c+dx)^3}{32a^3 f} + \frac{ie^{6ie+6ifx}(c+dx)^3}{48a^3 f} + \frac{(c+dx)^4}{32a^3 d}$$

```
[Out] 9/64*d^3*exp(2*I*e+2*I*f*x)/a^3/f^4-9/1024*d^3*exp(4*I*e+4*I*f*x)/a^3/f^4+1
/1728*d^3*exp(6*I*e+6*I*f*x)/a^3/f^4-9/32*I*d^2*exp(2*I*e+2*I*f*x)*(d*x+c)/
a^3/f^3+9/256*I*d^2*exp(4*I*e+4*I*f*x)*(d*x+c)/a^3/f^3-1/288*I*d^2*exp(6*I*
e+6*I*f*x)*(d*x+c)/a^3/f^3-9/32*d*exp(2*I*e+2*I*f*x)*(d*x+c)^2/a^3/f^2+9/12
8*d*exp(4*I*e+4*I*f*x)*(d*x+c)^2/a^3/f^2-1/96*d*exp(6*I*e+6*I*f*x)*(d*x+c)^
2/a^3/f^2+3/16*I*exp(2*I*e+2*I*f*x)*(d*x+c)^3/a^3/f-3/32*I*exp(4*I*e+4*I*f*
x)*(d*x+c)^3/a^3/f+1/48*I*exp(6*I*e+6*I*f*x)*(d*x+c)^3/a^3/f+1/32*(d*x+c)^4
/a^3/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3810, 2207, 2225}

$$\int \frac{(c + dx)^3}{(a + ia \cot(e + fx))^3} dx = -\frac{9id^2(c + dx)e^{2ie+2ifx}}{32a^3f^3} + \frac{9id^2(c + dx)e^{4ie+4ifx}}{256a^3f^3} - \frac{id^2(c + dx)e^{6ie+6ifx}}{288a^3f^3} - \frac{9d(c + dx)^2e^{2ie+2ifx}}{32a^3f^2} + \frac{9d(c + dx)^2e^{4ie+4ifx}}{128a^3f^2} - \frac{d(c + dx)^2e^{6ie+6ifx}}{96a^3f^2} + \frac{3i(c + dx)^3e^{2ie+2ifx}}{16a^3f} - \frac{3i(c + dx)^3e^{4ie+4ifx}}{32a^3f} + \frac{i(c + dx)^3e^{6ie+6ifx}}{48a^3f} + \frac{(c + dx)^4}{32a^3d} + \frac{9d^3e^{2ie+2ifx}}{64a^3f^4} - \frac{9d^3e^{4ie+4ifx}}{1024a^3f^4} + \frac{d^3e^{6ie+6ifx}}{1728a^3f^4}$$

[In] Int[(c + d*x)^3/(a + I*a*Cot[e + f*x])^3,x]

[Out] (9*d^3*E^((2*I)*e + (2*I)*f*x))/(64*a^3*f^4) - (9*d^3*E^((4*I)*e + (4*I)*f*x))/(1024*a^3*f^4) + (d^3*E^((6*I)*e + (6*I)*f*x))/(1728*a^3*f^4) - (((9*I)/32)*d^2*E^((2*I)*e + (2*I)*f*x)*(c + d*x))/(a^3*f^3) + (((9*I)/256)*d^2*E^((4*I)*e + (4*I)*f*x)*(c + d*x))/(a^3*f^3) - ((I/288)*d^2*E^((6*I)*e + (6*I)*f*x)*(c + d*x))/(a^3*f^3) - (9*d*E^((2*I)*e + (2*I)*f*x)*(c + d*x)^2)/(32*a^3*f^2) + (9*d*E^((4*I)*e + (4*I)*f*x)*(c + d*x)^2)/(128*a^3*f^2) - (d*E^((6*I)*e + (6*I)*f*x)*(c + d*x)^2)/(96*a^3*f^2) + (((3*I)/16)*E^((2*I)*e + (2*I)*f*x)*(c + d*x)^3)/(a^3*f) - (((3*I)/32)*E^((4*I)*e + (4*I)*f*x)*(c + d*x)^3)/(a^3*f) + ((I/48)*E^((6*I)*e + (6*I)*f*x)*(c + d*x)^3)/(a^3*f) + (c + d*x)^4/(32*a^3*d)

Rule 2207

Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[\$UseGamma]

Rule 2225

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 3810

Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*x)))/(2*a))^(n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(\frac{(c + dx)^3}{8a^3} - \frac{3e^{2ie+2ifx}(c + dx)^3}{8a^3} + \frac{3e^{4ie+4ifx}(c + dx)^3}{8a^3} - \frac{e^{6ie+6ifx}(c + dx)^3}{8a^3} \right) dx \\
 &= \frac{(c + dx)^4}{32a^3d} - \frac{\int e^{6ie+6ifx}(c + dx)^3 dx}{8a^3} - \frac{3 \int e^{2ie+2ifx}(c + dx)^3 dx}{8a^3} + \frac{3 \int e^{4ie+4ifx}(c + dx)^3 dx}{8a^3} \\
 &= \frac{3ie^{2ie+2ifx}(c + dx)^3}{16a^3f} - \frac{3ie^{4ie+4ifx}(c + dx)^3}{32a^3f} + \frac{ie^{6ie+6ifx}(c + dx)^3}{48a^3f} \\
 &\quad + \frac{(c + dx)^4}{32a^3d} - \frac{(id) \int e^{6ie+6ifx}(c + dx)^2 dx}{16a^3f} + \frac{(9id) \int e^{4ie+4ifx}(c + dx)^2 dx}{32a^3f} \\
 &\quad - \frac{(9id) \int e^{2ie+2ifx}(c + dx)^2 dx}{16a^3f} \\
 &= -\frac{9de^{2ie+2ifx}(c + dx)^2}{32a^3f^2} + \frac{9de^{4ie+4ifx}(c + dx)^2}{128a^3f^2} - \frac{de^{6ie+6ifx}(c + dx)^2}{96a^3f^2} \\
 &\quad + \frac{3ie^{2ie+2ifx}(c + dx)^3}{16a^3f} - \frac{3ie^{4ie+4ifx}(c + dx)^3}{32a^3f} \\
 &\quad + \frac{ie^{6ie+6ifx}(c + dx)^3}{48a^3f} + \frac{(c + dx)^4}{32a^3d} + \frac{d^2 \int e^{6ie+6ifx}(c + dx) dx}{48a^3f^2} \\
 &\quad - \frac{(9d^2) \int e^{4ie+4ifx}(c + dx) dx}{64a^3f^2} + \frac{(9d^2) \int e^{2ie+2ifx}(c + dx) dx}{16a^3f^2} \\
 &= -\frac{9id^2e^{2ie+2ifx}(c + dx)}{32a^3f^3} + \frac{9id^2e^{4ie+4ifx}(c + dx)}{256a^3f^3} - \frac{id^2e^{6ie+6ifx}(c + dx)}{288a^3f^3} \\
 &\quad - \frac{9de^{2ie+2ifx}(c + dx)^2}{32a^3f^2} + \frac{9de^{4ie+4ifx}(c + dx)^2}{128a^3f^2} - \frac{de^{6ie+6ifx}(c + dx)^2}{96a^3f^2} \\
 &\quad + \frac{3ie^{2ie+2ifx}(c + dx)^3}{16a^3f} - \frac{3ie^{4ie+4ifx}(c + dx)^3}{32a^3f} + \frac{ie^{6ie+6ifx}(c + dx)^3}{48a^3f} + \frac{(c + dx)^4}{32a^3d} \\
 &\quad + \frac{(id^3) \int e^{6ie+6ifx} dx}{288a^3f^3} - \frac{(9id^3) \int e^{4ie+4ifx} dx}{256a^3f^3} + \frac{(9id^3) \int e^{2ie+2ifx} dx}{32a^3f^3} \\
 &= \frac{9d^3e^{2ie+2ifx}}{64a^3f^4} - \frac{9d^3e^{4ie+4ifx}}{1024a^3f^4} + \frac{d^3e^{6ie+6ifx}}{1728a^3f^4} - \frac{9id^2e^{2ie+2ifx}(c + dx)}{32a^3f^3} + \frac{9id^2e^{4ie+4ifx}(c + dx)}{256a^3f^3} \\
 &\quad - \frac{id^2e^{6ie+6ifx}(c + dx)}{288a^3f^3} - \frac{9de^{2ie+2ifx}(c + dx)^2}{32a^3f^2} + \frac{9de^{4ie+4ifx}(c + dx)^2}{128a^3f^2} - \frac{de^{6ie+6ifx}(c + dx)^2}{96a^3f^2} \\
 &\quad + \frac{3ie^{2ie+2ifx}(c + dx)^3}{16a^3f} - \frac{3ie^{4ie+4ifx}(c + dx)^3}{32a^3f} + \frac{ie^{6ie+6ifx}(c + dx)^3}{48a^3f} + \frac{(c + dx)^4}{32a^3d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 3.03 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.68

$$\int \frac{(c + dx)^3}{(a + ia \cot(e + fx))^3} dx$$

$$= \frac{(\cos(3(e + fx)) + i \sin(3(e + fx))) (81i(32c^3 f^3 + 24c^2 df^2(3i + 4fx) + 12cd^2 f(-7 + 12ifx + 8f^2 x^2) + d^3))}{(27648a^3 f^4)}$$

[In] Integrate[(c + d*x)^3/(a + I*a*Cot[e + f*x])^3,x]

```
[Out] ((Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])*((81*I)*(32*c^3*f^3 + 24*c^2*d*f^2*(3*I + 4*f*x) + 12*c*d^2*f*(-7 + (12*I)*f*x + 8*f^2*x^2) + d^3*(-45*I - 84*f*x + (72*I)*f^2*x^2 + 32*f^3*x^3))*Cos[e + f*x] + 16*(36*c^3*f^3*(I + 6*f*x) + 18*c^2*d*f^2*(-1 + (6*I)*f*x + 18*f^2*x^2) + 6*c*d^2*f*(-I - 6*f*x + (18*I)*f^2*x^2 + 36*f^3*x^3) + d^3*(1 - (6*I)*f*x - 18*f^2*x^2 + (36*I)*f^3*x^3 + 54*f^4*x^4))*Cos[3*(e + f*x)] - (4131*I)*d^3*Sin[e + f*x] - 8748*c*d^2*f*Sin[e + f*x] + (9720*I)*c^2*d*f^2*Sin[e + f*x] + 7776*c^3*f^3*Sin[e + f*x] - 8748*d^3*f*x*Sin[e + f*x] + (19440*I)*c*d^2*f^2*x*Sin[e + f*x] + 23328*c^2*d*f^3*x*Sin[e + f*x] + (9720*I)*d^3*f^2*x^2*Sin[e + f*x] + 23328*c*d^2*f^3*x^2*Sin[e + f*x] + 7776*d^3*f^3*x^3*Sin[e + f*x] + (16*I)*d^3*Sin[3*(e + f*x)] + 96*c*d^2*f*Sin[3*(e + f*x)] - (288*I)*c^2*d*f^2*Sin[3*(e + f*x)] - 576*c^3*f^3*Sin[3*(e + f*x)] + 96*d^3*f*x*Sin[3*(e + f*x)] - (576*I)*c*d^2*f^2*x*Sin[3*(e + f*x)] - 1728*c^2*d*f^3*x*Sin[3*(e + f*x)] - (3456*I)*c^3*f^4*x*Sin[3*(e + f*x)] - (288*I)*d^3*f^2*x^2*Sin[3*(e + f*x)] - 1728*c*d^2*f^3*x^2*Sin[3*(e + f*x)] - (5184*I)*c^2*d*f^4*x^2*Sin[3*(e + f*x)] - 576*d^3*f^3*x^3*Sin[3*(e + f*x)] - (3456*I)*c*d^2*f^4*x^3*Sin[3*(e + f*x)] - (864*I)*d^3*f^4*x^4*Sin[3*(e + f*x)])))/(27648*a^3*f^4)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00

method	result
risch	$\frac{d^3 x^4}{32a^3} + \frac{d^2 c x^3}{8a^3} + \frac{3d c^2 x^2}{16a^3} + \frac{c^3 x}{8a^3} + \frac{c^4}{32a^3 d} + \frac{i(36d^3 x^3 f^3 + 108c d^2 f^3 x^2 + 18id^3 f^2 x^2 + 108c^2 d f^3 x + 36ic d^2 f^2 x + 36i^2 c^2 f^2 x + 36i^3 c^2 f^2 x)}{1728a^3 f^4}$
parallelrisch	$-6264 \left(-\frac{4 \left(\frac{dx}{2} + c \right) \left(\frac{1}{2} d^2 x^2 + cdx + c^2 \right) f^3}{29} + id \left(\frac{1}{3} d^2 x^2 + cdx + c^2 \right) f^2 - \frac{139 \left(\frac{dx}{2} + c \right) d^2 f}{174} - \frac{737id^3}{2088} \right) fx \tan(fx + e)^3 + \left(2592i \left(\frac{dx}{2} \right) \right)$
derivativedivides	Expression too large to display
default	Expression too large to display

[In] int((d*x+c)^3/(a+I*a*cot(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] $\frac{1}{32/a^3 d^3 x^4 + 1/8/a^3 d^2 c x^3 + 3/16/a^3 d c^2 x^2 + 1/8/a^3 c^3 x + 1/32/a^3 d^4 + 1/1728 I (36 d^3 x^3 f^3 + 18 I d^3 f^2 x^2 + 108 c d^2 f^3 x^2 + 36 I c d^2 f^2 x + 108 c^2 d f^3 x + 18 I c^2 d f^2 + 36 c^3 f^3 - 6 d^3 f x - I d^3 - 6 c d^2 f) / a^3 / f^4 \exp(6 I (f x + e)) - 3/1024 I (32 d^3 x^3 f^3 + 24 I d^3 f^2 x^2 + 96 c d^2 f^3 x^2 + 48 I c d^2 f^2 x + 96 c^2 d f^3 x + 24 I c^2 d f^2 + 32 c^3 f^3 - 12 d^3 f x - 3 I d^3 - 12 c d^2 f) / a^3 / f^4 \exp(4 I (f x + e)) + 3/64 I (4 d^3 x^3 f^3 + 6 I d^3 f^2 x^2 + 12 c d^2 f^3 x^2 + 12 I c d^2 f^2 x + 12 c^2 d f^3 x + 6 I c^2 d f^2 + 4 c^3 f^3 - 6 d^3 f x - 3 I d^3 - 6 c d^2 f) / a^3 / f^4 \exp(2 I (f x + e))$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.91

$$\int \frac{(c + dx)^3}{(a + ia \cot(e + fx))^3} dx$$

$$= \frac{864 d^3 f^4 x^4 + 3456 cd^2 f^4 x^3 + 5184 c^2 df^4 x^2 + 3456 c^3 f^4 x - 16(-36i d^3 f^3 x^3 - 36i c^3 f^3 + 18 c^2 df^2 + 6i cd^2 f)}{}$$

[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")

[Out] $\frac{1}{27648} (864 d^3 f^4 x^4 + 3456 c d^2 f^4 x^3 + 5184 c^2 d f^4 x^2 + 3456 c^3 f^4 x - 16(-36 I d^3 f^3 x^3 - 36 I c^3 f^3 + 18 c^2 d f^2 + 6 I c d^2 f - d^3 + 18(-6 I c d^2 f^3 + d^3 f^2) x^2 + 6(-18 I c^2 d f^3 + 6 c d^2 f^2 + I d^3 f) x) e^{(6 I f x + 6 I e)} - 81(32 I d^3 f^3 x^3 + 32 I c^3 f^3 - 24 c^2 d f^2 - 12 I c d^2 f + 3 d^3 + 24(4 I c d^2 f^3 - d^3 f^2) x^2 + 12(8 I c^2 d f^3 - 4 c d^2 f^2 - I d^3 f) x) e^{(4 I f x + 4 I e)} - 1296(-4 I d^3 f^3 x^3 - 4 I c^3 f^3 + 6 c^2 d f^2 + 6 I c d^2 f - 3 d^3 + 6(-2 I c d^2 f^3 + d^3 f^2) x^2 + 6(-2 I c^2 d f^3 + 2 c d^2 f^2 + I d^3 f) x) e^{(2 I f x + 2 I e)} / (a^3 f^4)$

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 932, normalized size of antiderivative = 2.35

$$\int \frac{(c + dx)^3}{(a + ia \cot(e + fx))^3} dx$$

$$= \left\{ \frac{(21233664 i a^6 c^3 f^{11} e^{2ie} + 63700992 i a^6 c^2 d f^{11} x e^{2ie} - 31850496 a^6 c^2 d f^{10} e^{2ie} + 63700992 i a^6 c d^2 f^{11} x^2 e^{2ie} - 63700992 a^6 c d^2 f^{10} x e^{2ie} - 31850496 i a^6 c d^2 f^9 e^{2ie})}{32 a^3} + \frac{x^3 (-c d^2 e^{6ie} + 3 c d^2 e^{4ie} - 3 c d^2 e^{2ie})}{8 a^3} + \frac{x^2 (-3 c^2 d e^{6ie} + 9 c^2 d e^{4ie} - 9 c^2 d e^{2ie})}{16 a^3} + \frac{x (-c^3 e^{6ie} + 3 c^3 e^{4ie} - 3 c^3 e^{2ie})}{8 a^3} \right.$$

$$\left. + \frac{c^3 x}{8 a^3} + \frac{3 c^2 d x^2}{16 a^3} + \frac{c d^2 x^3}{8 a^3} + \frac{d^3 x^4}{32 a^3} \right.$$

[In] integrate((d*x+c)**3/(a+I*a*cot(f*x+e))**3,x)

```
[Out] Piecewise((((21233664*I*a**6*c**3*f**11*exp(2*I*e) + 63700992*I*a**6*c**2*d
*f**11*x*exp(2*I*e) - 31850496*a**6*c**2*d*f**10*exp(2*I*e) + 63700992*I*a*
*6*c*d**2*f**11*x**2*exp(2*I*e) - 63700992*a**6*c*d**2*f**10*x*exp(2*I*e) -
31850496*I*a**6*c*d**2*f**9*exp(2*I*e) + 21233664*I*a**6*d**3*f**11*x**3*exp(2*I*e) -
31850496*a**6*d**3*f**10*x**2*exp(2*I*e) - 31850496*I*a**6*d**3
*f**9*x*exp(2*I*e) + 15925248*a**6*d**3*f**8*exp(2*I*e))*exp(2*I*f*x) + (-1
0616832*I*a**6*c**3*f**11*exp(4*I*e) - 31850496*I*a**6*c**2*d*f**11*x*exp(4
*I*e) + 7962624*a**6*c**2*d*f**10*exp(4*I*e) - 31850496*I*a**6*c*d**2*f**11
*x**2*exp(4*I*e) + 15925248*a**6*c*d**2*f**10*x*exp(4*I*e) + 3981312*I*a**6
*c*d**2*f**9*exp(4*I*e) - 10616832*I*a**6*d**3*f**11*x**3*exp(4*I*e) + 7962
624*a**6*d**3*f**10*x**2*exp(4*I*e) + 3981312*I*a**6*d**3*f**9*x*exp(4*I*e)
- 995328*a**6*d**3*f**8*exp(4*I*e))*exp(4*I*f*x) + (2359296*I*a**6*c**3*f*
*11*exp(6*I*e) + 7077888*I*a**6*c**2*d*f**11*x*exp(6*I*e) - 1179648*a**6*c*
*2*d*f**10*exp(6*I*e) + 7077888*I*a**6*c*d**2*f**11*x**2*exp(6*I*e) - 23592
96*a**6*c*d**2*f**10*x*exp(6*I*e) - 393216*I*a**6*c*d**2*f**9*exp(6*I*e) +
2359296*I*a**6*d**3*f**11*x**3*exp(6*I*e) - 1179648*a**6*d**3*f**10*x**2*exp(6*I*e) -
393216*I*a**6*d**3*f**9*x*exp(6*I*e) + 65536*a**6*d**3*f**8*exp(6*I*e))*exp(6*I*f*x)/(113246208*a**9*f**12), Ne(a**9*f**12, 0)), (x**4*(-d
**3*exp(6*I*e) + 3*d**3*exp(4*I*e) - 3*d**3*exp(2*I*e))/(32*a**3) + x**3*(-
c*d**2*exp(6*I*e) + 3*c*d**2*exp(4*I*e) - 3*c*d**2*exp(2*I*e))/(8*a**3) + x
**2*(-3*c**2*d*exp(6*I*e) + 9*c**2*d*exp(4*I*e) - 9*c**2*d*exp(2*I*e))/(16*
a**3) + x*(-c**3*exp(6*I*e) + 3*c**3*exp(4*I*e) - 3*c**3*exp(2*I*e))/(8*a**
3), True)) + c**3*x/(8*a**3) + 3*c**2*d*x**2/(16*a**3) + c*d**2*x**3/(8*a**
3) + d**3*x**4/(32*a**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + dx)^3}{(a + ia \cot(e + fx))^3} dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*x+c)^3/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```


Mupad [B] (verification not implemented)

Time = 13.93 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.06

$$\int \frac{(c + dx)^3}{(a + ia \cot(e + fx))^3} dx = e^{e^{2i+fx} 2i} \left(-\frac{(-12c^3 f^3 - c^2 d f^2 18i + 18c d^2 f + d^3 9i) 1i}{64 a^3 f^4} \right. \\ \left. + \frac{d^3 x^3 3i}{16 a^3 f} + \frac{dx (2c^2 f^2 + c d f 2i - d^2) 9i}{32 a^3 f^3} + \frac{d^2 x^2 (2c f + d 1i) 9i}{32 a^3 f^2} \right) \\ - e^{e^{4i+fx} 4i} \left(-\frac{(-96c^3 f^3 - c^2 d f^2 72i + 36c d^2 f + d^3 9i) 1i}{1024 a^3 f^4} \right. \\ \left. + \frac{d^3 x^3 3i}{32 a^3 f} + \frac{dx (8c^2 f^2 + c d f 4i - d^2) 9i}{256 a^3 f^3} + \frac{d^2 x^2 (4c f + d 1i) 9i}{128 a^3 f^2} \right) \\ + e^{e^{6i+fx} 6i} \left(-\frac{(-36c^3 f^3 - c^2 d f^2 18i + 6c d^2 f + d^3 1i) 1i}{1728 a^3 f^4} \right. \\ \left. + \frac{d^3 x^3 1i}{48 a^3 f} + \frac{dx (18c^2 f^2 + c d f 6i - d^2) 1i}{288 a^3 f^3} + \frac{d^2 x^2 (6c f + d 1i) 1i}{96 a^3 f^2} \right) + \frac{c^3 x}{8 a^3} + \frac{d^3 x^4}{32 a^3} + \frac{3c^2 d x^2}{16 a^3} + \frac{c d^2 x^3}{8 a^3}$$

[In] int((c + d*x)^3/(a + a*cot(e + f*x)*1i)^3,x)

```
[Out] exp(e*2i + f*x*2i)*((d^3*x^3*3i)/(16*a^3*f) - ((d^3*9i - 12*c^3*f^3 - c^2*d
*f^2*18i + 18*c*d^2*f)*1i)/(64*a^3*f^4) + (d*x*(2*c^2*f^2 - d^2 + c*d*f*2i)
*9i)/(32*a^3*f^3) + (d^2*x^2*(d*1i + 2*c*f)*9i)/(32*a^3*f^2)) - exp(e*4i +
f*x*4i)*((d^3*x^3*3i)/(32*a^3*f) - ((d^3*9i - 96*c^3*f^3 - c^2*d*f^2*72i +
36*c*d^2*f)*1i)/(1024*a^3*f^4) + (d*x*(8*c^2*f^2 - d^2 + c*d*f*4i)*9i)/(256
*a^3*f^3) + (d^2*x^2*(d*1i + 4*c*f)*9i)/(128*a^3*f^2)) + exp(e*6i + f*x*6i)
*((d^3*x^3*1i)/(48*a^3*f) - ((d^3*1i - 36*c^3*f^3 - c^2*d*f^2*18i + 6*c*d^2
*f)*1i)/(1728*a^3*f^4) + (d*x*(18*c^2*f^2 - d^2 + c*d*f*6i)*1i)/(288*a^3*f^
3) + (d^2*x^2*(d*1i + 6*c*f)*1i)/(96*a^3*f^2)) + (c^3*x)/(8*a^3) + (d^3*x^4
)/(32*a^3) + (3*c^2*d*x^2)/(16*a^3) + (c*d^2*x^3)/(8*a^3)
```

3.28 $\int \frac{(c+dx)^2}{(a+ia \cot(e+fx))^3} dx$

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Optimal result

Integrand size = 23, antiderivative size = 294

$$\int \frac{(c+dx)^2}{(a+ia \cot(e+fx))^3} dx = -\frac{3id^2e^{2ie+2ifx}}{32a^3f^3} + \frac{3id^2e^{4ie+4ifx}}{256a^3f^3} - \frac{id^2e^{6ie+6ifx}}{864a^3f^3} - \frac{3de^{2ie+2ifx}(c+dx)}{16a^3f^2} + \frac{3de^{4ie+4ifx}(c+dx)}{64a^3f^2} - \frac{de^{6ie+6ifx}(c+dx)}{144a^3f^2} + \frac{3ie^{2ie+2ifx}(c+dx)^2}{16a^3f} - \frac{3ie^{4ie+4ifx}(c+dx)^2}{32a^3f} + \frac{ie^{6ie+6ifx}(c+dx)^2}{48a^3f} + \frac{(c+dx)^3}{24a^3d}$$

[Out] $-3/32*I*d^2*\exp(2*I*e+2*I*f*x)/a^3/f^3+3/256*I*d^2*\exp(4*I*e+4*I*f*x)/a^3/f^3-1/864*I*d^2*\exp(6*I*e+6*I*f*x)/a^3/f^3-3/16*d*\exp(2*I*e+2*I*f*x)*(d*x+c)/a^3/f^2+3/64*d*\exp(4*I*e+4*I*f*x)*(d*x+c)/a^3/f^2-1/144*d*\exp(6*I*e+6*I*f*x)*(d*x+c)/a^3/f^2+3/16*I*\exp(2*I*e+2*I*f*x)*(d*x+c)^2/a^3/f-3/32*I*\exp(4*I*e+4*I*f*x)*(d*x+c)^2/a^3/f+1/48*I*\exp(6*I*e+6*I*f*x)*(d*x+c)^2/a^3/f+1/24*(d*x+c)^3/a^3/d$

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used

= {3810, 2207, 2225}

$$\int \frac{(c+dx)^2}{(a+ia \cot(e+fx))^3} dx = -\frac{3d(c+dx)e^{2ie+2ifx}}{16a^3f^2} + \frac{3d(c+dx)e^{4ie+4ifx}}{64a^3f^2} - \frac{d(c+dx)e^{6ie+6ifx}}{144a^3f^2} + \frac{3i(c+dx)^2e^{2ie+2ifx}}{16a^3f} - \frac{3i(c+dx)^2e^{4ie+4ifx}}{32a^3f} + \frac{i(c+dx)^2e^{6ie+6ifx}}{48a^3f} + \frac{(c+dx)^3}{24a^3d} - \frac{3id^2e^{2ie+2ifx}}{32a^3f^3} + \frac{3id^2e^{4ie+4ifx}}{256a^3f^3} - \frac{id^2e^{6ie+6ifx}}{864a^3f^3}$$

[In] Int[(c + d*x)^2/(a + I*a*Cot[e + f*x])^3,x]

[Out] (((-3*I)/32)*d^2*E^((2*I)*e + (2*I)*f*x))/(a^3*f^3) + (((3*I)/256)*d^2*E^((4*I)*e + (4*I)*f*x))/(a^3*f^3) - ((I/864)*d^2*E^((6*I)*e + (6*I)*f*x))/(a^3*f^3) - (3*d*E^((2*I)*e + (2*I)*f*x)*(c + d*x))/(16*a^3*f^2) + (3*d*E^((4*I)*e + (4*I)*f*x)*(c + d*x))/(64*a^3*f^2) - (d*E^((6*I)*e + (6*I)*f*x)*(c + d*x))/(144*a^3*f^2) + (((3*I)/16)*E^((2*I)*e + (2*I)*f*x)*(c + d*x)^2)/(a^3*f) - (((3*I)/32)*E^((4*I)*e + (4*I)*f*x)*(c + d*x)^2)/(a^3*f) + ((I/48)*E^((6*I)*e + (6*I)*f*x)*(c + d*x)^2)/(a^3*f) + (c + d*x)^3/(24*a^3*d)

Rule 2207

Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[\$UseGamma]

Rule 2225

Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 3810

Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*x)))/(2*a)]^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, 0]

Rubi steps

$$\text{integral} = \int \left(\frac{(c+dx)^2}{8a^3} - \frac{3e^{2ie+2ifx}(c+dx)^2}{8a^3} + \frac{3e^{4ie+4ifx}(c+dx)^2}{8a^3} - \frac{e^{6ie+6ifx}(c+dx)^2}{8a^3} \right) dx$$

$$\begin{aligned}
&= \frac{(c+dx)^3}{24a^3d} - \frac{\int e^{6ie+6ifx}(c+dx)^2 dx}{8a^3} - \frac{3 \int e^{2ie+2ifx}(c+dx)^2 dx}{8a^3} + \frac{3 \int e^{4ie+4ifx}(c+dx)^2 dx}{8a^3} \\
&= \frac{3ie^{2ie+2ifx}(c+dx)^2}{16a^3f} - \frac{3ie^{4ie+4ifx}(c+dx)^2}{32a^3f} + \frac{ie^{6ie+6ifx}(c+dx)^2}{48a^3f} \\
&\quad + \frac{(c+dx)^3}{24a^3d} - \frac{(id) \int e^{6ie+6ifx}(c+dx) dx}{24a^3f} + \frac{(3id) \int e^{4ie+4ifx}(c+dx) dx}{16a^3f} \\
&\quad - \frac{(3id) \int e^{2ie+2ifx}(c+dx) dx}{8a^3f} \\
&= -\frac{3de^{2ie+2ifx}(c+dx)}{16a^3f^2} + \frac{3de^{4ie+4ifx}(c+dx)}{64a^3f^2} - \frac{de^{6ie+6ifx}(c+dx)}{144a^3f^2} \\
&\quad + \frac{3ie^{2ie+2ifx}(c+dx)^2}{16a^3f} - \frac{3ie^{4ie+4ifx}(c+dx)^2}{32a^3f} + \frac{ie^{6ie+6ifx}(c+dx)^2}{48a^3f} \\
&\quad + \frac{(c+dx)^3}{24a^3d} + \frac{d^2 \int e^{6ie+6ifx} dx}{144a^3f^2} - \frac{(3d^2) \int e^{4ie+4ifx} dx}{64a^3f^2} + \frac{(3d^2) \int e^{2ie+2ifx} dx}{16a^3f^2} \\
&= -\frac{3id^2e^{2ie+2ifx}}{32a^3f^3} + \frac{3id^2e^{4ie+4ifx}}{256a^3f^3} - \frac{id^2e^{6ie+6ifx}}{864a^3f^3} - \frac{3de^{2ie+2ifx}(c+dx)}{16a^3f^2} \\
&\quad + \frac{3de^{4ie+4ifx}(c+dx)}{64a^3f^2} - \frac{de^{6ie+6ifx}(c+dx)}{144a^3f^2} + \frac{3ie^{2ie+2ifx}(c+dx)^2}{16a^3f} \\
&\quad - \frac{3ie^{4ie+4ifx}(c+dx)^2}{32a^3f} + \frac{ie^{6ie+6ifx}(c+dx)^2}{48a^3f} + \frac{(c+dx)^3}{24a^3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)^2}{(a+ia \cot(e+fx))^3} dx$$

$$= \frac{288f^3x(3c^2+3cdx+d^2x^2)+648((1+i)cf+d(-1+(1+i)fx))((1+i)cf+d(i+(1+i)fx))\cos(2fx)}{
}$$

[In] Integrate[(c + d*x)^2/(a + I*a*Cot[e + f*x])^3,x]

[Out] (288*f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 648*((1 + I)*c*f + d*(-1 + (1 + I)*f*x))*((1 + I)*c*f + d*(I + (1 + I)*f*x))*Cos[2*f*x]*(Cos[2*e] + I*Sin[2*e]) - 81*((2 + 2*I)*c*f + d*(-1 + (2 + 2*I)*f*x))*((2 + 2*I)*c*f + d*(I + (2 + 2*I)*f*x))*Cos[4*f*x]*(Cos[4*e] + I*Sin[4*e]) + 8*((3 + 3*I)*c*f + d*(-1 + (3 + 3*I)*f*x))*((3 + 3*I)*c*f + d*(I + (3 + 3*I)*f*x))*Cos[6*f*x]*(Cos[6*e] + I*Sin[6*e]) + (648*I)*((1 + I)*c*f + d*(-1 + (1 + I)*f*x))*((1 + I)*c*f + d*(I + (1 + I)*f*x))*(Cos[2*e] + I*Sin[2*e])*Sin[2*f*x] - 81*(d - (2 + 2*I)*c*f - (2 + 2*I)*d*f*x)*(d + (2 - 2*I)*c*f + (2 - 2*I)*d*f*x)*(Cos[4*e] + I*Sin[4*e])*Sin[4*f*x] + (8*I)*((3 + 3*I)*c*f + d*(-1 + (3 + 3*I)*f*x))*((3 + 3*I)*c*f + d*(I + (3 + 3*I)*f*x))*(Cos[6*e] + I*Sin[6*e])*Sin[6*f*x])/ (6912*a^3*f^3)

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.81

method	result
risch	$\frac{d^2x^3}{24a^3} + \frac{dcx^2}{8a^3} + \frac{c^2x}{8a^3} + \frac{c^3}{24a^3d} + \frac{i(18d^2x^2f^2+36cdf^2x+6id^2fx+18c^2f^2+6icdf-d^2)e^{6i(fx+e)}}{864a^3f^3} - \frac{3i(8d^2x^2f^2+18c^2f^2+6icdf-d^2)e^{6i(fx+e)}}{864a^3f^3}$
parallelrisch	$\frac{3(24(d^2x^3+3dx^2c+3xc^2)f^3+6i(-29d^2x^2-58cdx-36c^2)f^2+(139d^2x+180cd)f+81id^2)\tan(fx+e)^3+6\left(108i\left(\frac{1}{3}d^2x^2+dx+c\right)\tan(fx+e)^2+36i(d^2x+cd)\tan(fx+e)+36icd\right)}{864a^3f^3}$
derivativdivides	Expression too large to display
default	Expression too large to display

[In] int((d*x+c)^2/(a+I*a*cot(f*x+e))^3,x,method=_RETURNVERBOSE)

```
[Out] 1/24/a^3*d^2*x^3+1/8/a^3*d*c*x^2+1/8/a^3*c^2*x+1/24/a^3/d*c^3+1/864*I*(18*d^2*x^2*f^2+6*I*d^2*f*x+36*c*d*f^2*x+6*I*c*d*f+18*c^2*f^2-d^2)/a^3/f^3*exp(6*I*(f*x+e))-3/256*I*(8*d^2*x^2*f^2+4*I*d^2*f*x+16*c*d*f^2*x+4*I*c*d*f+8*c^2*f^2-d^2)/a^3/f^3*exp(4*I*(f*x+e))+3/32*I*(2*d^2*x^2*f^2+2*I*d^2*f*x+4*c*d*f^2*x+2*I*c*d*f+2*c^2*f^2-d^2)/a^3/f^3*exp(2*I*(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.72

$$\int \frac{(c + dx)^2}{(a + ia \cot(e + fx))^3} dx$$

$$= \frac{288 d^2 f^3 x^3 + 864 c d f^3 x^2 + 864 c^2 f^3 x - 8(-18i d^2 f^2 x^2 - 18i c^2 f^2 + 6 c d f + i d^2 + 6(-6i c d f^2 + d^2 f)x)e^{6i(fx+e)}}{864 a^3 f^3}$$

[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")

```
[Out] 1/6912*(288*d^2*f^3*x^3 + 864*c*d*f^3*x^2 + 864*c^2*f^3*x - 8*(-18*I*d^2*f^2*x^2 - 18*I*c^2*f^2 + 6*c*d*f + I*d^2 + 6*(-6*I*c*d*f^2 + d^2*f)*x)*e^(6*I*f*x + 6*I*e) - 81*(8*I*d^2*f^2*x^2 + 8*I*c^2*f^2 - 4*c*d*f - I*d^2 + 4*(4*I*c*d*f^2 - d^2*f)*x)*e^(4*I*f*x + 4*I*e) - 648*(-2*I*d^2*f^2*x^2 - 2*I*c^2*f^2 + 2*c*d*f + I*d^2 + 2*(-2*I*c*d*f^2 + d^2*f)*x)*e^(2*I*f*x + 2*I*e))/(a^3*f^3)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.96

$$\int \frac{(c + dx)^2}{(a + ia \cot(e + fx))^3} dx$$

$$= \left\{ \frac{(1327104ia^6c^2f^8e^{2ie} + 2654208ia^6cdf^8xe^{2ie} - 1327104a^6cdf^7e^{2ie} + 1327104ia^6d^2f^8x^2e^{2ie} - 1327104a^6d^2f^7xe^{2ie} - 663552ia^6d^2f^6e^{2ie})e^{2ifx} + (-d^2e^{6ie} + 3d^2e^{4ie} - 3d^2e^{2ie})}{24a^3} + \frac{x^2(-cde^{6ie} + 3cde^{4ie} - 3cde^{2ie})}{8a^3} + \frac{x(-c^2e^{6ie} + 3c^2e^{4ie} - 3c^2e^{2ie})}{8a^3} \right.$$

$$\left. + \frac{c^2x}{8a^3} + \frac{cdx^2}{8a^3} + \frac{d^2x^3}{24a^3} \right.$$

[In] integrate((d*x+c)**2/(a+I*a*cot(f*x+e))**3,x)

[Out] Piecewise((((1327104*I*a**6*c**2*f**8*exp(2*I*e) + 2654208*I*a**6*c*d*f**8*x*exp(2*I*e) - 1327104*a**6*c*d*f**7*exp(2*I*e) + 1327104*I*a**6*d**2*f**8*x**2*exp(2*I*e) - 1327104*a**6*d**2*f**7*x*exp(2*I*e) - 663552*I*a**6*d**2*f**6*exp(2*I*e))*exp(2*I*f*x) + (-663552*I*a**6*c**2*f**8*exp(4*I*e) - 1327104*I*a**6*c*d*f**8*x*exp(4*I*e) + 331776*a**6*c*d*f**7*exp(4*I*e) - 663552*I*a**6*d**2*f**8*x**2*exp(4*I*e) + 331776*a**6*d**2*f**7*x*exp(4*I*e) + 82944*I*a**6*d**2*f**6*exp(4*I*e))*exp(4*I*f*x) + (147456*I*a**6*c**2*f**8*exp(6*I*e) + 294912*I*a**6*c*d*f**8*x*exp(6*I*e) - 49152*a**6*c*d*f**7*exp(6*I*e) + 147456*I*a**6*d**2*f**8*x**2*exp(6*I*e) - 49152*a**6*d**2*f**7*x*exp(6*I*e) - 8192*I*a**6*d**2*f**6*exp(6*I*e))*exp(6*I*f*x))/(7077888*a**9*f**9), Ne(a**9*f**9, 0)), (x**3*(-d**2*exp(6*I*e) + 3*d**2*exp(4*I*e) - 3*d**2*exp(2*I*e))/(24*a**3) + x**2*(-c*d*exp(6*I*e) + 3*c*d*exp(4*I*e) - 3*c*d*exp(2*I*e))/(8*a**3) + x*(-c**2*exp(6*I*e) + 3*c**2*exp(4*I*e) - 3*c**2*exp(2*I*e))/(8*a**3), True)) + c**2*x/(8*a**3) + c*d*x**2/(8*a**3) + d**2*x**3/(24*a**3)

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + dx)^2}{(a + ia \cot(e + fx))^3} dx = \text{Exception raised: RuntimeError}$$

[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.13

$$\int \frac{(c + dx)^2}{(a + ia \cot(e + fx))^3} dx$$

$$= \frac{288 d^2 f^3 x^3 + 864 c d f^3 x^2 + 144 i d^2 f^2 x^2 e^{(6i fx + 6i e)} - 648 i d^2 f^2 x^2 e^{(4i fx + 4i e)} + 1296 i d^2 f^2 x^2 e^{(2i fx + 2i e)} + 864$$

[In] integrate((d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")

[Out] 1/6912*(288*d^2*f^3*x^3 + 864*c*d*f^3*x^2 + 144*I*d^2*f^2*x^2*e^(6*I*f*x + 6*I*e) - 648*I*d^2*f^2*x^2*e^(4*I*f*x + 4*I*e) + 1296*I*d^2*f^2*x^2*e^(2*I*f*x + 2*I*e) + 864*c^2*f^3*x + 288*I*c*d*f^2*x*e^(6*I*f*x + 6*I*e) - 1296*I*c*d*f^2*x*e^(4*I*f*x + 4*I*e) + 2592*I*c*d*f^2*x*e^(2*I*f*x + 2*I*e) + 144*I*c^2*f^2*e^(6*I*f*x + 6*I*e) - 48*d^2*f*x*e^(6*I*f*x + 6*I*e) - 648*I*c^2*f^2*e^(4*I*f*x + 4*I*e) + 324*d^2*f*x*e^(4*I*f*x + 4*I*e) + 1296*I*c^2*f^2*e^(2*I*f*x + 2*I*e) - 1296*d^2*f*x*e^(2*I*f*x + 2*I*e) - 48*c*d*f*e^(6*I*f*x + 6*I*e) + 324*c*d*f*e^(4*I*f*x + 4*I*e) - 1296*c*d*f*e^(2*I*f*x + 2*I*e) - 8*I*d^2*e^(6*I*f*x + 6*I*e) + 81*I*d^2*e^(4*I*f*x + 4*I*e) - 648*I*d^2*e^(2*I*f*x + 2*I*e))/(a^3*f^3)

Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.89

$$\int \frac{(c + dx)^2}{(a + ia \cot(e + fx))^3} dx = e^{e^{2i+fx} 2i} \left(\frac{(6c^2 f^2 + c d f 6i - 3d^2) 1i}{32 a^3 f^3} + \frac{d^2 x^2 3i}{16 a^3 f} + \frac{d x (2 c f + d 1i) 3i}{16 a^3 f^2} \right)$$

$$- e^{e^{4i+fx} 4i} \left(\frac{(24c^2 f^2 + c d f 12i - 3d^2) 1i}{256 a^3 f^3} + \frac{d^2 x^2 3i}{32 a^3 f} + \frac{d x (4 c f + d 1i) 3i}{64 a^3 f^2} \right)$$

$$+ e^{e^{6i+fx} 6i} \left(\frac{(18c^2 f^2 + c d f 6i - d^2) 1i}{864 a^3 f^3} + \frac{d^2 x^2 1i}{48 a^3 f} + \frac{d x (6 c f + d 1i) 1i}{144 a^3 f^2} \right) + \frac{c^2 x}{8 a^3} + \frac{d^2 x^3}{24 a^3} + \frac{c d x^2}{8 a^3}$$

[In] int((c + d*x)^2/(a + a*cot(e + f*x)*1i)^3,x)

[Out] exp(e*2i + f*x*2i)*(((6*c^2*f^2 - 3*d^2 + c*d*f*6i)*1i)/(32*a^3*f^3) + (d^2*x^2*3i)/(16*a^3*f) + (d*x*(d*1i + 2*c*f)*3i)/(16*a^3*f^2)) - exp(e*4i + f*

$$\begin{aligned}
& x^4 i) * (((24 * c^2 * f^2 - 3 * d^2 + c * d * f * 12 i) * 1 i) / (256 * a^3 * f^3) + (d^2 * x^2 * 3 i) / (\\
& 32 * a^3 * f) + (d * x * (d * 1 i + 4 * c * f) * 3 i) / (64 * a^3 * f^2)) + \exp(e * 6 i + f * x * 6 i) * (((1 \\
& 8 * c^2 * f^2 - d^2 + c * d * f * 6 i) * 1 i) / (864 * a^3 * f^3) + (d^2 * x^2 * 1 i) / (48 * a^3 * f) + (\\
& d * x * (d * 1 i + 6 * c * f) * 1 i) / (144 * a^3 * f^2)) + (c^2 * x) / (8 * a^3) + (d^2 * x^3) / (24 * a^3 \\
&) + (c * d * x^2) / (8 * a^3)
\end{aligned}$$

3.29 $\int \frac{c+dx}{(a+ia \cot(e+fx))^3} dx$

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Optimal result

Integrand size = 21, antiderivative size = 209

$$\int \frac{c+dx}{(a+ia \cot(e+fx))^3} dx = \frac{11idx}{96a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia \cot(e+fx))^3}$$

$$- \frac{i(c+dx)}{6f(a+ia \cot(e+fx))^3} + \frac{5d}{96af^2(a+ia \cot(e+fx))^2}$$

$$- \frac{i(c+dx)}{8af(a+ia \cot(e+fx))^2} + \frac{11d}{96f^2(a^3+ia^3 \cot(e+fx))}$$

$$- \frac{i(c+dx)}{8f(a^3+ia^3 \cot(e+fx))}$$

[Out] 11/96*I*d*x/a^3/f-1/16*d*x^2/a^3+1/8*x*(d*x+c)/a^3+1/36*d/f^2/(a+I*a*cot(f*x+e))^3-1/6*I*(d*x+c)/f/(a+I*a*cot(f*x+e))^3+5/96*d/a/f^2/(a+I*a*cot(f*x+e))^2-1/8*I*(d*x+c)/a/f/(a+I*a*cot(f*x+e))^2+11/96*d/f^2/(a^3+I*a^3*cot(f*x+e))-1/8*I*(d*x+c)/f/(a^3+I*a^3*cot(f*x+e))

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used

= {3560, 8, 3811}

$$\int \frac{c + dx}{(a + ia \cot(e + fx))^3} dx = -\frac{i(c + dx)}{8f(a^3 + ia^3 \cot(e + fx))} + \frac{x(c + dx)}{8a^3}$$

$$+ \frac{11d}{96f^2(a^3 + ia^3 \cot(e + fx))} + \frac{11idx}{96a^3f} - \frac{dx^2}{16a^3}$$

$$- \frac{i(c + dx)}{8af(a + ia \cot(e + fx))^2} - \frac{i(c + dx)}{6f(a + ia \cot(e + fx))^3}$$

$$+ \frac{5d}{96af^2(a + ia \cot(e + fx))^2} + \frac{d}{36f^2(a + ia \cot(e + fx))^3}$$

[In] Int[(c + d*x)/(a + I*a*Cot[e + f*x])^3,x]

[Out] (((11*I)/96)*d*x)/(a^3*f) - (d*x^2)/(16*a^3) + (x*(c + d*x))/(8*a^3) + d/(36*f^2*(a + I*a*Cot[e + f*x])^3) - ((I/6)*(c + d*x))/(f*(a + I*a*Cot[e + f*x])^3) + (5*d)/(96*a*f^2*(a + I*a*Cot[e + f*x])^2) - ((I/8)*(c + d*x))/(a*f*(a + I*a*Cot[e + f*x])^2) + (11*d)/(96*f^2*(a^3 + I*a^3*Cot[e + f*x])) - ((I/8)*(c + d*x))/(f*(a^3 + I*a^3*Cot[e + f*x]))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3560

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[a*((a + b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 3811

Int[((c_) + (d_)*(x_))^(m_)*((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[Dist[(c + d*x)^(m - 1), u, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]

Rubi steps

$$\text{integral} = \frac{x(c + dx)}{8a^3} - \frac{i(c + dx)}{6f(a + ia \cot(e + fx))^3} - \frac{i(c + dx)}{8af(a + ia \cot(e + fx))^2}$$

$$- \frac{i(c + dx)}{8f(a^3 + ia^3 \cot(e + fx))} - d \int \left(\frac{x}{8a^3} - \frac{i}{6f(a + ia \cot(e + fx))^3} \right.$$

$$\left. - \frac{i}{8af(a + ia \cot(e + fx))^2} - \frac{i}{8f(a^3 + ia^3 \cot(e + fx))} \right) dx$$

$$\begin{aligned}
&= -\frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{8af(a+ia\cot(e+fx))^2} \\
&\quad - \frac{i(c+dx)}{8f(a^3+ia^3\cot(e+fx))} + \frac{(id)\int \frac{1}{a^3+ia^3\cot(e+fx)} dx}{8f} \\
&\quad + \frac{(id)\int \frac{1}{(a+ia\cot(e+fx))^3} dx}{6f} + \frac{(id)\int \frac{1}{(a+ia\cot(e+fx))^2} dx}{8af} \\
&= -\frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3} \\
&\quad + \frac{d}{32af^2(a+ia\cot(e+fx))^2} - \frac{i(c+dx)}{8af(a+ia\cot(e+fx))^2} \\
&\quad + \frac{d}{16f^2(a^3+ia^3\cot(e+fx))} - \frac{i(c+dx)}{8f(a^3+ia^3\cot(e+fx))} \\
&\quad + \frac{(id)\int 1 dx}{16a^3f} + \frac{(id)\int \frac{1}{a+ia\cot(e+fx)} dx}{16a^2f} + \frac{(id)\int \frac{1}{(a+ia\cot(e+fx))^2} dx}{12af} \\
&= \frac{id x}{16a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\cot(e+fx))^3} \\
&\quad - \frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3} + \frac{5d}{96af^2(a+ia\cot(e+fx))^2} \\
&\quad - \frac{i(c+dx)}{8af(a+ia\cot(e+fx))^2} + \frac{32f^2(a^3+ia^3\cot(e+fx))}{3d} \\
&\quad - \frac{i(c+dx)}{8f(a^3+ia^3\cot(e+fx))} + \frac{(id)\int 1 dx}{32a^3f} + \frac{(id)\int \frac{1}{a+ia\cot(e+fx)} dx}{24a^2f} \\
&= \frac{3id x}{32a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3} \\
&\quad + \frac{5d}{96af^2(a+ia\cot(e+fx))^2} - \frac{i(c+dx)}{8af(a+ia\cot(e+fx))^2} \\
&\quad + \frac{11d}{96f^2(a^3+ia^3\cot(e+fx))} - \frac{i(c+dx)}{8f(a^3+ia^3\cot(e+fx))} + \frac{(id)\int 1 dx}{48a^3f} \\
&= \frac{11id x}{96a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} + \frac{d}{36f^2(a+ia\cot(e+fx))^3} - \frac{i(c+dx)}{6f(a+ia\cot(e+fx))^3} \\
&\quad + \frac{5d}{96af^2(a+ia\cot(e+fx))^2} - \frac{i(c+dx)}{8af(a+ia\cot(e+fx))^2} \\
&\quad + \frac{11d}{96f^2(a^3+ia^3\cot(e+fx))} - \frac{i(c+dx)}{8f(a^3+ia^3\cot(e+fx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.17

$$\int \frac{c + dx}{(a + ia \cot(e + fx))^3} dx$$

$$= \frac{-72de^2 + 144cef + 144cf^2x + 72df^2x^2 + 108i(2cf + d(i + 2fx)) \cos(2(e + fx)) + 27(d - 4icf - 4idf)x \cos(4(e + fx)) - 4d \cos(6(e + fx)) + (24i)cf \cos(6(e + fx)) + (24i)d \cos(6(e + fx)) - (108i)d \sin(2(e + fx)) - 216cf \sin(2(e + fx)) - 216d \sin(2(e + fx)) + (27i)d \sin(4(e + fx)) + 108cf \sin(4(e + fx)) + 108d \sin(4(e + fx)) - (4i)d \sin(6(e + fx)) - 24cf \sin(6(e + fx)) - 24d \sin(6(e + fx))}{(1152a^3f^2)}$$

[In] Integrate[(c + d*x)/(a + I*a*Cot[e + f*x])^3,x]

```
[Out] (-72*d*e^2 + 144*c*e*f + 144*c*f^2*x + 72*d*f^2*x^2 + (108*I)*(2*c*f + d*(I + 2*f*x))*Cos[2*(e + f*x)] + 27*(d - (4*I)*c*f - (4*I)*d*f*x)*Cos[4*(e + f*x)] - 4*d*Cos[6*(e + f*x)] + (24*I)*c*f*Cos[6*(e + f*x)] + (24*I)*d*f*x*Cos[6*(e + f*x)] - (108*I)*d*Sin[2*(e + f*x)] - 216*c*f*Sin[2*(e + f*x)] - 216*d*f*x*Sin[2*(e + f*x)] + (27*I)*d*Sin[4*(e + f*x)] + 108*c*f*Sin[4*(e + f*x)] + 108*d*f*x*Sin[4*(e + f*x)] - (4*I)*d*Sin[6*(e + f*x)] - 24*c*f*Sin[6*(e + f*x)] - 24*d*f*x*Sin[6*(e + f*x)])/(1152*a^3*f^2)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.55

method	result
risch	$\frac{dx^2}{16a^3} + \frac{xc}{8a^3} + \frac{i(6dfx+6cf+id)e^{6i(fx+e)}}{288a^3f^2} - \frac{3i(4dfx+4cf+id)e^{4i(fx+e)}}{128a^3f^2} + \frac{3i(2dfx+2cf+id)e^{2i(fx+e)}}{32a^3f^2}$
parallelrisc	$\frac{3(6(dx^2+2xc)f^2+i(-29dx-36c)f+15d) \tan(fx+e)^3 + 3\left(36i\left(\frac{dx}{2}+c\right)xf^2+3f(dx+8c)+16id\right) \tan(fx+e)^2 - 63\left(\frac{6(dx+2c)f+id}{7}\right) \tan(fx+e)}{288f^2a^3(-i-3 \tan(fx+e)+3i \tan(fx+e)^2+\tan(fx+e)^3)}$

[In] int((d*x+c)/(a+I*a*cot(f*x+e))^3,x,method=_RETURNVERBOSE)

```
[Out] 1/16*d*x^2/a^3+1/8/a^3*x*c+1/288*I*(6*d*f*x+I*d+6*c*f)/a^3/f^2*exp(6*I*(f*x+e))-3/128*I*(4*d*f*x+I*d+4*c*f)/a^3/f^2*exp(4*I*(f*x+e))+3/32*I*(2*d*f*x+I*d+2*c*f)/a^3/f^2*exp(2*I*(f*x+e))
```


Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.44

$$\int \frac{c + dx}{(a + ia \cot(e + fx))^3} dx$$

$$= \frac{72 df^2 x^2 + 144 cf^2 x - 4(-6i dfx - 6i cf + d)e^{(6i fx + 6i e)} - 27(4i dfx + 4i cf - d)e^{(4i fx + 4i e)} - 108(-2i dfx - 2i cf + d)e^{(2i fx + 2i e)}}{1152 a^3 f^2}$$

[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")

```
[Out] 1/1152*(72*d*f^2*x^2 + 144*c*f^2*x - 4*(-6*I*d*f*x - 6*I*c*f + d)*e^(6*I*f*x + 6*I*e) - 27*(4*I*d*f*x + 4*I*c*f - d)*e^(4*I*f*x + 4*I*e) - 108*(-2*I*d*f*x - 2*I*c*f + d)*e^(2*I*f*x + 2*I*e))/(a^3*f^2)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.43

$$\int \frac{c + dx}{(a + ia \cot(e + fx))^3} dx$$

$$= \left\{ \frac{(221184ia^6 cf^5 e^{2ie} + 221184ia^6 df^5 x e^{2ie} - 110592a^6 df^4 e^{2ie})e^{2ifx} + (-110592ia^6 cf^5 e^{4ie} - 110592ia^6 df^5 x e^{4ie} + 27648a^6 df^4 e^{4ie})e^{4ifx} + (24576ia^6 cf^5 e^{6ie} + 24576ia^6 df^5 x e^{6ie} - 110592a^6 df^4 e^{6ie})e^{6ifx}}{1179648a^9 f^6} \right.$$

$$+ \frac{x^2(-de^{6ie} + 3de^{4ie} - 3de^{2ie})}{16a^3} + \frac{x(-ce^{6ie} + 3ce^{4ie} - 3ce^{2ie})}{8a^3}$$

$$+ \frac{cx}{8a^3} + \frac{dx^2}{16a^3}$$

[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))**3,x)

```
[Out] Piecewise((((221184*I*a**6*c*f**5*exp(2*I*e) + 221184*I*a**6*d*f**5*x*exp(2*I*e) - 110592*a**6*d*f**4*exp(2*I*e))*exp(2*I*f*x) + (-110592*I*a**6*c*f**5*exp(4*I*e) - 110592*I*a**6*d*f**5*x*exp(4*I*e) + 27648*a**6*d*f**4*exp(4*I*e))*exp(4*I*f*x) + (24576*I*a**6*c*f**5*exp(6*I*e) + 24576*I*a**6*d*f**5*x*exp(6*I*e) - 110592*a**6*d*f**4*exp(6*I*e))*exp(6*I*f*x))/(1179648*a**9*f**6), Ne(a**9*f**6, 0)), (x**2*(-d*exp(6*I*e) + 3*d*exp(4*I*e) - 3*d*exp(2*I*e))/(16*a**3) + x*(-c*exp(6*I*e) + 3*c*exp(4*I*e) - 3*c*exp(2*I*e))/(8*a**3), True)) + c*x/(8*a**3) + d*x**2/(16*a**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{c + dx}{(a + ia \cot(e + fx))^3} dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.68

$$\int \frac{c + dx}{(a + ia \cot(e + fx))^3} dx = \frac{72df^2x^2 + 144cf^2x + 24i d f x e^{(6i f x + 6i e)} - 108i d f x e^{(4i f x + 4i e)} + 216i d f x e^{(2i f x + 2i e)} + 24i c f e^{(6i f x + 6i e)} - 108i c f e^{(4i f x + 4i e)} + 216i c f e^{(2i f x + 2i e)}}{1152 a^3 f^2}$$

```
[In] integrate((d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/1152*(72*d*f^2*x^2 + 144*c*f^2*x + 24*I*d*f*x*e^(6*I*f*x + 6*I*e) - 108*I*d*f*x*e^(4*I*f*x + 4*I*e) + 216*I*d*f*x*e^(2*I*f*x + 2*I*e) + 24*I*c*f*e^(6*I*f*x + 6*I*e) - 108*I*c*f*e^(4*I*f*x + 4*I*e) + 216*I*c*f*e^(2*I*f*x + 2*I*e) - 4*d*e^(6*I*f*x + 6*I*e) + 27*d*e^(4*I*f*x + 4*I*e) - 108*d*e^(2*I*f*x + 2*I*e))/(a^3*f^2)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.69

$$\int \frac{c + dx}{(a + ia \cot(e + fx))^3} dx = e^{e^{2i+fx} 2i} \left(\frac{(6cf + d3i) li}{32a^3 f^2} + \frac{dx 3i}{16a^3 f} \right) - e^{e^{4i+fx} 4i} \left(\frac{(12cf + d3i) li}{128a^3 f^2} + \frac{dx 3i}{32a^3 f} \right) + e^{e^{6i+fx} 6i} \left(\frac{(6cf + d1i) li}{288a^3 f^2} + \frac{dx li}{48a^3 f} \right) + \frac{dx^2}{16a^3} + \frac{cx}{8a^3}$$

```
[In] int((c + d*x)/(a + a*cot(e + f*x)*1i)^3,x)
```

```
[Out] exp(e*2i + f*x*2i)*(((d*3i + 6*c*f)*1i)/(32*a^3*f^2) + (d*x*3i)/(16*a^3*f)) - exp(e*4i + f*x*4i)*(((d*3i + 12*c*f)*1i)/(128*a^3*f^2) + (d*x*3i)/(32*a^3*f)) + exp(e*6i + f*x*6i)*(((d*1i + 6*c*f)*1i)/(288*a^3*f^2) + (d*x*1i)/(48*a^3*f)) + (d*x^2)/(16*a^3) + (c*x)/(8*a^3)
```

$$3.30 \quad \int \frac{1}{(c+dx)(a+ia \cot(e+fx))^3} dx$$

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Optimal result

Integrand size = 23, antiderivative size = 449

$$\begin{aligned} \int \frac{1}{(c+dx)(a+ia \cot(e+fx))^3} dx = & -\frac{3 \cos \left(2e - \frac{2cf}{d}\right) \operatorname{CosIntegral} \left(\frac{2cf}{d} + 2fx\right)}{8a^3d} \\ & + \frac{3 \cos \left(4e - \frac{4cf}{d}\right) \operatorname{CosIntegral} \left(\frac{4cf}{d} + 4fx\right)}{8a^3d} \\ & - \frac{\cos \left(6e - \frac{6cf}{d}\right) \operatorname{CosIntegral} \left(\frac{6cf}{d} + 6fx\right)}{8a^3d} \\ & + \frac{\log(c+dx)}{8a^3d} \\ & - \frac{i \operatorname{CosIntegral} \left(\frac{6cf}{d} + 6fx\right) \sin \left(6e - \frac{6cf}{d}\right)}{8a^3d} \\ & + \frac{3i \operatorname{CosIntegral} \left(\frac{4cf}{d} + 4fx\right) \sin \left(4e - \frac{4cf}{d}\right)}{8a^3d} \\ & - \frac{3i \operatorname{CosIntegral} \left(\frac{2cf}{d} + 2fx\right) \sin \left(2e - \frac{2cf}{d}\right)}{8a^3d} \\ & - \frac{3i \cos \left(2e - \frac{2cf}{d}\right) \operatorname{Si} \left(\frac{2cf}{d} + 2fx\right)}{8a^3d} \\ & + \frac{3 \sin \left(2e - \frac{2cf}{d}\right) \operatorname{Si} \left(\frac{2cf}{d} + 2fx\right)}{8a^3d} \\ & + \frac{3i \cos \left(4e - \frac{4cf}{d}\right) \operatorname{Si} \left(\frac{4cf}{d} + 4fx\right)}{8a^3d} \\ & - \frac{3 \sin \left(4e - \frac{4cf}{d}\right) \operatorname{Si} \left(\frac{4cf}{d} + 4fx\right)}{8a^3d} \\ & - \frac{i \cos \left(6e - \frac{6cf}{d}\right) \operatorname{Si} \left(\frac{6cf}{d} + 6fx\right)}{8a^3d} \\ & + \frac{\sin \left(6e - \frac{6cf}{d}\right) \operatorname{Si} \left(\frac{6cf}{d} + 6fx\right)}{8a^3d} \end{aligned}$$

[Out] $-1/8 \text{Ci}(6cf/d+6fx) \cos(-6e+6cf/d)/a^3/d+3/8 \text{Ci}(4cf/d+4fx) \cos(-4e+4cf/d)/a^3/d-3/8 \text{Ci}(2cf/d+2fx) \cos(-2e+2cf/d)/a^3/d+1/8 \ln(dx+c)/a^3/d-3/8 I \cos(-2e+2cf/d) \text{Si}(2cf/d+2fx)/a^3/d+3/8 I \cos(-4e+4cf/d) \text{Si}(4cf/d+4fx)/a^3/d-1/8 I \cos(-6e+6cf/d) \text{Si}(6cf/d+6fx)/a^3/d+1/8 I \text{Ci}(6cf/d+6fx) \sin(-6e+6cf/d)/a^3/d-1/8 \text{Si}(6cf/d+6fx) \sin(-6e+6cf/d)/a^3/d-3/8 I \text{Ci}(4cf/d+4fx) \sin(-4e+4cf/d)/a^3/d+3/8 \text{Si}(4cf/d+4fx) \sin(-4e+4cf/d)/a^3/d+3/8 I \text{Ci}(2cf/d+2fx) \sin(-2e+2cf/d)/a^3/d-3/8 \text{Si}(2cf/d+2fx) \sin(-2e+2cf/d)/a^3/d$

Rubi [A] (verified)

Time = 2.21 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 53, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3809, 3384, 3380, 3383, 3393, 4491, 4513}

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))^3} dx = -\frac{3i \text{CosIntegral}\left(2xf + \frac{2cf}{d}\right) \sin\left(2e - \frac{2cf}{d}\right)}{8a^3d} - \frac{i \text{CosIntegral}\left(6xf + \frac{6cf}{d}\right) \sin\left(6e - \frac{6cf}{d}\right)}{8a^3d} + \frac{3i \text{CosIntegral}\left(4xf + \frac{4cf}{d}\right) \sin\left(4e - \frac{4cf}{d}\right)}{8a^3d} - \frac{3 \text{CosIntegral}\left(2xf + \frac{2cf}{d}\right) \cos\left(2e - \frac{2cf}{d}\right)}{8a^3d} + \frac{3 \text{CosIntegral}\left(4xf + \frac{4cf}{d}\right) \cos\left(4e - \frac{4cf}{d}\right)}{8a^3d} - \frac{\text{CosIntegral}\left(6xf + \frac{6cf}{d}\right) \cos\left(6e - \frac{6cf}{d}\right)}{8a^3d} + \frac{3 \sin\left(2e - \frac{2cf}{d}\right) \text{Si}\left(2xf + \frac{2cf}{d}\right)}{8a^3d} - \frac{3 \sin\left(4e - \frac{4cf}{d}\right) \text{Si}\left(4xf + \frac{4cf}{d}\right)}{8a^3d} + \frac{\sin\left(6e - \frac{6cf}{d}\right) \text{Si}\left(6xf + \frac{6cf}{d}\right)}{8a^3d} - \frac{3i \cos\left(2e - \frac{2cf}{d}\right) \text{Si}\left(2xf + \frac{2cf}{d}\right)}{8a^3d} + \frac{3i \cos\left(4e - \frac{4cf}{d}\right) \text{Si}\left(4xf + \frac{4cf}{d}\right)}{8a^3d} - \frac{i \cos\left(6e - \frac{6cf}{d}\right) \text{Si}\left(6xf + \frac{6cf}{d}\right)}{8a^3d} + \frac{\log(c+dx)}{8a^3d}$$

[In] Int[1/((c + d*x)*(a + I*a*Cot[e + f*x])^3),x]

[Out] $(-3 \text{Cos}[2e - (2cf)/d] \text{CosIntegral}[(2cf)/d + 2fx])/(8a^3d) + (3 \text{Cos}[4e - (4cf)/d] \text{CosIntegral}[(4cf)/d + 4fx])/(8a^3d) - (\text{Cos}[6e - (6$

```
*c*f)/d]*CosIntegral[(6*c*f)/d + 6*f*x]]/(8*a^3*d) + Log[c + d*x]/(8*a^3*d)
- ((I/8)*CosIntegral[(6*c*f)/d + 6*f*x]*Sin[6*e - (6*c*f)/d]]/(a^3*d) + ((
(3*I)/8)*CosIntegral[(4*c*f)/d + 4*f*x]*Sin[4*e - (4*c*f)/d]]/(a^3*d) - (((
3*I)/8)*CosIntegral[(2*c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d]]/(a^3*d) - (((3
*I)/8)*Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x]]/(a^3*d) + (3*Si
n[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x]]/(8*a^3*d) + (((3*I)/8)*C
os[4*e - (4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x]]/(a^3*d) - (3*Sin[4*e -
(4*c*f)/d]*SinIntegral[(4*c*f)/d + 4*f*x]]/(8*a^3*d) - ((I/8)*Cos[6*e - (6*
c*f)/d]*SinIntegral[(6*c*f)/d + 6*f*x]]/(a^3*d) + (Sin[6*e - (6*c*f)/d]*Sin
Integral[(6*c*f)/d + 6*f*x]]/(8*a^3*d)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3809

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_),
x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

tQ[p, 0]

Rule 4513

Int[((e_.) + (f_.)*(x_.))^(m_.)*Sin[(a_.) + (b_.)*(x_.)]^(p_.)*Sin[(c_.) + (d_.)*(x_.)]^(q_.), x_Symbol] := Int[ExpandTrigReduce[(e + f*x)^m, Sin[a + b*x]^(p)*Sin[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ[q, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(\frac{1}{8a^3(c+dx)} - \frac{3\cos(2e+2fx)}{8a^3(c+dx)} + \frac{3\cos^2(2e+2fx)}{8a^3(c+dx)} - \frac{\cos^3(2e+2fx)}{8a^3(c+dx)} \right. \\
 &\quad - \frac{3i\sin(2e+2fx)}{8a^3(c+dx)} - \frac{3i\cos^2(2e+2fx)\sin(2e+2fx)}{8a^3(c+dx)} - \frac{3\sin^2(2e+2fx)}{8a^3(c+dx)} \\
 &\quad \left. + \frac{i\sin^3(2e+2fx)}{8a^3(c+dx)} + \frac{3i\sin(4e+4fx)}{8a^3(c+dx)} + \frac{3\sin(2e+2fx)\sin(4e+4fx)}{16a^3(c+dx)} \right) dx \\
 &= \frac{\log(c+dx)}{8a^3d} + \frac{i\int\frac{\sin^3(2e+2fx)}{c+dx}dx}{8a^3} - \frac{(3i)\int\frac{\sin(2e+2fx)}{c+dx}dx}{8a^3} \\
 &\quad - \frac{(3i)\int\frac{\cos^2(2e+2fx)\sin(2e+2fx)}{c+dx}dx}{8a^3} + \frac{(3i)\int\frac{\sin(4e+4fx)}{c+dx}dx}{8a^3} \\
 &\quad - \frac{\int\frac{\cos^3(2e+2fx)}{c+dx}dx}{8a^3} + \frac{3\int\frac{\sin(2e+2fx)\sin(4e+4fx)}{c+dx}dx}{16a^3} \\
 &\quad - \frac{3\int\frac{\cos(2e+2fx)}{c+dx}dx}{8a^3} + \frac{3\int\frac{\cos^2(2e+2fx)}{c+dx}dx}{8a^3} - \frac{3\int\frac{\sin^2(2e+2fx)}{c+dx}dx}{8a^3} \\
 &= \frac{\log(c+dx)}{8a^3d} + \frac{i\int\left(\frac{3\sin(2e+2fx)}{4(c+dx)} - \frac{\sin(6e+6fx)}{4(c+dx)}\right)dx}{8a^3} \\
 &\quad - \frac{(3i)\int\left(\frac{\sin(2e+2fx)}{4(c+dx)} + \frac{\sin(6e+6fx)}{4(c+dx)}\right)dx}{8a^3} - \frac{\int\left(\frac{3\cos(2e+2fx)}{4(c+dx)} + \frac{\cos(6e+6fx)}{4(c+dx)}\right)dx}{8a^3} \\
 &\quad + \frac{3\int\left(\frac{\cos(2e+2fx)}{2(c+dx)} - \frac{\cos(6e+6fx)}{2(c+dx)}\right)dx}{16a^3} - \frac{3\int\left(\frac{1}{2(c+dx)} - \frac{\cos(4e+4fx)}{2(c+dx)}\right)dx}{8a^3} \\
 &\quad + \frac{3\int\left(\frac{1}{2(c+dx)} + \frac{\cos(4e+4fx)}{2(c+dx)}\right)dx}{8a^3} + \frac{(3i\cos\left(4e - \frac{4cf}{d}\right))\int\frac{\sin\left(\frac{4cf}{d} + 4fx\right)}{c+dx}dx}{8a^3} \\
 &\quad - \frac{(3i\cos\left(2e - \frac{2cf}{d}\right))\int\frac{\sin\left(\frac{2cf}{d} + 2fx\right)}{c+dx}dx}{8a^3} \\
 &\quad - \frac{(3\cos\left(2e - \frac{2cf}{d}\right))\int\frac{\cos\left(\frac{2cf}{d} + 2fx\right)}{c+dx}dx}{8a^3} + \frac{(3i\sin\left(4e - \frac{4cf}{d}\right))\int\frac{\cos\left(\frac{4cf}{d} + 4fx\right)}{c+dx}dx}{8a^3} \\
 &\quad - \frac{(3i\sin\left(2e - \frac{2cf}{d}\right))\int\frac{\cos\left(\frac{2cf}{d} + 2fx\right)}{c+dx}dx}{8a^3} + \frac{(3\sin\left(2e - \frac{2cf}{d}\right))\int\frac{\sin\left(\frac{2cf}{d} + 2fx\right)}{c+dx}dx}{8a^3}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{3 \cos\left(2e - \frac{2cf}{d}\right) \text{CosIntegral}\left(\frac{2cf}{d} + 2fx\right)}{8a^3d} + \frac{\log(c + dx)}{8a^3d} \\
&\quad + \frac{3i \text{CosIntegral}\left(\frac{4cf}{d} + 4fx\right) \sin\left(4e - \frac{4cf}{d}\right)}{8a^3d} \\
&\quad - \frac{3i \text{CosIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(2e - \frac{2cf}{d}\right)}{8a^3d} \\
&\quad - \frac{3i \cos\left(2e - \frac{2cf}{d}\right) \text{Si}\left(\frac{2cf}{d} + 2fx\right)}{8a^3d} + \frac{3 \sin\left(2e - \frac{2cf}{d}\right) \text{Si}\left(\frac{2cf}{d} + 2fx\right)}{8a^3d} \\
&\quad + \frac{3i \cos\left(4e - \frac{4cf}{d}\right) \text{Si}\left(\frac{4cf}{d} + 4fx\right)}{8a^3d} - \frac{i \int \frac{\sin(6e+6fx)}{c+dx} dx}{32a^3} - \frac{(3i) \int \frac{\sin(6e+6fx)}{c+dx} dx}{32a^3} \\
&\quad - \frac{\int \frac{\cos(6e+6fx)}{c+dx} dx}{32a^3} - \frac{3 \int \frac{\cos(6e+6fx)}{c+dx} dx}{32a^3} + 2 \frac{3 \int \frac{\cos(4e+4fx)}{c+dx} dx}{16a^3} \\
&= -\frac{3 \cos\left(2e - \frac{2cf}{d}\right) \text{CosIntegral}\left(\frac{2cf}{d} + 2fx\right)}{8a^3d} + \frac{\log(c + dx)}{8a^3d} \\
&\quad + \frac{3i \text{CosIntegral}\left(\frac{4cf}{d} + 4fx\right) \sin\left(4e - \frac{4cf}{d}\right)}{8a^3d} \\
&\quad - \frac{3i \text{CosIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(2e - \frac{2cf}{d}\right)}{8a^3d} - \frac{3i \cos\left(2e - \frac{2cf}{d}\right) \text{Si}\left(\frac{2cf}{d} + 2fx\right)}{8a^3d} \\
&\quad + \frac{3 \sin\left(2e - \frac{2cf}{d}\right) \text{Si}\left(\frac{2cf}{d} + 2fx\right)}{8a^3d} + \frac{3i \cos\left(4e - \frac{4cf}{d}\right) \text{Si}\left(\frac{4cf}{d} + 4fx\right)}{8a^3d} \\
&\quad - \frac{(i \cos\left(6e - \frac{6cf}{d}\right)) \int \frac{\sin\left(\frac{6cf}{d} + 6fx\right)}{c+dx} dx}{32a^3} - \frac{(3i \cos\left(6e - \frac{6cf}{d}\right)) \int \frac{\sin\left(\frac{6cf}{d} + 6fx\right)}{c+dx} dx}{32a^3} \\
&\quad - \frac{\cos\left(6e - \frac{6cf}{d}\right) \int \frac{\cos\left(\frac{6cf}{d} + 6fx\right)}{c+dx} dx}{32a^3} - \frac{(3 \cos\left(6e - \frac{6cf}{d}\right)) \int \frac{\cos\left(\frac{6cf}{d} + 6fx\right)}{c+dx} dx}{32a^3} \\
&\quad - \frac{(i \sin\left(6e - \frac{6cf}{d}\right)) \int \frac{\cos\left(\frac{6cf}{d} + 6fx\right)}{c+dx} dx}{32a^3} - \frac{(3i \sin\left(6e - \frac{6cf}{d}\right)) \int \frac{\cos\left(\frac{6cf}{d} + 6fx\right)}{c+dx} dx}{32a^3} \\
&\quad + \frac{\sin\left(6e - \frac{6cf}{d}\right) \int \frac{\sin\left(\frac{6cf}{d} + 6fx\right)}{c+dx} dx}{32a^3} + \frac{(3 \sin\left(6e - \frac{6cf}{d}\right)) \int \frac{\sin\left(\frac{6cf}{d} + 6fx\right)}{c+dx} dx}{32a^3} \\
&\quad + 2 \left(\frac{(3 \cos\left(4e - \frac{4cf}{d}\right)) \int \frac{\cos\left(\frac{4cf}{d} + 4fx\right)}{c+dx} dx}{16a^3} - \frac{(3 \sin\left(4e - \frac{4cf}{d}\right)) \int \frac{\sin\left(\frac{4cf}{d} + 4fx\right)}{c+dx} dx}{16a^3} \right)
\end{aligned}$$

$$\begin{aligned}
&= -\frac{3 \cos\left(2e - \frac{2cf}{d}\right) \operatorname{CosIntegral}\left(\frac{2cf}{d} + 2fx\right)}{8a^3d} \\
&\quad - \frac{\cos\left(6e - \frac{6cf}{d}\right) \operatorname{CosIntegral}\left(\frac{6cf}{d} + 6fx\right)}{8a^3d} + \frac{\log(c + dx)}{8a^3d} \\
&\quad - \frac{i \operatorname{CosIntegral}\left(\frac{6cf}{d} + 6fx\right) \sin\left(6e - \frac{6cf}{d}\right)}{8a^3d} \\
&\quad + \frac{3i \operatorname{CosIntegral}\left(\frac{4cf}{d} + 4fx\right) \sin\left(4e - \frac{4cf}{d}\right)}{8a^3d} \\
&\quad - \frac{3i \operatorname{CosIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(2e - \frac{2cf}{d}\right)}{8a^3d} - \frac{3i \cos\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(\frac{2cf}{d} + 2fx\right)}{8a^3d} \\
&\quad + \frac{3 \sin\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(\frac{2cf}{d} + 2fx\right)}{8a^3d} + \frac{3i \cos\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(\frac{4cf}{d} + 4fx\right)}{8a^3d} \\
&\quad + 2 \left(\frac{3 \cos\left(4e - \frac{4cf}{d}\right) \operatorname{CosIntegral}\left(\frac{4cf}{d} + 4fx\right)}{16a^3d} - \frac{3 \sin\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(\frac{4cf}{d} + 4fx\right)}{16a^3d} \right) \\
&\quad - \frac{i \cos\left(6e - \frac{6cf}{d}\right) \operatorname{Si}\left(\frac{6cf}{d} + 6fx\right)}{8a^3d} + \frac{\sin\left(6e - \frac{6cf}{d}\right) \operatorname{Si}\left(\frac{6cf}{d} + 6fx\right)}{8a^3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.44

$$\int \frac{1}{(c + dx)(a + ia \cot(e + fx))^3} dx$$

$$= \frac{\log(c + dx) - 3\left(\cos\left(2e - \frac{2cf}{d}\right) + i \sin\left(2e - \frac{2cf}{d}\right)\right) \left(\operatorname{CosIntegral}\left(\frac{2f(c+dx)}{d}\right) + i \operatorname{Si}\left(\frac{2f(c+dx)}{d}\right)\right) + 3\left(\cos\left(4e - \frac{4cf}{d}\right) + i \sin\left(4e - \frac{4cf}{d}\right)\right) \left(\operatorname{CosIntegral}\left(\frac{4f(c+dx)}{d}\right) + i \operatorname{Si}\left(\frac{4f(c+dx)}{d}\right)\right) - 3\left(\cos\left(6e - \frac{6cf}{d}\right) + i \sin\left(6e - \frac{6cf}{d}\right)\right) \left(\operatorname{CosIntegral}\left(\frac{6f(c+dx)}{d}\right) + i \operatorname{Si}\left(\frac{6f(c+dx)}{d}\right)\right)}{8a^3d}$$

[In] Integrate[1/((c + d*x)*(a + I*a*Cot[e + f*x])^3),x]

[Out] (Log[c + d*x] - 3*(Cos[2*e - (2*c*f)/d] + I*Sin[2*e - (2*c*f)/d])*(CosIntegral[(2*f*(c + d*x))/d] + I*SinIntegral[(2*f*(c + d*x))/d]) + 3*(Cos[4*e - (4*c*f)/d] + I*Sin[4*e - (4*c*f)/d])*(CosIntegral[(4*f*(c + d*x))/d] + I*SinIntegral[(4*f*(c + d*x))/d]) - (Cos[6*e - (6*c*f)/d] + I*Sin[6*e - (6*c*f)/d])*(CosIntegral[(6*f*(c + d*x))/d] + I*SinIntegral[(6*f*(c + d*x))/d])/(8*a^3*d)

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.38

method	result
risch	$\frac{\ln(dx+c)}{8a^3d} + \frac{3e^{-\frac{2i(cf-de)}{d}} \operatorname{Ei}_1\left(-2ifx-2ie-\frac{2(icf-ide)}{d}\right)}{8a^3d} - \frac{3e^{-\frac{4i(cf-de)}{d}} \operatorname{Ei}_1\left(-4ifx-4ie-\frac{4(icf-ide)}{d}\right)}{8a^3d} + \frac{e^{-\frac{6i(cf-de)}{d}} \operatorname{Ei}_1\left(-6ifx-6ie-\frac{6(icf-ide)}{d}\right)}{8a^3d}$

[In] int(1/(d*x+c)/(a+I*a*cot(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] $\frac{1}{8} \ln(dx+c)/a^3/d + 3/8/a^3/d \exp(-2*I*(c*f-d*e)/d) * \operatorname{Ei}(1, -2*I*f*x - 2*I*e - 2*(I*c*f - I*d*e)/d) - 3/8/a^3/d \exp(-4*I*(c*f-d*e)/d) * \operatorname{Ei}(1, -4*I*f*x - 4*I*e - 4*(I*c*f - I*d*e)/d) + 1/8/a^3/d \exp(-6*I*(c*f-d*e)/d) * \operatorname{Ei}(1, -6*I*f*x - 6*I*e - 6*(I*c*f - I*d*e)/d)$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.27

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))^3} dx = \frac{3 \operatorname{Ei}\left(-\frac{2(-idf_x-icf)}{d}\right) e^{\left(-\frac{2(-ide+icf)}{d}\right)} - 3 \operatorname{Ei}\left(-\frac{4(-idf_x-icf)}{d}\right) e^{\left(-\frac{4(-ide+icf)}{d}\right)} + \operatorname{Ei}\left(-\frac{6(-idf_x-icf)}{d}\right) e^{\left(-\frac{6(-ide+icf)}{d}\right)}}{8a^3d}$$

[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")

[Out] $-1/8*(3*\operatorname{Ei}(-2*(-I*d*f*x - I*c*f)/d)*e^{(-2*(-I*d*e + I*c*f)/d)} - 3*\operatorname{Ei}(-4*(-I*d*f*x - I*c*f)/d)*e^{(-4*(-I*d*e + I*c*f)/d)} + \operatorname{Ei}(-6*(-I*d*f*x - I*c*f)/d)*e^{(-6*(-I*d*e + I*c*f)/d)} - \log((d*x + c)/d))/(a^3*d)$

Sympy [F]

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))^3} dx = \frac{i \int \frac{1}{c \cot^3(e+fx) - 3ic \cot^2(e+fx) - 3c \cot(e+fx) + ic + dx \cot^3(e+fx) - 3idx \cot^2(e+fx) - 3dx \cot(e+fx) + idx} dx}{a^3}$$

[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))**3,x)

[Out] $I*\operatorname{Integral}(1/(c*\cot(e + f*x)**3 - 3*I*c*\cot(e + f*x)**2 - 3*c*\cot(e + f*x) + I*c + d*x*\cot(e + f*x)**3 - 3*I*d*x*\cot(e + f*x)**2 - 3*d*x*\cot(e + f*x) + I*d*x), x)/a**3$

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.61

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))^3} dx$$

$$= \frac{f \cos\left(-\frac{6(de-cf)}{d}\right) E_1\left(\frac{6(-i(fx+e)d+ide-icf)}{d}\right) - 3f \cos\left(-\frac{4(de-cf)}{d}\right) E_1\left(\frac{4(-i(fx+e)d+ide-icf)}{d}\right) + 3f \cos\left(-\frac{2(de-cf)}{d}\right) E_1\left(\frac{2(-i(fx+e)d+ide-icf)}{d}\right)}{a^3}$$

[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")

```
[Out] 1/8*(f*cos(-6*(d*e - c*f)/d)*exp_integral_e(1, 6*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) - 3*f*cos(-4*(d*e - c*f)/d)*exp_integral_e(1, 4*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) + 3*f*cos(-2*(d*e - c*f)/d)*exp_integral_e(1, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) - 3*I*f*exp_integral_e(1, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d)*sin(-2*(d*e - c*f)/d) + 3*I*f*exp_integral_e(1, 4*(-I*(f*x + e)*d + I*d*e - I*c*f)/d)*sin(-4*(d*e - c*f)/d) - I*f*exp_integral_e(1, 6*(-I*(f*x + e)*d + I*d*e - I*c*f)/d)*sin(-6*(d*e - c*f)/d) + f*log((f*x + e)*d - d*e + c*f)/(a^3*d*f)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1791 vs. 2(411) = 822.

Time = 0.39 (sec) , antiderivative size = 1791, normalized size of antiderivative = 3.99

$$\int \frac{1}{(c+dx)(a+ia \cot(e+fx))^3} dx = \text{Too large to display}$$

[In] integrate(1/(d*x+c)/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")

```
[Out] -1/8*(cos(e)^6*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d) + 6*I*cos(e)^5*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e) - 15*cos(e)^4*cos(6*c*f/d)/d*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^2 - 20*I*cos(e)^3*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^3 + 15*cos(e)^2*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^4 + 6*I*cos(e)*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^5 - cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^6 - I*cos(e)^6*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d) + 6*cos(e)^5*cos_integral(6*(d*f*x + c*f)/d)*sin(e)*sin(6*c*f/d) + 15*I*cos(e)^4*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^2*sin(6*c*f/d) - 20*cos(e)^3*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^3*sin(6*c*f/d) - 15*I*cos(e)^2*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^4*sin(6*c*f/d) + 6*cos(e)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^5*sin(6*c*f/d) + I*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^6*sin(6*c*f/d)
```

$$\begin{aligned}
& (6*c*f/d) + I*cos(e)^6*cos(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) - 6*cos \\
& (e)^5*cos(6*c*f/d)*sin(e)*sin_integral(6*(d*f*x + c*f)/d) - 15*I*cos(e)^4*c \\
& os(6*c*f/d)*sin(e)^2*sin_integral(6*(d*f*x + c*f)/d) + 20*cos(e)^3*cos(6*c* \\
& f/d)*sin(e)^3*sin_integral(6*(d*f*x + c*f)/d) + 15*I*cos(e)^2*cos(6*c*f/d)* \\
& sin(e)^4*sin_integral(6*(d*f*x + c*f)/d) - 6*cos(e)*cos(6*c*f/d)*sin(e)^5*s \\
& in_integral(6*(d*f*x + c*f)/d) - I*cos(6*c*f/d)*sin(e)^6*sin_integral(6*(d* \\
& f*x + c*f)/d) + cos(e)^6*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 6*I \\
& *cos(e)^5*sin(e)*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) - 15*cos(e)^4 \\
& *sin(e)^2*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) - 20*I*cos(e)^3*sin(\\
& e)^3*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 15*cos(e)^2*sin(e)^4*si \\
& n(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 6*I*cos(e)*sin(e)^5*sin(6*c*f/ \\
& d)*sin_integral(6*(d*f*x + c*f)/d) - sin(e)^6*sin(6*c*f/d)*sin_integral(6*(\\
& d*f*x + c*f)/d) - 3*cos(e)^4*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d) - \\
& 12*I*cos(e)^3*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e) + 18*cos \\
& (e)^2*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2 + 12*I*cos(e)*c \\
& os(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^3 - 3*cos(4*c*f/d)*cos_i \\
& ntegral(4*(d*f*x + c*f)/d)*sin(e)^4 + 3*I*cos(e)^4*cos_integral(4*(d*f*x + \\
& c*f)/d)*sin(4*c*f/d) - 12*cos(e)^3*cos_integral(4*(d*f*x + c*f)/d)*sin(e)*s \\
& in(4*c*f/d) - 18*I*cos(e)^2*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2*sin(4* \\
& c*f/d) + 12*cos(e)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^3*sin(4*c*f/d) + \\
& 3*I*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4*sin(4*c*f/d) - 3*I*cos(e)^4*co \\
& s(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 12*cos(e)^3*cos(4*c*f/d)*sin(e \\
&)*sin_integral(4*(d*f*x + c*f)/d) + 18*I*cos(e)^2*cos(4*c*f/d)*sin(e)^2*sin \\
& _integral(4*(d*f*x + c*f)/d) - 12*cos(e)*cos(4*c*f/d)*sin(e)^3*sin_integral \\
& (4*(d*f*x + c*f)/d) - 3*I*cos(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*f \\
&)/d) - 3*cos(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 12*I*cos(e \\
&)^3*sin(e)*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 18*cos(e)^2*sin(e \\
&)^2*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 12*I*cos(e)*sin(e)^3*sin \\
& (4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 3*sin(e)^4*sin(4*c*f/d)*sin_int \\
& egral(4*(d*f*x + c*f)/d) + 3*cos(e)^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + \\
& c*f)/d) + 6*I*cos(e)*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) - \\
& 3*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2 - 3*I*cos(e)^2*cos_ \\
& ntegral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) + 6*cos(e)*cos_integral(2*(d*f*x + \\
& c*f)/d)*sin(e)*sin(2*c*f/d) + 3*I*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2 \\
& *sin(2*c*f/d) + 3*I*cos(e)^2*cos(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - \\
& 6*cos(e)*cos(2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*f)/d) - 3*I*cos(2*c \\
& *f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) + 3*cos(e)^2*sin(2*c*f/d)*si \\
& n_integral(2*(d*f*x + c*f)/d) + 6*I*cos(e)*sin(e)*sin(2*c*f/d)*sin_integral \\
& (2*(d*f*x + c*f)/d) - 3*sin(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/ \\
& d) - \log(d*x + c))/(a^3*d)
\end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c + dx)(a + ia \cot(e + fx))^3} dx = \int \frac{1}{(a + a \cot(e + fx) 1i)^3 (c + dx)} dx$$

```
[In] int(1/((a + a*cot(e + f*x)*1i)^3*(c + d*x)),x)
```

```
[Out] int(1/((a + a*cot(e + f*x)*1i)^3*(c + d*x)), x)
```

$$3.31 \quad \int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))^3} dx$$

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Optimal result

Integrand size = 23, antiderivative size = 712

$$\begin{aligned}
 \int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))^3} dx = & -\frac{1}{8a^3d(c+dx)} + \frac{9 \cos(2e+2fx)}{32a^3d(c+dx)} - \frac{3 \cos^2(2e+2fx)}{8a^3d(c+dx)} \\
 & + \frac{\cos^3(2e+2fx)}{8a^3d(c+dx)} + \frac{3 \cos(6e+6fx)}{32a^3d(c+dx)} \\
 & - \frac{3if \cos\left(2e - \frac{2cf}{d}\right) \operatorname{CosIntegral}\left(\frac{2cf}{d} + 2fx\right)}{4a^3d^2} \\
 & + \frac{3if \cos\left(4e - \frac{4cf}{d}\right) \operatorname{CosIntegral}\left(\frac{4cf}{d} + 4fx\right)}{2a^3d^2} \\
 & - \frac{3if \cos\left(6e - \frac{6cf}{d}\right) \operatorname{CosIntegral}\left(\frac{6cf}{d} + 6fx\right)}{4a^3d^2} \\
 & + \frac{3f \operatorname{CosIntegral}\left(\frac{6cf}{d} + 6fx\right) \sin\left(6e - \frac{6cf}{d}\right)}{4a^3d^2} \\
 & - \frac{3f \operatorname{CosIntegral}\left(\frac{4cf}{d} + 4fx\right) \sin\left(4e - \frac{4cf}{d}\right)}{2a^3d^2} \\
 & + \frac{3f \operatorname{CosIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(2e - \frac{2cf}{d}\right)}{4a^3d^2} \\
 & + \frac{15i \sin(2e+2fx)}{32a^3d(c+dx)} + \frac{3 \sin^2(2e+2fx)}{8a^3d(c+dx)} \\
 & - \frac{i \sin^3(2e+2fx)}{8a^3d(c+dx)} - \frac{3i \sin(4e+4fx)}{8a^3d(c+dx)} \\
 & + \frac{3i \sin(6e+6fx)}{32a^3d(c+dx)} + \frac{3f \cos\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(\frac{2cf}{d} + 2fx\right)}{4a^3d^2} \\
 & + \frac{3if \sin\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(\frac{2cf}{d} + 2fx\right)}{4a^3d^2} \\
 & - \frac{3f \cos\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(\frac{4cf}{d} + 4fx\right)}{2a^3d^2} \\
 & - \frac{3if \sin\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(\frac{4cf}{d} + 4fx\right)}{2a^3d^2} \\
 & + \frac{3f \cos\left(6e - \frac{6cf}{d}\right) \operatorname{Si}\left(\frac{6cf}{d} + 6fx\right)}{4a^3d^2} \\
 & + \frac{3if \sin\left(6e - \frac{6cf}{d}\right) \operatorname{Si}\left(\frac{6cf}{d} + 6fx\right)}{4a^3d^2}
 \end{aligned}$$

[Out] -1/8/a^3/d/(d*x+c)-3/8*I*sin(4*f*x+4*e)/a^3/d/(d*x+c)-3/4*I*f*Ci(6*c*f/d+6*f*x)*cos(-6*e+6*c*f/d)/a^3/d^2+15/32*I*sin(2*f*x+2*e)/a^3/d/(d*x+c)+9/32*cos(2*f*x+2*e)/a^3/d/(d*x+c)-3/8*cos(2*f*x+2*e)^2/a^3/d/(d*x+c)+1/8*cos(2*f*x+2*e)^3/a^3/d/(d*x+c)+3/32*cos(6*f*x+6*e)/a^3/d/(d*x+c)+3/4*f*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/a^3/d^2-3/2*f*cos(-4*e+4*c*f/d)*Si(4*c*f/d+4*f*x)/a^3/d^2+3/4*f*cos(-6*e+6*c*f/d)*Si(6*c*f/d+6*f*x)/a^3/d^2-3/4*f*Ci(6*c*f/d+6*f*x)

$$\begin{aligned}
& f*x)*\sin(-6*e+6*c*f/d)/a^3/d^2-3/4*I*f*Si(6*c*f/d+6*f*x)*\sin(-6*e+6*c*f/d)/ \\
& a^3/d^2+3/2*f*Ci(4*c*f/d+4*f*x)*\sin(-4*e+4*c*f/d)/a^3/d^2+3/32*I*\sin(6*f*x+ \\
& 6*e)/a^3/d/(d*x+c)-3/4*f*Ci(2*c*f/d+2*f*x)*\sin(-2*e+2*c*f/d)/a^3/d^2-1/8*I* \\
& \sin(2*f*x+2*e)^3/a^3/d/(d*x+c)-3/4*I*f*Si(2*c*f/d+2*f*x)*\sin(-2*e+2*c*f/d)/ \\
& a^3/d^2+3/8*\sin(2*f*x+2*e)^2/a^3/d/(d*x+c)+3/2*I*f*Si(4*c*f/d+4*f*x)*\sin(-4 \\
& *e+4*c*f/d)/a^3/d^2-3/4*I*f*Ci(2*c*f/d+2*f*x)*\cos(-2*e+2*c*f/d)/a^3/d^2+3/2 \\
& *I*f*Ci(4*c*f/d+4*f*x)*\cos(-4*e+4*c*f/d)/a^3/d^2
\end{aligned}$$

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 60, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used

= {3809, 3378, 3384, 3380, 3383, 3394, 12, 4491, 4513}

$$\int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))^3} dx = \frac{3f \operatorname{CosIntegral}\left(6xf + \frac{6cf}{d}\right) \sin\left(6e - \frac{6cf}{d}\right)}{4a^3d^2} - \frac{3f \operatorname{CosIntegral}\left(4xf + \frac{4cf}{d}\right) \sin\left(4e - \frac{4cf}{d}\right)}{2a^3d^2} + \frac{3f \operatorname{CosIntegral}\left(2xf + \frac{2cf}{d}\right) \sin\left(2e - \frac{2cf}{d}\right)}{4a^3d^2} - \frac{3if \operatorname{CosIntegral}\left(2xf + \frac{2cf}{d}\right) \cos\left(2e - \frac{2cf}{d}\right)}{4a^3d^2} + \frac{3if \operatorname{CosIntegral}\left(4xf + \frac{4cf}{d}\right) \cos\left(4e - \frac{4cf}{d}\right)}{2a^3d^2} - \frac{3if \operatorname{CosIntegral}\left(6xf + \frac{6cf}{d}\right) \cos\left(6e - \frac{6cf}{d}\right)}{4a^3d^2} + \frac{3if \sin\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(2xf + \frac{2cf}{d}\right)}{4a^3d^2} - \frac{3if \sin\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(4xf + \frac{4cf}{d}\right)}{2a^3d^2} + \frac{3if \sin\left(6e - \frac{6cf}{d}\right) \operatorname{Si}\left(6xf + \frac{6cf}{d}\right)}{4a^3d^2} + \frac{3f \cos\left(2e - \frac{2cf}{d}\right) \operatorname{Si}\left(2xf + \frac{2cf}{d}\right)}{4a^3d^2} - \frac{3f \cos\left(4e - \frac{4cf}{d}\right) \operatorname{Si}\left(4xf + \frac{4cf}{d}\right)}{2a^3d^2} + \frac{3f \cos\left(6e - \frac{6cf}{d}\right) \operatorname{Si}\left(6xf + \frac{6cf}{d}\right)}{4a^3d^2} - \frac{i \sin^3(2e+2fx)}{8a^3d(c+dx)} + \frac{3 \sin^2(2e+2fx)}{8a^3d(c+dx)} + \frac{15i \sin(2e+2fx)}{32a^3d(c+dx)} - \frac{3i \sin(4e+4fx)}{8a^3d(c+dx)} + \frac{3i \sin(6e+6fx)}{32a^3d(c+dx)} + \frac{\cos^3(2e+2fx)}{8a^3d(c+dx)} - \frac{3 \cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{9 \cos(2e+2fx)}{32a^3d(c+dx)} + \frac{3 \cos(6e+6fx)}{32a^3d(c+dx)} - \frac{1}{8a^3d(c+dx)}$$

[In] Int[1/((c + d*x)^2*(a + I*a*Cot[e + f*x])^3),x]

[Out] -1/8*1/(a^3*d*(c + d*x)) + (9*Cos[2*e + 2*f*x])/(32*a^3*d*(c + d*x)) - (3*Cos[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) + Cos[2*e + 2*f*x]^3/(8*a^3*d*(c + d*x)) + (3*Cos[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) - (((3*I)/4)*f*Cos[2*e - (2*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(a^3*d^2) + (((3*I)/2)*f*Cos[4*e - (4*c*f)/d]*CosIntegral[(4*c*f)/d + 4*f*x])/(a^3*d^2) - (((3*I)/4)*f*Cos[6*e - (6*c*f)/d]*CosIntegral[(6*c*f)/d + 6*f*x])/(a^3*d^2) + (3*f*CosIntegra

$$1[(6cf)/d + 6fx] \sin[6e - (6cf)/d] / (4a^3d^2) - (3f \operatorname{CosIntegral}[(4cf)/d + 4fx] \sin[4e - (4cf)/d]) / (2a^3d^2) + (3f \operatorname{CosIntegral}[(2cf)/d + 2fx] \sin[2e - (2cf)/d]) / (4a^3d^2) + ((15I)/32) \sin[2e + 2fx] / (a^3d(c + dx)) + (3 \sin[2e + 2fx]^2) / (8a^3d(c + dx)) - ((I/8) \sin[2e + 2fx]^3) / (a^3d(c + dx)) - (((3I)/8) \sin[4e + 4fx]) / (a^3d(c + dx)) + (((3I)/32) \sin[6e + 6fx]) / (a^3d(c + dx)) + (3f \operatorname{Cos}[2e - (2cf)/d] \operatorname{SinIntegral}[(2cf)/d + 2fx]) / (4a^3d^2) + (((3I)/4) f \sin[2e - (2cf)/d] \operatorname{SinIntegral}[(2cf)/d + 2fx]) / (a^3d^2) - (3f \operatorname{Cos}[4e - (4cf)/d] \operatorname{SinIntegral}[(4cf)/d + 4fx]) / (2a^3d^2) - (((3I)/2) f \sin[4e - (4cf)/d] \operatorname{SinIntegral}[(4cf)/d + 4fx]) / (a^3d^2) + (3f \operatorname{Cos}[6e - (6cf)/d] \operatorname{SinIntegral}[(6cf)/d + 6fx]) / (4a^3d^2) + (((3I)/4) f \sin[6e - (6cf)/d] \operatorname{SinIntegral}[(6cf)/d + 6fx]) / (a^3d^2)$$
Rule 12

$$\operatorname{Int}[(a_*) (u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match} Q[u, (b_*) (v_)] /; \operatorname{FreeQ}[b, x]$$
Rule 3378

$$\operatorname{Int}[(c_*) + (d_*) (x_)]^{(m_*)} \sin[(e_*) + (f_*) (x_)], x_Symbol] \rightarrow \operatorname{Simp}[(c + dx)^{(m+1)} (\sin[e + fx] / (d(m+1))), x] - \operatorname{Dist}[f / (d(m+1)), \operatorname{Int}[(c + dx)^{(m+1)} \operatorname{Cos}[e + fx], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \ \&\& \operatorname{LtQ}[m, -1]$$
Rule 3380

$$\operatorname{Int}[\sin[(e_*) + (f_*) (x_)] / ((c_*) + (d_*) (x_)), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{SinIntegral}[e + fx] / d, x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \ \&\& \operatorname{EqQ}[d*e - c*f, 0]$$
Rule 3383

$$\operatorname{Int}[\sin[(e_*) + (f_*) (x_)] / ((c_*) + (d_*) (x_)), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CosIntegral}[e - \pi/2 + fx] / d, x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \ \&\& \operatorname{EqQ}[d*(e - \pi/2) - c*f, 0]$$
Rule 3384

$$\operatorname{Int}[\sin[(e_*) + (f_*) (x_)] / ((c_*) + (d_*) (x_)), x_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Cos}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Sin}[c*(f/d) + fx] / (c + dx), x], x] + \operatorname{Dist}[\operatorname{Sin}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Cos}[c*(f/d) + fx] / (c + dx), x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \ \&\& \operatorname{NeQ}[d*e - c*f, 0]$$
Rule 3394

$$\operatorname{Int}[(c_*) + (d_*) (x_)]^{(m_*)} \sin[(e_*) + (f_*) (x_)]^{(n_*)}, x_Symbol] \rightarrow \operatorname{Simp}[(c + dx)^{(m+1)} (\sin[e + fx]^n / (d(m+1))), x] - \operatorname{Dist}[f*(n / (d(m+1))$$

)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n - 1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

Rule 3809

Int[((c_.) + (d_.)*(x_))^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x])/(2*a) + Sin[2*e + 2*f*x]/(2*b)]^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]

Rule 4491

Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4513

Int[((e_.) + (f_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol] := Int[ExpandTrigReduce[(e + f*x)^m, Sin[a + b*x]]^p*Sin[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ[q, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(\frac{1}{8a^3(c+dx)^2} - \frac{3\cos(2e+2fx)}{8a^3(c+dx)^2} + \frac{3\cos^2(2e+2fx)}{8a^3(c+dx)^2} - \frac{\cos^3(2e+2fx)}{8a^3(c+dx)^2} \right. \\
 &\quad - \frac{3i\sin(2e+2fx)}{8a^3(c+dx)^2} - \frac{3i\cos^2(2e+2fx)\sin(2e+2fx)}{8a^3(c+dx)^2} - \frac{3\sin^2(2e+2fx)}{8a^3(c+dx)^2} \\
 &\quad \left. + \frac{i\sin^3(2e+2fx)}{8a^3(c+dx)^2} + \frac{3i\sin(4e+4fx)}{8a^3(c+dx)^2} + \frac{3\sin(2e+2fx)\sin(4e+4fx)}{16a^3(c+dx)^2} \right) dx \\
 &= -\frac{1}{8a^3d(c+dx)} + \frac{i\int\frac{\sin^3(2e+2fx)}{(c+dx)^2}dx}{8a^3} - \frac{(3i)\int\frac{\sin(2e+2fx)}{(c+dx)^2}dx}{8a^3} \\
 &\quad - \frac{(3i)\int\frac{\cos^2(2e+2fx)\sin(2e+2fx)}{(c+dx)^2}dx}{8a^3} + \frac{(3i)\int\frac{\sin(4e+4fx)}{(c+dx)^2}dx}{8a^3} \\
 &\quad - \frac{\int\frac{\cos^3(2e+2fx)}{(c+dx)^2}dx}{8a^3} + \frac{3\int\frac{\sin(2e+2fx)\sin(4e+4fx)}{(c+dx)^2}dx}{16a^3} \\
 &\quad - \frac{3\int\frac{\cos(2e+2fx)}{(c+dx)^2}dx}{8a^3} + \frac{3\int\frac{\cos^2(2e+2fx)}{(c+dx)^2}dx}{8a^3} - \frac{3\int\frac{\sin^2(2e+2fx)}{(c+dx)^2}dx}{8a^3}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{8a^3d(c+dx)} + \frac{3\cos(2e+2fx)}{8a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+2fx)}{8a^3d(c+dx)} \\
&+ \frac{3i\sin(2e+2fx)}{8a^3d(c+dx)} + \frac{3\sin^2(2e+2fx)}{8a^3d(c+dx)} - \frac{i\sin^3(2e+2fx)}{8a^3d(c+dx)} \\
&- \frac{3i\sin(4e+4fx)}{8a^3d(c+dx)} - \frac{(3i)\int\left(\frac{\sin(2e+2fx)}{4(c+dx)^2} + \frac{\sin(6e+6fx)}{4(c+dx)^2}\right)dx}{8a^3} \\
&+ \frac{3\int\left(\frac{\cos(2e+2fx)}{2(c+dx)^2} - \frac{\cos(6e+6fx)}{2(c+dx)^2}\right)dx}{16a^3} - \frac{(3if)\int\frac{\cos(2e+2fx)}{c+dx}dx}{4a^3d} \\
&+ \frac{(3if)\int\left(\frac{\cos(2e+2fx)}{4(c+dx)} - \frac{\cos(6e+6fx)}{4(c+dx)}\right)dx}{4a^3d} + \frac{(3if)\int\frac{\cos(4e+4fx)}{c+dx}dx}{2a^3d} \\
&+ \frac{(3f)\int\frac{\sin(2e+2fx)}{c+dx}dx}{4a^3d} - \frac{(3f)\int\left(-\frac{\sin(2e+2fx)}{4(c+dx)} - \frac{\sin(6e+6fx)}{4(c+dx)}\right)dx}{4a^3d} \\
&+ \frac{(3f)\int-\frac{\sin(4e+4fx)}{2(c+dx)}dx}{2a^3d} - \frac{(3f)\int\frac{\sin(4e+4fx)}{2(c+dx)}dx}{2a^3d} \\
&= -\frac{1}{8a^3d(c+dx)} + \frac{3\cos(2e+2fx)}{8a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+2fx)}{8a^3d(c+dx)} \\
&+ \frac{3i\sin(2e+2fx)}{8a^3d(c+dx)} + \frac{3\sin^2(2e+2fx)}{8a^3d(c+dx)} - \frac{i\sin^3(2e+2fx)}{8a^3d(c+dx)} - \frac{3i\sin(4e+4fx)}{8a^3d(c+dx)} \\
&- \frac{(3i)\int\frac{\sin(2e+2fx)}{(c+dx)^2}dx}{32a^3} - \frac{(3i)\int\frac{\sin(6e+6fx)}{(c+dx)^2}dx}{32a^3} + \frac{3\int\frac{\cos(2e+2fx)}{(c+dx)^2}dx}{32a^3} - \frac{3\int\frac{\cos(6e+6fx)}{(c+dx)^2}dx}{32a^3} \\
&+ \frac{(3if)\int\frac{\cos(2e+2fx)}{c+dx}dx}{16a^3d} - \frac{(3if)\int\frac{\cos(6e+6fx)}{c+dx}dx}{16a^3d} + \frac{(3f)\int\frac{\sin(2e+2fx)}{c+dx}dx}{16a^3d} \\
&+ \frac{(3f)\int\frac{\sin(6e+6fx)}{c+dx}dx}{16a^3d} - 2\frac{(3f)\int\frac{\sin(4e+4fx)}{c+dx}dx}{4a^3d} + \frac{(3if\cos(4e-\frac{4cf}{d}))\int\frac{\cos(\frac{4cf}{d}+4fx)}{c+dx}dx}{2a^3d} \\
&- \frac{(3if\cos(2e-\frac{2cf}{d}))\int\frac{\cos(\frac{2cf}{d}+2fx)}{c+dx}dx}{4a^3d} + \frac{(3f\cos(2e-\frac{2cf}{d}))\int\frac{\sin(\frac{2cf}{d}+2fx)}{c+dx}dx}{4a^3d} \\
&- \frac{(3if\sin(4e-\frac{4cf}{d}))\int\frac{\sin(\frac{4cf}{d}+4fx)}{c+dx}dx}{2a^3d} + \frac{(3if\sin(2e-\frac{2cf}{d}))\int\frac{\sin(\frac{2cf}{d}+2fx)}{c+dx}dx}{4a^3d} \\
&+ \frac{(3f\sin(2e-\frac{2cf}{d}))\int\frac{\cos(\frac{2cf}{d}+2fx)}{c+dx}dx}{4a^3d}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{8a^3d(c+dx)} + \frac{9\cos(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+2fx)}{8a^3d(c+dx)} \\
&+ \frac{3\cos(6e+6fx)}{32a^3d(c+dx)} - \frac{3if\cos(2e-\frac{2cf}{d})\text{CosIntegral}(\frac{2cf}{d}+2fx)}{4a^3d^2} \\
&+ \frac{3if\cos(4e-\frac{4cf}{d})\text{CosIntegral}(\frac{4cf}{d}+4fx)}{2a^3d^2} \\
&+ \frac{3f\text{CosIntegral}(\frac{2cf}{d}+2fx)\sin(2e-\frac{2cf}{d})}{4a^3d^2} + \frac{15i\sin(2e+2fx)}{32a^3d(c+dx)} \\
&+ \frac{3\sin^2(2e+2fx)}{8a^3d(c+dx)} - \frac{i\sin^3(2e+2fx)}{8a^3d(c+dx)} - \frac{3i\sin(4e+4fx)}{8a^3d(c+dx)} + \frac{3i\sin(6e+6fx)}{32a^3d(c+dx)} \\
&+ \frac{3f\cos(2e-\frac{2cf}{d})\text{Si}(\frac{2cf}{d}+2fx)}{4a^3d^2} + \frac{3if\sin(2e-\frac{2cf}{d})\text{Si}(\frac{2cf}{d}+2fx)}{4a^3d^2} \\
&- \frac{3if\sin(4e-\frac{4cf}{d})\text{Si}(\frac{4cf}{d}+4fx)}{2a^3d^2} - \frac{(3if)\int\frac{\cos(2e+2fx)}{c+dx}dx}{16a^3d} \\
&- \frac{(9if)\int\frac{\cos(6e+6fx)}{c+dx}dx}{16a^3d} - \frac{(3f)\int\frac{\sin(2e+2fx)}{c+dx}dx}{16a^3d} + \frac{(9f)\int\frac{\sin(6e+6fx)}{c+dx}dx}{16a^3d} \\
&- \frac{(3if\cos(6e-\frac{6cf}{d}))\int\frac{\cos(\frac{6cf}{d}+6fx)}{c+dx}dx}{16a^3d} + \frac{(3f\cos(6e-\frac{6cf}{d}))\int\frac{\sin(\frac{6cf}{d}+6fx)}{c+dx}dx}{16a^3d} \\
&+ \frac{(3if\cos(2e-\frac{2cf}{d}))\int\frac{\cos(\frac{2cf}{d}+2fx)}{c+dx}dx}{16a^3d} + \frac{(3f\cos(2e-\frac{2cf}{d}))\int\frac{\sin(\frac{2cf}{d}+2fx)}{c+dx}dx}{16a^3d} \\
&+ \frac{(3if\sin(6e-\frac{6cf}{d}))\int\frac{\sin(\frac{6cf}{d}+6fx)}{c+dx}dx}{16a^3d} + \frac{(3f\sin(6e-\frac{6cf}{d}))\int\frac{\cos(\frac{6cf}{d}+6fx)}{c+dx}dx}{16a^3d} \\
&- 2\left(\frac{(3f\cos(4e-\frac{4cf}{d}))\int\frac{\sin(\frac{4cf}{d}+4fx)}{c+dx}dx}{4a^3d} + \frac{(3f\sin(4e-\frac{4cf}{d}))\int\frac{\cos(\frac{4cf}{d}+4fx)}{c+dx}dx}{4a^3d}\right) \\
&- \frac{(3if\sin(2e-\frac{2cf}{d}))\int\frac{\sin(\frac{2cf}{d}+2fx)}{c+dx}dx}{16a^3d} + \frac{(3f\sin(2e-\frac{2cf}{d}))\int\frac{\cos(\frac{2cf}{d}+2fx)}{c+dx}dx}{16a^3d}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{8a^3d(c+dx)} + \frac{9\cos(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+2fx)}{8a^3d(c+dx)} \\
&+ \frac{3\cos(6e+6fx)}{32a^3d(c+dx)} - \frac{9if\cos(2e-\frac{2cf}{d})\text{CosIntegral}(\frac{2cf}{d}+2fx)}{16a^3d^2} \\
&+ \frac{3if\cos(4e-\frac{4cf}{d})\text{CosIntegral}(\frac{4cf}{d}+4fx)}{2a^3d^2} \\
&- \frac{3if\cos(6e-\frac{6cf}{d})\text{CosIntegral}(\frac{6cf}{d}+6fx)}{16a^3d^2} \\
&+ \frac{3f\text{CosIntegral}(\frac{6cf}{d}+6fx)\sin(6e-\frac{6cf}{d})}{16a^3d^2} \\
&+ \frac{15f\text{CosIntegral}(\frac{2cf}{d}+2fx)\sin(2e-\frac{2cf}{d})}{16a^3d^2} + \frac{15i\sin(2e+2fx)}{32a^3d(c+dx)} \\
&+ \frac{3\sin^2(2e+2fx)}{8a^3d(c+dx)} - \frac{i\sin^3(2e+2fx)}{8a^3d(c+dx)} - \frac{3i\sin(4e+4fx)}{8a^3d(c+dx)} + \frac{3i\sin(6e+6fx)}{32a^3d(c+dx)} \\
&+ \frac{15f\cos(2e-\frac{2cf}{d})\text{Si}(\frac{2cf}{d}+2fx)}{16a^3d^2} + \frac{9if\sin(2e-\frac{2cf}{d})\text{Si}(\frac{2cf}{d}+2fx)}{16a^3d^2} \\
&- \frac{3if\sin(4e-\frac{4cf}{d})\text{Si}(\frac{4cf}{d}+4fx)}{2a^3d^2} - 2\left(\frac{3f\text{CosIntegral}(\frac{4cf}{d}+4fx)\sin(4e-\frac{4cf}{d})}{4a^3d^2} \right. \\
&\quad \left. + \frac{3f\cos(4e-\frac{4cf}{d})\text{Si}(\frac{4cf}{d}+4fx)}{4a^3d^2}\right) \\
&+ \frac{3f\cos(6e-\frac{6cf}{d})\text{Si}(\frac{6cf}{d}+6fx)}{16a^3d^2} + \frac{3if\sin(6e-\frac{6cf}{d})\text{Si}(\frac{6cf}{d}+6fx)}{16a^3d^2} \\
&- \frac{(9if\cos(6e-\frac{6cf}{d}))\int\frac{\cos(\frac{6cf}{d}+6fx)}{c+dx}dx}{16a^3d} + \frac{(9f\cos(6e-\frac{6cf}{d}))\int\frac{\sin(\frac{6cf}{d}+6fx)}{c+dx}dx}{16a^3d} \\
&- \frac{(3if\cos(2e-\frac{2cf}{d}))\int\frac{\cos(\frac{2cf}{d}+2fx)}{c+dx}dx}{16a^3d} - \frac{(3f\cos(2e-\frac{2cf}{d}))\int\frac{\sin(\frac{2cf}{d}+2fx)}{c+dx}dx}{16a^3d} \\
&+ \frac{(9if\sin(6e-\frac{6cf}{d}))\int\frac{\sin(\frac{6cf}{d}+6fx)}{c+dx}dx}{16a^3d} + \frac{(9f\sin(6e-\frac{6cf}{d}))\int\frac{\cos(\frac{6cf}{d}+6fx)}{c+dx}dx}{16a^3d} \\
&+ \frac{(3if\sin(2e-\frac{2cf}{d}))\int\frac{\sin(\frac{2cf}{d}+2fx)}{c+dx}dx}{16a^3d} - \frac{(3f\sin(2e-\frac{2cf}{d}))\int\frac{\cos(\frac{2cf}{d}+2fx)}{c+dx}dx}{16a^3d}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{8a^3d(c+dx)} + \frac{9\cos(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cos^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cos^3(2e+2fx)}{8a^3d(c+dx)} \\
&+ \frac{3\cos(6e+6fx)}{32a^3d(c+dx)} - \frac{3if\cos(2e-\frac{2cf}{d})\operatorname{CosIntegral}(\frac{2cf}{d}+2fx)}{4a^3d^2} \\
&+ \frac{3if\cos(4e-\frac{4cf}{d})\operatorname{CosIntegral}(\frac{4cf}{d}+4fx)}{2a^3d^2} \\
&- \frac{3if\cos(6e-\frac{6cf}{d})\operatorname{CosIntegral}(\frac{6cf}{d}+6fx)}{4a^3d^2} \\
&+ \frac{3f\operatorname{CosIntegral}(\frac{6cf}{d}+6fx)\sin(6e-\frac{6cf}{d})}{4a^3d^2} \\
&+ \frac{3f\operatorname{CosIntegral}(\frac{2cf}{d}+2fx)\sin(2e-\frac{2cf}{d})}{4a^3d^2} + \frac{15i\sin(2e+2fx)}{32a^3d(c+dx)} \\
&+ \frac{3\sin^2(2e+2fx)}{8a^3d(c+dx)} - \frac{i\sin^3(2e+2fx)}{8a^3d(c+dx)} - \frac{3i\sin(4e+4fx)}{8a^3d(c+dx)} + \frac{3i\sin(6e+6fx)}{32a^3d(c+dx)} \\
&+ \frac{3f\cos(2e-\frac{2cf}{d})\operatorname{Si}(\frac{2cf}{d}+2fx)}{4a^3d^2} + \frac{3if\sin(2e-\frac{2cf}{d})\operatorname{Si}(\frac{2cf}{d}+2fx)}{4a^3d^2} \\
&- \frac{3if\sin(4e-\frac{4cf}{d})\operatorname{Si}(\frac{4cf}{d}+4fx)}{2a^3d^2} - 2\left(\frac{3f\operatorname{CosIntegral}(\frac{4cf}{d}+4fx)\sin(4e-\frac{4cf}{d})}{4a^3d^2} \right. \\
&\quad \left. + \frac{3f\cos(4e-\frac{4cf}{d})\operatorname{Si}(\frac{4cf}{d}+4fx)}{4a^3d^2}\right) \\
&+ \frac{3f\cos(6e-\frac{6cf}{d})\operatorname{Si}(\frac{6cf}{d}+6fx)}{4a^3d^2} + \frac{3if\sin(6e-\frac{6cf}{d})\operatorname{Si}(\frac{6cf}{d}+6fx)}{4a^3d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.41

$$\int \frac{1}{(c+dx)^2(a+ia\cot(e+fx))^3} dx$$

$$= \frac{-d + 3d(\cos(2(e+fx)) + i\sin(2(e+fx))) - 3d(\cos(4(e+fx)) + i\sin(4(e+fx))) + d(\cos(6(e+fx)) + i\sin(6(e+fx)))}{(8a^3d^2(c+dx))}$$

[In] Integrate[1/((c+d*x)^2*(a+I*a*Cot[e+f*x])^3),x]

[Out] (-d + 3*d*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]) - 3*d*(Cos[4*(e + f*x)] + I*Sin[4*(e + f*x)]) + d*(Cos[6*(e + f*x)] + I*Sin[6*(e + f*x)]) + 6*f*(c + d*x)*((-I)*Cos[2*e - (2*c*f)/d] + Sin[2*e - (2*c*f)/d])*(CosIntegral[(2*f*(c + d*x))/d] + I*SinIntegral[(2*f*(c + d*x))/d]) + (12*I)*f*(c + d*x)*(Cos[4*e - (4*c*f)/d] + I*Sin[4*e - (4*c*f)/d])*(CosIntegral[(4*f*(c + d*x))/d] + I*SinIntegral[(4*f*(c + d*x))/d]) + 6*f*(c + d*x)*((-I)*Cos[6*e - (6*c*f)/d] + Sin[6*e - (6*c*f)/d])*(CosIntegral[(6*f*(c + d*x))/d] + I*SinIntegral[(6*f*(c + d*x))/d])/(8*a^3*d^2*(c + d*x))

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.39

method	result
risch	$-\frac{1}{8a^3d(dx+c)} + \frac{3ife^{2i(fx+e)}}{8a^3d^2\left(ifax+\frac{icf}{d}\right)} + \frac{3ife^{-\frac{2i(cf-de)}{d}} \operatorname{Ei}_1\left(-2ifx-2ie-\frac{2(icf-ide)}{d}\right)}{4a^3d^2} - \frac{3ife^{4i(fx+e)}}{8a^3d^2\left(ifax+\frac{icf}{d}\right)} - \frac{3ife^{-\frac{4i(cf-de)}{d}}}{8a^3d^2\left(ifax+\frac{icf}{d}\right)}$

[In] int(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out]
$$-1/8/a^3/d/(d*x+c)+3/8*I/a^3*f/d^2*\exp(2*I*(f*x+e))/(I*f*x+I/d*c*f)+3/4*I/a^3*f/d^2*\exp(-2*I*(c*f-d*e)/d)*\operatorname{Ei}(1,-2*I*f*x-2*I*e-2*(I*c*f-I*d*e)/d)-3/8*I/a^3*f/d^2*\exp(4*I*(f*x+e))/(I*f*x+I/d*c*f)-3/2*I/a^3*f/d^2*\exp(-4*I*(c*f-d*e)/d)*\operatorname{Ei}(1,-4*I*f*x-4*I*e-4*(I*c*f-I*d*e)/d)+1/8*I/a^3*f/d^2*\exp(6*I*(f*x+e))/(I*f*x+I/d*c*f)+3/4*I/a^3*f/d^2*\exp(-6*I*(c*f-d*e)/d)*\operatorname{Ei}(1,-6*I*f*x-6*I*e-6*(I*c*f-I*d*e)/d)$$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.26

$$\int \frac{1}{(c+dx)^2(a+ia \cot(e+fx))^3} dx = \frac{6(i d f x + i c f) \operatorname{Ei}\left(-\frac{2(-i d f x - i c f)}{d}\right) e^{\left(-\frac{2(-i d e + i c f)}{d}\right)} + 12(-i d f x - i c f) \operatorname{Ei}\left(-\frac{4(-i d f x - i c f)}{d}\right) e^{\left(-\frac{4(-i d e + i c f)}{d}\right)}}{8(a^3 d^3 x + \dots)}$$

[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")

[Out]
$$-1/8*(6*(I*d*f*x + I*c*f)*\operatorname{Ei}(-2*(-I*d*f*x - I*c*f)/d)*e^{(-2*(-I*d*e + I*c*f)/d)} + 12*(-I*d*f*x - I*c*f)*\operatorname{Ei}(-4*(-I*d*f*x - I*c*f)/d)*e^{(-4*(-I*d*e + I*c*f)/d)} + 6*(I*d*f*x + I*c*f)*\operatorname{Ei}(-6*(-I*d*f*x - I*c*f)/d)*e^{(-6*(-I*d*e + I*c*f)/d)} - d*e^{(6*I*f*x + 6*I*e)} + 3*d*e^{(4*I*f*x + 4*I*e)} - 3*d*e^{(2*I*f*x + 2*I*e)} + d)/(a^3*d^3*x + a^3*c*d^2)$$

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c + dx)^2(a + ia \cot(e + fx))^3} dx = \text{Timed out}$$

[In] integrate(1/(d*x+c)**2/(a+I*a*cot(f*x+e))**3,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.42

$$\int \frac{1}{(c + dx)^2(a + ia \cot(e + fx))^3} dx$$

$$= \frac{f^2 \cos\left(-\frac{6(de - cf)}{d}\right) E_2\left(\frac{6(-i(fx + e)d + ide - icf)}{d}\right) - 3f^2 \cos\left(-\frac{4(de - cf)}{d}\right) E_2\left(\frac{4(-i(fx + e)d + ide - icf)}{d}\right) + 3f^2 \cos\left(-\frac{2(de - cf)}{d}\right) E_2\left(\frac{2(-i(fx + e)d + ide - icf)}{d}\right) - f^2 \cos\left(-\frac{de - cf}{d}\right) E_2\left(\frac{-i(fx + e)d + ide - icf}{d}\right)}{(c + dx)^2(a + ia \cot(e + fx))^3}$$

[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")

[Out] 1/8*(f^2*cos(-6*(d*e - c*f)/d)*exp_integral_e(2, 6*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) - 3*f^2*cos(-4*(d*e - c*f)/d)*exp_integral_e(2, 4*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) + 3*f^2*cos(-2*(d*e - c*f)/d)*exp_integral_e(2, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) - 3*I*f^2*exp_integral_e(2, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d)*sin(-2*(d*e - c*f)/d) + 3*I*f^2*exp_integral_e(2, 4*(-I*(f*x + e)*d + I*d*e - I*c*f)/d)*sin(-4*(d*e - c*f)/d) - I*f^2*exp_integral_e(2, 6*(-I*(f*x + e)*d + I*d*e - I*c*f)/d)*sin(-6*(d*e - c*f)/d) - f^2/(((f*x + e)*a^3*d^2 - a^3*d^2*e + a^3*c*d*f)*f)

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4284 vs. 2(648) = 1296.

Time = 1.37 (sec) , antiderivative size = 4284, normalized size of antiderivative = 6.02

$$\int \frac{1}{(c + dx)^2(a + ia \cot(e + fx))^3} dx = \text{Too large to display}$$

[In] integrate(1/(d*x+c)^2/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")

[Out] 1/8*(-6*I*d*f*x*cos(e)^6*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d) + 36*d*f*x*cos(e)^5*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e) + 90*I*d


```

*f*x*cos(e)^4*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^2 - 120*d
*f*x*cos(e)^3*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^3 - 90*I*
d*f*x*cos(e)^2*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^4 + 36*d
*f*x*cos(e)*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^5 + 6*I*d*f
*x*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^6 - 6*d*f*x*cos(e)^6
*cos_integral(6*(d*f*x + c*f)/d)*sin(6*c*f/d) - 36*I*d*f*x*cos(e)^5*cos_int
egral(6*(d*f*x + c*f)/d)*sin(e)*sin(6*c*f/d) + 90*d*f*x*cos(e)^4*cos_integr
al(6*(d*f*x + c*f)/d)*sin(e)^2*sin(6*c*f/d) + 120*I*d*f*x*cos(e)^3*cos_inte
gral(6*(d*f*x + c*f)/d)*sin(e)^3*sin(6*c*f/d) - 90*d*f*x*cos(e)^2*cos_integ
ral(6*(d*f*x + c*f)/d)*sin(e)^4*sin(6*c*f/d) - 36*I*d*f*x*cos(e)*cos_integr
al(6*(d*f*x + c*f)/d)*sin(e)^5*sin(6*c*f/d) + 6*d*f*x*cos_integral(6*(d*f*x
+ c*f)/d)*sin(e)^6*sin(6*c*f/d) + 6*d*f*x*cos(e)^6*cos(6*c*f/d)*sin_integr
al(6*(d*f*x + c*f)/d) + 36*I*d*f*x*cos(e)^5*cos(6*c*f/d)*sin(e)*sin_integra
l(6*(d*f*x + c*f)/d) - 90*d*f*x*cos(e)^4*cos(6*c*f/d)*sin(e)^2*sin_integral
(6*(d*f*x + c*f)/d) - 120*I*d*f*x*cos(e)^3*cos(6*c*f/d)*sin(e)^3*sin_integr
al(6*(d*f*x + c*f)/d) + 90*d*f*x*cos(e)^2*cos(6*c*f/d)*sin(e)^4*sin_integra
l(6*(d*f*x + c*f)/d) + 36*I*d*f*x*cos(e)*cos(6*c*f/d)*sin(e)^5*sin_integral
(6*(d*f*x + c*f)/d) - 6*d*f*x*cos(6*c*f/d)*sin(e)^6*sin_integral(6*(d*f*x +
c*f)/d) - 6*I*d*f*x*cos(e)^6*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d)
+ 36*d*f*x*cos(e)^5*sin(e)*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 9
0*I*d*f*x*cos(e)^4*sin(e)^2*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) -
120*d*f*x*cos(e)^3*sin(e)^3*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) -
90*I*d*f*x*cos(e)^2*sin(e)^4*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) +
36*d*f*x*cos(e)*sin(e)^5*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 6*
I*d*f*x*sin(e)^6*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) - 6*I*c*f*cos
(e)^6*cos(6*c*f/d)*cos_integral(6*(d*f*x + c*f)/d) + 36*c*f*cos(e)^5*cos(6*
c*f/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e) + 90*I*c*f*cos(e)^4*cos(6*c*f
/d)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^2 - 120*c*f*cos(e)^3*cos(6*c*f/d
)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^3 - 90*I*c*f*cos(e)^2*cos(6*c*f/d)
*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^4 + 36*c*f*cos(e)*cos(6*c*f/d)*cos_
integral(6*(d*f*x + c*f)/d)*sin(e)^5 + 6*I*c*f*cos(6*c*f/d)*cos_integral(6*
(d*f*x + c*f)/d)*sin(e)^6 - 6*c*f*cos(e)^6*cos_integral(6*(d*f*x + c*f)/d)*
sin(6*c*f/d) - 36*I*c*f*cos(e)^5*cos_integral(6*(d*f*x + c*f)/d)*sin(e)*sin
(6*c*f/d) + 90*c*f*cos(e)^4*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^2*sin(6*
c*f/d) + 120*I*c*f*cos(e)^3*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^3*sin(6*
c*f/d) - 90*c*f*cos(e)^2*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^4*sin(6*c*f
/d) - 36*I*c*f*cos(e)*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^5*sin(6*c*f/d)
+ 6*c*f*cos_integral(6*(d*f*x + c*f)/d)*sin(e)^6*sin(6*c*f/d) + 6*c*f*cos(
e)^6*cos(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 36*I*c*f*cos(e)^5*cos(6
*c*f/d)*sin(e)*sin_integral(6*(d*f*x + c*f)/d) - 90*c*f*cos(e)^4*cos(6*c*f/
d)*sin(e)^2*sin_integral(6*(d*f*x + c*f)/d) - 120*I*c*f*cos(e)^3*cos(6*c*f/
d)*sin(e)^3*sin_integral(6*(d*f*x + c*f)/d) + 90*c*f*cos(e)^2*cos(6*c*f/d)*
sin(e)^4*sin_integral(6*(d*f*x + c*f)/d) + 36*I*c*f*cos(e)*cos(6*c*f/d)*sin
(e)^5*sin_integral(6*(d*f*x + c*f)/d) - 6*c*f*cos(6*c*f/d)*sin(e)^6*sin_int
egral(6*(d*f*x + c*f)/d) - 6*I*c*f*cos(e)^6*sin(6*c*f/d)*sin_integral(6*(d*

```

$$\begin{aligned}
& f*x + c*f)/d) + 36*c*f*cos(e)^5*sin(e)*sin(6*c*f/d)*sin_integral(6*(d*f*x + \\
& c*f)/d) + 90*I*c*f*cos(e)^4*sin(e)^2*sin(6*c*f/d)*sin_integral(6*(d*f*x + \\
& c*f)/d) - 120*c*f*cos(e)^3*sin(e)^3*sin(6*c*f/d)*sin_integral(6*(d*f*x + c* \\
& f)/d) - 90*I*c*f*cos(e)^2*sin(e)^4*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f \\
&)/d) + 36*c*f*cos(e)*sin(e)^5*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) \\
& + 6*I*c*f*sin(e)^6*sin(6*c*f/d)*sin_integral(6*(d*f*x + c*f)/d) + 12*I*d*f*x \\
& *cos(e)^4*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d) - 48*d*f*x*cos(e)^3 \\
& *cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e) - 72*I*d*f*x*cos(e)^2* \\
& cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2 + 48*d*f*x*cos(e)*cos \\
& (4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^3 + 12*I*d*f*x*cos(4*c*f/d \\
&)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4 + 12*d*f*x*cos(e)^4*cos_integral \\
& (4*(d*f*x + c*f)/d)*sin(4*c*f/d) + 48*I*d*f*x*cos(e)^3*cos_integral(4*(d*f* \\
& x + c*f)/d)*sin(e)*sin(4*c*f/d) - 72*d*f*x*cos(e)^2*cos_integral(4*(d*f*x + \\
& c*f)/d)*sin(e)^2*sin(4*c*f/d) - 48*I*d*f*x*cos(e)*cos_integral(4*(d*f*x + \\
& c*f)/d)*sin(e)^3*sin(4*c*f/d) + 12*d*f*x*cos_integral(4*(d*f*x + c*f)/d)*si \\
& n(e)^4*sin(4*c*f/d) - 12*d*f*x*cos(e)^4*cos(4*c*f/d)*sin_integral(4*(d*f*x \\
& + c*f)/d) - 48*I*d*f*x*cos(e)^3*cos(4*c*f/d)*sin(e)*sin_integral(4*(d*f*x + \\
& c*f)/d) + 72*d*f*x*cos(e)^2*cos(4*c*f/d)*sin(e)^2*sin_integral(4*(d*f*x + \\
& c*f)/d) + 48*I*d*f*x*cos(e)*cos(4*c*f/d)*sin(e)^3*sin_integral(4*(d*f*x + c \\
& *f)/d) - 12*d*f*x*cos(4*c*f/d)*sin(e)^4*sin_integral(4*(d*f*x + c*f)/d) + 1 \\
& 2*I*d*f*x*cos(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 48*d*f*x* \\
& cos(e)^3*sin(e)*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 72*I*d*f*x*c \\
& os(e)^2*sin(e)^2*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 48*d*f*x*co \\
& s(e)*sin(e)^3*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + 12*I*d*f*x*sin \\
& (e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) + d*cos(6*f*x)*cos(e)^6 \\
& + 12*I*c*f*cos(e)^4*cos(4*c*f/d)*cos_integral(4*(d*f*x + c*f)/d) + I*d*cos(\\
& e)^6*sin(6*f*x) + 6*I*d*cos(6*f*x)*cos(e)^5*sin(e) - 48*c*f*cos(e)^3*cos(4* \\
& c*f/d)*cos_integral(4*(d*f*x + c*f)/d)*sin(e) - 6*d*cos(e)^5*sin(6*f*x)*sin \\
& (e) - 15*d*cos(6*f*x)*cos(e)^4*sin(e)^2 - 72*I*c*f*cos(e)^2*cos(4*c*f/d)*co \\
& s_integral(4*(d*f*x + c*f)/d)*sin(e)^2 - 15*I*d*cos(e)^4*sin(6*f*x)*sin(e)^ \\
& 2 - 20*I*d*cos(6*f*x)*cos(e)^3*sin(e)^3 + 48*c*f*cos(e)*cos(4*c*f/d)*cos_in \\
& tegral(4*(d*f*x + c*f)/d)*sin(e)^3 + 20*d*cos(e)^3*sin(6*f*x)*sin(e)^3 + 15 \\
& *d*cos(6*f*x)*cos(e)^2*sin(e)^4 + 12*I*c*f*cos(4*c*f/d)*cos_integral(4*(d*f \\
& *x + c*f)/d)*sin(e)^4 + 15*I*d*cos(e)^2*sin(6*f*x)*sin(e)^4 + 6*I*d*cos(6*f \\
& *x)*cos(e)*sin(e)^5 - 6*d*cos(e)*sin(6*f*x)*sin(e)^5 - d*cos(6*f*x)*sin(e)^ \\
& 6 - I*d*sin(6*f*x)*sin(e)^6 + 12*c*f*cos(e)^4*cos_integral(4*(d*f*x + c*f)/ \\
& d)*sin(4*c*f/d) + 48*I*c*f*cos(e)^3*cos_integral(4*(d*f*x + c*f)/d)*sin(e)* \\
& sin(4*c*f/d) - 72*c*f*cos(e)^2*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^2*sin \\
& (4*c*f/d) - 48*I*c*f*cos(e)*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^3*sin(4* \\
& c*f/d) + 12*c*f*cos_integral(4*(d*f*x + c*f)/d)*sin(e)^4*sin(4*c*f/d) - 12* \\
& c*f*cos(e)^4*cos(4*c*f/d)*sin_integral(4*(d*f*x + c*f)/d) - 48*I*c*f*cos(e) \\
& ^3*cos(4*c*f/d)*sin(e)*sin_integral(4*(d*f*x + c*f)/d) + 72*c*f*cos(e)^2*co \\
& s(4*c*f/d)*sin(e)^2*sin_integral(4*(d*f*x + c*f)/d) + 48*I*c*f*cos(e)*cos(4 \\
& *c*f/d)*sin(e)^3*sin_integral(4*(d*f*x + c*f)/d) - 12*c*f*cos(4*c*f/d)*sin(\\
& e)^4*sin_integral(4*(d*f*x + c*f)/d) + 12*I*c*f*cos(e)^4*sin(4*c*f/d)*sin_i
\end{aligned}$$

```

ntegral(4*(d*f*x + c*f)/d) - 48*c*f*cos(e)^3*sin(e)*sin(4*c*f/d)*sin_integr
al(4*(d*f*x + c*f)/d) - 72*I*c*f*cos(e)^2*sin(e)^2*sin(4*c*f/d)*sin_integra
l(4*(d*f*x + c*f)/d) + 48*c*f*cos(e)*sin(e)^3*sin(4*c*f/d)*sin_integral(4*(
d*f*x + c*f)/d) + 12*I*c*f*sin(e)^4*sin(4*c*f/d)*sin_integral(4*(d*f*x + c*
f)/d) - 6*I*d*f*x*cos(e)^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d) + 1
2*d*f*x*cos(e)*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) + 6*I*d*
f*x*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2 - 6*d*f*x*cos(e)^
2*cos_integral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) - 12*I*d*f*x*cos(e)*cos_inte
gral(2*(d*f*x + c*f)/d)*sin(e)*sin(2*c*f/d) + 6*d*f*x*cos_integral(2*(d*f*x
+ c*f)/d)*sin(e)^2*sin(2*c*f/d) + 6*d*f*x*cos(e)^2*cos(2*c*f/d)*sin_integr
al(2*(d*f*x + c*f)/d) + 12*I*d*f*x*cos(e)*cos(2*c*f/d)*sin(e)*sin_integral(
2*(d*f*x + c*f)/d) - 6*d*f*x*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x +
c*f)/d) - 6*I*d*f*x*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) +
12*d*f*x*cos(e)*sin(e)*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 6*I*
d*f*x*sin(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) - 3*d*cos(4*f*x
)*cos(e)^4 - 6*I*c*f*cos(e)^2*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)
- 3*I*d*cos(e)^4*sin(4*f*x) - 12*I*d*cos(4*f*x)*cos(e)^3*sin(e) + 12*c*f*co
s(e)*cos(2*c*f/d)*cos_integral(2*(d*f*x + c*f)/d)*sin(e) + 12*d*cos(e)^3*si
n(4*f*x)*sin(e) + 18*d*cos(4*f*x)*cos(e)^2*sin(e)^2 + 6*I*c*f*cos(2*c*f/d)*
cos_integral(2*(d*f*x + c*f)/d)*sin(e)^2 + 18*I*d*cos(e)^2*sin(4*f*x)*sin(e
)^2 + 12*I*d*cos(4*f*x)*cos(e)*sin(e)^3 - 12*d*cos(e)*sin(4*f*x)*sin(e)^3 -
3*d*cos(4*f*x)*sin(e)^4 - 3*I*d*sin(4*f*x)*sin(e)^4 - 6*c*f*cos(e)^2*cos_i
ntegral(2*(d*f*x + c*f)/d)*sin(2*c*f/d) - 12*I*c*f*cos(e)*cos_integral(2*(d
*f*x + c*f)/d)*sin(e)*sin(2*c*f/d) + 6*c*f*cos_integral(2*(d*f*x + c*f)/d)*
sin(e)^2*sin(2*c*f/d) + 6*c*f*cos(e)^2*cos(2*c*f/d)*sin_integral(2*(d*f*x +
c*f)/d) + 12*I*c*f*cos(e)*cos(2*c*f/d)*sin(e)*sin_integral(2*(d*f*x + c*f)
/d) - 6*c*f*cos(2*c*f/d)*sin(e)^2*sin_integral(2*(d*f*x + c*f)/d) - 6*I*c*f
*cos(e)^2*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 12*c*f*cos(e)*sin(
e)*sin(2*c*f/d)*sin_integral(2*(d*f*x + c*f)/d) + 6*I*c*f*sin(e)^2*sin(2*c*
f/d)*sin_integral(2*(d*f*x + c*f)/d) + 3*d*cos(2*f*x)*cos(e)^2 + 3*I*d*cos(
e)^2*sin(2*f*x) + 6*I*d*cos(2*f*x)*cos(e)*sin(e) - 6*d*cos(e)*sin(2*f*x)*si
n(e) - 3*d*cos(2*f*x)*sin(e)^2 - 3*I*d*sin(2*f*x)*sin(e)^2 - d)/(a^3*d^3*x
+ a^3*c*d^2)

```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c + dx)^2(a + ia \cot(e + fx))^3} dx = \int \frac{1}{(a + a \cot(e + fx) li)^3 (c + dx)^2} dx$$

[In] int(1/((a + a*cot(e + f*x)*1i)^3*(c + d*x)^2), x)

[Out] int(1/((a + a*cot(e + f*x)*1i)^3*(c + d*x)^2), x)

3.32 $\int (c + dx)^m (a + ia \cot(e + fx))^2 dx$

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Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (c + dx)^m (a + ia \cot(e + fx))^2 dx = \text{Int}((c + dx)^m (a + ia \cot(e + fx))^2, x)$$

[Out] Unintegrable((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x)

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c + dx)^m (a + ia \cot(e + fx))^2 dx = \int (c + dx)^m (a + ia \cot(e + fx))^2 dx$$

[In] Int[(c + d*x)^m*(a + I*a*Cot[e + f*x])^2,x]

[Out] Defer[Int][(c + d*x)^m*(a + I*a*Cot[e + f*x])^2, x]

Rubi steps

$$\text{integral} = \int (c + dx)^m (a + ia \cot(e + fx))^2 dx$$

Mathematica [N/A]

Not integrable

Time = 13.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (c + dx)^m (a + ia \cot(e + fx))^2 dx = \int (c + dx)^m (a + ia \cot(e + fx))^2 dx$$

[In] Integrate[(c + d*x)^m*(a + I*a*Cot[e + f*x])^2,x]

[Out] Integrate[(c + d*x)^m*(a + I*a*Cot[e + f*x])^2, x]

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (dx + c)^m (a + ia \cot (fx + e))^2 dx$$

[In] int((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x)

[Out] int((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x)

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 4.78

$$\int (c + dx)^m (a + ia \cot(e + fx))^2 dx = \int (ia \cot (fx + e) + a)^2 (dx + c)^m dx$$

[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")

```
[Out] (2*I*(d*x + c)^m*a^2 + (f*e^(2*I*f*x + 2*I*e) - f)*integral(2*(2*a^2*d*f*x
+ 2*a^2*c*f + I*a^2*d*m)*(d*x + c)^m/(d*f*x + c*f - (d*f*x + c*f)*e^(2*I*f*
x + 2*I*e)), x))/(f*e^(2*I*f*x + 2*I*e) - f)
```

Sympy [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int (c + dx)^m (a + ia \cot(e + fx))^2 dx = -a^2 \left(\int (c + dx)^m \cot^2(e + fx) dx \right. \\ \left. + \int (-2i(c + dx)^m \cot(e + fx)) dx \right. \\ \left. + \int -(c + dx)^m dx \right)$$

```
[In] integrate((d*x+c)**m*(a+I*a*cot(f*x+e))**2,x)
```

```
[Out] -a**2*(Integral((c + d*x)**m*cot(e + f*x)**2, x) + Integral(-2*I*(c + d*x)*
**m*cot(e + f*x), x) + Integral(-(c + d*x)**m, x))
```

Maxima [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 496, normalized size of antiderivative = 21.57

$$\int (c + dx)^m (a + ia \cot(e + fx))^2 dx = \int (i a \cot(fx + e) + a)^2 (dx + c)^m dx$$

```
[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (d*x + c)^(m + 1)*a^2/(d*(m + 1)) - integrate(-((d*x + c)^m*a^2*cos(4*f*x +
4*e)^2 + 4*(d*x + c)^m*a^2*cos(2*f*x + 2*e)^2 + (d*x + c)^m*a^2*sin(4*f*x
+ 4*e)^2 - 4*(d*x + c)^m*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(d*x + c
)^m*a^2*sin(2*f*x + 2*e)^2 + 4*(d*x + c)^m*a^2*cos(2*f*x + 2*e) - 3*(d*x +
c)^m*a^2 - 2*(2*(d*x + c)^m*a^2*cos(2*f*x + 2*e) + (d*x + c)^m*a^2)*cos(4*f
*x + 4*e))/(2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^
2 - 4*cos(2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*
x + 2*e) - 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1), x) - I*integrate
(-4*((d*x + c)^m*a^2*sin(4*f*x + 4*e) - 2*(d*x + c)^m*a^2*sin(2*f*x + 2*e))
/(2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - 4*cos(
2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) -
4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (c + dx)^m (a + ia \cot(e + fx))^2 dx = \int (ia \cot(fx + e) + a)^2 (dx + c)^m dx$$

[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e))^2,x, algorithm="giac")

[Out] integrate((I*a*cot(f*x + e) + a)^2*(d*x + c)^m, x)

Mupad [N/A]

Not integrable

Time = 12.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int (c + dx)^m (a + ia \cot(e + fx))^2 dx = \int (a + a \cot(e + fx) 1i)^2 (c + dx)^m dx$$

[In] int((a + a*cot(e + f*x)*1i)^2*(c + d*x)^m,x)

[Out] int((a + a*cot(e + f*x)*1i)^2*(c + d*x)^m, x)

3.33 $\int (c + dx)^m (a + ia \cot(e + fx)) dx$

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Maple [N/A] (verified)	233
Fricas [N/A]	233
Sympy [N/A]	233
Maxima [N/A]	234
Giac [N/A]	234
Mupad [N/A]	234

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (c + dx)^m (a + ia \cot(e + fx)) dx = \text{Int}((c + dx)^m (a + ia \cot(e + fx)), x)$$

[Out] Unintegrable((d*x+c)^m*(a+I*a*cot(f*x+e)),x)

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c + dx)^m (a + ia \cot(e + fx)) dx = \int (c + dx)^m (a + ia \cot(e + fx)) dx$$

[In] Int[(c + d*x)^m*(a + I*a*Cot[e + f*x]),x]

[Out] Defer[Int][(c + d*x)^m*(a + I*a*Cot[e + f*x]), x]

Rubi steps

$$\text{integral} = \int (c + dx)^m (a + ia \cot(e + fx)) dx$$

Mathematica [N/A]

Not integrable

Time = 6.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (c + dx)^m (a + ia \cot(e + fx)) dx = \int (c + dx)^m (a + ia \cot(e + fx)) dx$$

[In] Integrate[(c + d*x)^m*(a + I*a*Cot[e + f*x]),x]

[Out] Integrate[(c + d*x)^m*(a + I*a*Cot[e + f*x]), x]

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int (dx + c)^m (a + ia \cot(fx + e)) dx$$

[In] int((d*x+c)^m*(a+I*a*cot(f*x+e)),x)

[Out] int((d*x+c)^m*(a+I*a*cot(f*x+e)),x)

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int (c + dx)^m (a + ia \cot(e + fx)) dx = \int (ia \cot(fx + e) + a)(dx + c)^m dx$$

[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e)),x, algorithm="fricas")

[Out] integral(-2*(d*x + c)^m*a/(e^(2*I*f*x + 2*I*e) - 1), x)

Sympy [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int (c + dx)^m (a + ia \cot(e + fx)) dx = ia \left(\int (-i(c + dx)^m) dx + \int (c + dx)^m \cot(e + fx) dx \right)$$

[In] integrate((d*x+c)**m*(a+I*a*cot(f*x+e)),x)

[Out] I*a*(Integral(-I*(c + d*x)**m, x) + Integral((c + d*x)**m*cot(e + f*x), x))

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.76

$$\int (c + dx)^m (a + ia \cot(e + fx)) dx = \int (ia \cot(fx + e) + a)(dx + c)^m dx$$

[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e)),x, algorithm="maxima")

[Out] 2*I*a*integrate((d*x + c)^m*sin(2*f*x + 2*e)/(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 - 2*cos(2*f*x + 2*e) + 1), x) + (d*x + c)^(m + 1)*a/(d*(m + 1))

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (c + dx)^m (a + ia \cot(e + fx)) dx = \int (ia \cot(fx + e) + a)(dx + c)^m dx$$

[In] integrate((d*x+c)^m*(a+I*a*cot(f*x+e)),x, algorithm="giac")

[Out] integrate((I*a*cot(f*x + e) + a)*(d*x + c)^m, x)

Mupad [N/A]

Not integrable

Time = 12.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int (c + dx)^m (a + ia \cot(e + fx)) dx = \int (a + a \cot(e + fx) 1i) (c + dx)^m dx$$

[In] int((a + a*cot(e + f*x)*1i)*(c + d*x)^m,x)

[Out] int((a + a*cot(e + f*x)*1i)*(c + d*x)^m, x)

3.34 $\int \frac{(c+dx)^m}{a+ia \cot(e+fx)} dx$

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Rubi [A] (verified)	235
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Fricas [A] (verification not implemented)	237
Sympy [F]	237
Maxima [F]	238
Giac [F]	238
Mupad [F(-1)]	238

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{(c+dx)^m}{a+ia \cot(e+fx)} dx = \frac{(c+dx)^{1+m}}{2ad(1+m)} + \frac{i2^{-2-m} e^{2i(e-\frac{cf}{d})} (c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{2if(c+dx)}{d}\right)}{af}$$

[Out] 1/2*(d*x+c)^(1+m)/a/d/(1+m)+I*2^(-2-m)*exp(2*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-2*I*f*(d*x+c)/d)/a/f/((-I*f*(d*x+c)/d)^m)

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3808, 2212}

$$\int \frac{(c+dx)^m}{a+ia \cot(e+fx)} dx = \frac{(c+dx)^{m+1}}{2ad(m+1)} + \frac{i2^{-m-2} e^{2i(e-\frac{cf}{d})} (c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2if(c+dx)}{d}\right)}{af}$$

[In] Int[(c + d*x)^m/(a + I*a*Cot[e + f*x]),x]

[Out] (c + d*x)^(1 + m)/(2*a*d*(1 + m)) + (I*2^(-2 - m)*E^((2*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(a*f*(((I)*f*(c + d*x))/d)^m)

Rule 2212

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d))
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d)^FracPart[m])*Gamma[m + 1,
((-f)*g*(Log[F]/d)*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3808

```
Int[((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Sym
bol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + Dist[1/(2*a), Int[(c +
d*x)^m*E^(2*(a/b)*(e + f*x)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 + b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(c + dx)^{1+m}}{2ad(1+m)} + \frac{\int e^{2i(e+\frac{\pi}{2}+fx)}(c + dx)^m dx}{2a} \\ &= \frac{(c + dx)^{1+m}}{2ad(1+m)} + \frac{i2^{-2-m}e^{2i(e-\frac{cf}{d})}(c + dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{2if(c+dx)}{d}\right)}{af} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\begin{aligned} &\int \frac{(c + dx)^m}{a + ia \cot(e + fx)} dx \\ &= -\frac{(c + dx)^m \left(-\frac{2(c+dx)}{d(1+m)} - \frac{i2^{-m}e^{2i(e-\frac{cf}{d})} \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{2if(c+dx)}{d}\right)}{f} \right)}{4a} \end{aligned}$$

```
[In] Integrate[(c + d*x)^m/(a + I*a*Cot[e + f*x]),x]
```

```
[Out] -1/4*((c + d*x)^m*((-2*(c + d*x))/(d*(1 + m)) - (I*E^((2*I)*(e - (c*f)/d))*
Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(2^m*f*(((-I)*f*(c + d*x))/d)^m))/a
```

Maple [F]

$$\int \frac{(dx + c)^m}{a + ia \cot(fx + e)} dx$$

[In] int((d*x+c)^m/(a+I*a*cot(f*x+e)),x)

[Out] int((d*x+c)^m/(a+I*a*cot(f*x+e)),x)

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.87

$$\int \frac{(c + dx)^m}{a + ia \cot(e + fx)} dx$$

$$= \frac{(i dm + i d) e^{\left(-\frac{dm \log\left(-\frac{2if}{d}\right) - 2ide + 2icf}{d}\right)} \Gamma\left(m + 1, -\frac{2(idfx + icf)}{d}\right) + 2(df x + cf)(dx + c)^m}{4(adfm + adf)}$$

[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e)),x, algorithm="fricas")

[Out] 1/4*((I*d*m + I*d)*e^(-(d*m*log(-2*I*f/d) - 2*I*d*e + 2*I*c*f)/d)*gamma(m + 1, -2*(I*d*f*x + I*c*f)/d) + 2*(d*f*x + c*f)*(d*x + c)^m)/(a*d*f*m + a*d*f)

Sympy [F]

$$\int \frac{(c + dx)^m}{a + ia \cot(e + fx)} dx = -\frac{i \int \frac{(c+dx)^m}{\cot(e+fx)-i} dx}{a}$$

[In] integrate((d*x+c)**m/(a+I*a*cot(f*x+e)),x)

[Out] -I*Integral((c + d*x)**m/(cot(e + f*x) - I), x)/a

Maxima [F]

$$\int \frac{(c + dx)^m}{a + ia \cot(e + fx)} dx = \int \frac{(dx + c)^m}{i a \cot(fx + e) + a} dx$$

[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e)),x, algorithm="maxima")

[Out] -1/2*((d*m + d)*integrate((d*x + c)^m*cos(2*f*x + 2*e), x) + (I*d*m + I*d)*integrate((d*x + c)^m*sin(2*f*x + 2*e), x) - e^(m*log(d*x + c) + log(d*x + c)))/(a*d*m + a*d)

Giac [F]

$$\int \frac{(c + dx)^m}{a + ia \cot(e + fx)} dx = \int \frac{(dx + c)^m}{i a \cot(fx + e) + a} dx$$

[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e)),x, algorithm="giac")

[Out] integrate((d*x + c)^m/(I*a*cot(f*x + e) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx)^m}{a + ia \cot(e + fx)} dx = \int \frac{(c + dx)^m}{a + a \cot(e + fx) 1i} dx$$

[In] int((c + d*x)^m/(a + a*cot(e + f*x)*1i),x)

[Out] int((c + d*x)^m/(a + a*cot(e + f*x)*1i), x)

3.35 $\int \frac{(c+dx)^m}{(a+ia \cot(e+fx))^2} dx$

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Mathematica [A] (verified)	240
Maple [F]	241
Fricas [A] (verification not implemented)	241
Sympy [F]	241
Maxima [F]	242
Giac [F]	242
Mupad [F(-1)]	242

Optimal result

Integrand size = 23, antiderivative size = 171

$$\int \frac{(c+dx)^m}{(a+ia \cot(e+fx))^2} dx$$

$$= \frac{(c+dx)^{1+m}}{4a^2d(1+m)} + \frac{i2^{-2-m}e^{2i\left(\frac{e-cf}{d}\right)}(c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{2if(c+dx)}{d}\right)}{a^2f}$$

$$- \frac{i4^{-2-m}e^{4i\left(\frac{e-cf}{d}\right)}(c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{4if(c+dx)}{d}\right)}{a^2f}$$

[Out] $1/4*(d*x+c)^{(1+m)}/a^2/d/(1+m)+I*2^{(-2-m)}*\exp(2*I*(e-c*f/d))*(d*x+c)^m*\text{GAMMA}(1+m, -2*I*f*(d*x+c)/d)/a^2/f/((-I*f*(d*x+c)/d)^m - I*4^{(-2-m)}*\exp(4*I*(e-c*f/d))*(d*x+c)^m*\text{GAMMA}(1+m, -4*I*f*(d*x+c)/d)/a^2/f/((-I*f*(d*x+c)/d)^m$

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3810, 2212}

$$\int \frac{(c+dx)^m}{(a+ia \cot(e+fx))^2} dx$$

$$= \frac{i2^{-m-2}e^{2i\left(\frac{e-cf}{d}\right)}(c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2if(c+dx)}{d}\right)}{a^2f}$$

$$- \frac{i4^{-m-2}e^{4i\left(\frac{e-cf}{d}\right)}(c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{4if(c+dx)}{d}\right)}{a^2f} + \frac{(c+dx)^{m+1}}{4a^2d(m+1)}$$

[In] Int[(c + d*x)^m/(a + I*a*Cot[e + f*x])^2,x]

[Out] (c + d*x)^(1 + m)/(4*a^2*d*(1 + m)) + (I*2^(-2 - m)*E^((2*I)*(e - (c*f)/d)) * (c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(a^2*f*(((-I)*f*(c + d*x))/d)^m) - (I*4^(-2 - m)*E^((4*I)*(e - (c*f)/d)) * (c + d*x)^m*Gamma[1 + m, ((-4*I)*f*(c + d*x))/d])/(a^2*f*(((-I)*f*(c + d*x))/d)^m)

Rule 2212

Int[(F_)^((g_.)*(e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)))^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]])*Gamma[m + 1, ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

Rule 3810

Int[((c_.) + (d_.)*(x_))^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{(c + dx)^m}{4a^2} - \frac{e^{2ie+2ifx}(c + dx)^m}{2a^2} + \frac{e^{4ie+4ifx}(c + dx)^m}{4a^2} \right) dx \\ &= \frac{(c + dx)^{1+m}}{4a^2d(1+m)} + \frac{\int e^{4ie+4ifx}(c + dx)^m dx}{4a^2} - \frac{\int e^{2ie+2ifx}(c + dx)^m dx}{2a^2} \\ &= \frac{(c + dx)^{1+m}}{4a^2d(1+m)} + \frac{i2^{-2-m}e^{2i\left(e-\frac{cf}{d}\right)}(c + dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{2if(c+dx)}{d}\right)}{a^2f} \\ &\quad - \frac{i4^{-2-m}e^{4i\left(e-\frac{cf}{d}\right)}(c + dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{4if(c+dx)}{d}\right)}{a^2f} \end{aligned}$$

Mathematica [A] (verified)

Time = 4.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.89

$$\begin{aligned} &\int \frac{(c + dx)^m}{(a + ia \cot(e + fx))^2} dx \\ &= \frac{(c + dx)^m \left(\frac{4f(c+dx)}{d(1+m)} + i2^{2-m}e^{2i\left(e-\frac{cf}{d}\right)} \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{2if(c+dx)}{d}\right) - i4^{-m}e^{4i\left(e-\frac{cf}{d}\right)} \left(-\frac{if(c+dx)}{d}\right)^{-m} \right)}{16a^2f} \end{aligned}$$

[In] Integrate[(c + d*x)^m/(a + I*a*Cot[e + f*x])^2,x]

[Out] ((c + d*x)^m*((4*f*(c + d*x))/(d*(1 + m)) + (I*2^(2 - m)*E^((2*I)*(e - (c*f)/d))*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(((-I)*f*(c + d*x))/d)^m - (I*E^((4*I)*(e - (c*f)/d))*Gamma[1 + m, ((-4*I)*f*(c + d*x))/d])/(4^m*((-I)*f*(c + d*x))/d^m)))/(16*a^2*f)

Maple [F]

$$\int \frac{(dx + c)^m}{(a + ia \cot(fx + e))^2} dx$$

[In] int((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x)

[Out] int((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

$$\int \frac{(c + dx)^m}{(a + ia \cot(e + fx))^2} dx = \frac{4(-i dm - i d)e^{\left(-\frac{dm \log\left(-\frac{2if}{d}\right) - 2ide + 2icf}{d}\right)} \Gamma\left(m + 1, -\frac{2(idfx + icf)}{d}\right) - (-i dm - i d)e^{\left(-\frac{dm \log\left(-\frac{4if}{d}\right) - 4ide + 4icf}{d}\right)} \Gamma\left(m + 1, -\frac{2(idfx + icf)}{d}\right)}{16(a^2 dfm + a^2 df)}$$

[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x, algorithm="fricas")

[Out] -1/16*(4*(-I*d*m - I*d)*e^(-(d*m*log(-2*I*f/d) - 2*I*d*e + 2*I*c*f)/d)*gamma(m + 1, -2*(I*d*f*x + I*c*f)/d) - (-I*d*m - I*d)*e^(-(d*m*log(-4*I*f/d) - 4*I*d*e + 4*I*c*f)/d)*gamma(m + 1, -4*(I*d*f*x + I*c*f)/d) - 4*(d*f*x + c*f)*(d*x + c)^m)/(a^2*d*f*m + a^2*d*f)

Sympy [F]

$$\int \frac{(c + dx)^m}{(a + ia \cot(e + fx))^2} dx = -\frac{\int \frac{(c+dx)^m}{\cot^2(e+fx) - 2i \cot(e+fx) - 1} dx}{a^2}$$

[In] integrate((d*x+c)**m/(a+I*a*cot(f*x+e))**2,x)

[Out] -Integral((c + d*x)**m/(cot(e + f*x)**2 - 2*I*cot(e + f*x) - 1), x)/a**2

Maxima [F]

$$\int \frac{(c + dx)^m}{(a + ia \cot(e + fx))^2} dx = \int \frac{(dx + c)^m}{(ia \cot(fx + e) + a)^2} dx$$

[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x, algorithm="maxima")

[Out] 1/4*((d*m + d)*integrate((d*x + c)^m*cos(4*f*x + 4*e), x) - 2*(d*m + d)*integrate((d*x + c)^m*cos(2*f*x + 2*e), x) + (I*d*m + I*d)*integrate((d*x + c)^m*sin(4*f*x + 4*e), x) - 2*(I*d*m + I*d)*integrate((d*x + c)^m*sin(2*f*x + 2*e), x) + e^(m*log(d*x + c) + log(d*x + c)))/(a^2*d*m + a^2*d)

Giac [F]

$$\int \frac{(c + dx)^m}{(a + ia \cot(e + fx))^2} dx = \int \frac{(dx + c)^m}{(ia \cot(fx + e) + a)^2} dx$$

[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^2,x, algorithm="giac")

[Out] integrate((d*x + c)^m/(I*a*cot(f*x + e) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx)^m}{(a + ia \cot(e + fx))^2} dx = \int \frac{(c + dx)^m}{(a + a \cot(e + fx) li)^2} dx$$

[In] int((c + d*x)^m/(a + a*cot(e + f*x)*li)^2,x)

[Out] int((c + d*x)^m/(a + a*cot(e + f*x)*li)^2, x)

3.36 $\int \frac{(c+dx)^m}{(a+ia \cot(e+fx))^3} dx$

Optimal result	243
Rubi [A] (verified)	243
Mathematica [A] (verified)	245
Maple [F]	245
Fricas [A] (verification not implemented)	246
Sympy [F(-1)]	246
Maxima [F]	246
Giac [F]	247
Mupad [F(-1)]	247

Optimal result

Integrand size = 23, antiderivative size = 251

$$\int \frac{(c+dx)^m}{(a+ia \cot(e+fx))^3} dx$$

$$= \frac{(c+dx)^{1+m}}{8a^3d(1+m)} + \frac{3i2^{-4-m}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{2if(c+dx)}{d}\right)}{a^3f}$$

$$- \frac{3i2^{-5-2m}e^{4i\left(e-\frac{cf}{d}\right)}(c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{4if(c+dx)}{d}\right)}{a^3f}$$

$$+ \frac{i2^{-4-m}3^{-1-m}e^{6i\left(e-\frac{cf}{d}\right)}(c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{6if(c+dx)}{d}\right)}{a^3f}$$

```
[Out] 1/8*(d*x+c)^(1+m)/a^3/d/(1+m)+3*I*2^(-4-m)*exp(2*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-2*I*f*(d*x+c)/d)/a^3/f/((-I*f*(d*x+c)/d)^m)-3*I*2^(-5-2*m)*exp(4*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-4*I*f*(d*x+c)/d)/a^3/f/((-I*f*(d*x+c)/d)^m)+I*2^(-4-m)*3^(-1-m)*exp(6*I*(e-c*f/d))*(d*x+c)^m*GAMMA(1+m,-6*I*f*(d*x+c)/d)/a^3/f/((-I*f*(d*x+c)/d)^m)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used

= {3810, 2212}

$$\int \frac{(c+dx)^m}{(a+ia \cot(e+fx))^3} dx$$

$$= \frac{3i2^{-m-4} e^{2i(e-\frac{cf}{d})} (c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2if(c+dx)}{d}\right)}{a^3 f}$$

$$- \frac{3i2^{-2m-5} e^{4i(e-\frac{cf}{d})} (c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{4if(c+dx)}{d}\right)}{a^3 f}$$

$$+ \frac{i2^{-m-4} 3^{-m-1} e^{6i(e-\frac{cf}{d})} (c+dx)^m \left(-\frac{if(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{6if(c+dx)}{d}\right)}{a^3 f} + \frac{(c+dx)^{m+1}}{8a^3 d(m+1)}$$

[In] Int[(c + d*x)^m/(a + I*a*Cot[e + f*x])^3,x]

[Out] (c + d*x)^(1 + m)/(8*a^3*d*(1 + m)) + ((3*I)*2^(-4 - m)*E^((2*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d])/(a^3*f*(((I)*f*(c + d*x))/d)^m) - ((3*I)*2^(-5 - 2*m)*E^((4*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*f*(c + d*x))/d])/(a^3*f*(((I)*f*(c + d*x))/d)^m) + (I*2^(-4 - m)*3^(-1 - m)*E^((6*I)*(e - (c*f)/d))*(c + d*x)^m*Gamma[1 + m, ((-6*I)*f*(c + d*x))/d])/(a^3*f*(((I)*f*(c + d*x))/d)^m)

Rule 2212

Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol] :> Simp[(-F^(g*(e - c*(f/d))))*(c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d))^(IntPart[m] + 1))*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]]*Gamma[m + 1, ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

Rule 3810

Int[((c_) + (d_)*(x_))^(m_)*((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, 0]

Rubi steps

$$\text{integral} = \int \left(\frac{(c+dx)^m}{8a^3} - \frac{3e^{2ie+2ifx}(c+dx)^m}{8a^3} + \frac{3e^{4ie+4ifx}(c+dx)^m}{8a^3} - \frac{e^{6ie+6ifx}(c+dx)^m}{8a^3} \right) dx$$

$$= \frac{(c+dx)^{1+m}}{8a^3 d(1+m)} - \frac{\int e^{6ie+6ifx}(c+dx)^m dx}{8a^3}$$

$$- \frac{3 \int e^{2ie+2ifx}(c+dx)^m dx}{8a^3} + \frac{3 \int e^{4ie+4ifx}(c+dx)^m dx}{8a^3}$$

$$\begin{aligned}
&= \frac{(c+dx)^{1+m}}{8a^3d(1+m)} + \frac{3i2^{-4-m}e^{2i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2if(c+dx)}{d}\right)}{a^3f} \\
&\quad - \frac{3i2^{-5-2m}e^{4i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{4if(c+dx)}{d}\right)}{a^3f} \\
&\quad + \frac{i2^{-4-m}3^{-1-m}e^{6i\left(e-\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{6if(c+dx)}{d}\right)}{a^3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 6.05 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^m}{(a+ia\cot(e+fx))^3} dx$$

$$2^{-5-2m}3^{-1-m}e^{-\frac{6icf}{d}}(c+dx)^m\left(-\frac{if(c+dx)}{d}\right)^{-m}\left(12^{1+m}e^{\frac{6icf}{d}}f(c+dx)\left(-\frac{if(c+dx)}{d}\right)^m+i2^{1+m}3^{2+m}de^{2ie+\frac{4icf}{d}}\right)$$

[In] Integrate[(c + d*x)^m/(a + I*a*Cot[e + f*x])^3,x]

[Out] (2^(-5 - 2*m)*3^(-1 - m)*(c + d*x)^m*(12^(1 + m)*E^(((6*I)*c*f)/d)*f*(c + d*x)*(((- I)*f*(c + d*x))/d)^m + I*2^(1 + m)*3^(2 + m)*d*E^((2*I)*e + ((4*I)*c*f)/d)*(1 + m)*Gamma[1 + m, ((-2*I)*f*(c + d*x))/d] - I*3^(2 + m)*d*E^((2*I)*(2*e + (c*f)/d))*(1 + m)*Gamma[1 + m, ((-4*I)*f*(c + d*x))/d] + I*2^(1 + m)*d*E^((6*I)*e)*(1 + m)*Gamma[1 + m, ((-6*I)*f*(c + d*x))/d])/ (a^3*d*E^(((6*I)*c*f)/d)*f*(1 + m)*(((- I)*f*(c + d*x))/d)^m)

Maple [F]

$$\int \frac{(dx+c)^m}{(a+ia\cot(fx+e))^3} dx$$

[In] int((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x)

[Out] int((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x)

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.79

$$\int \frac{(c + dx)^m}{(a + ia \cot(e + fx))^3} dx = \frac{18(-i dm - i d)e^{\left(-\frac{dm \log\left(-\frac{2i f}{d}\right) - 2i de + 2i cf}{d}\right)} \Gamma\left(m + 1, -\frac{2(i dfx + i cf)}{d}\right) + 9(i dm + i d)e^{\left(-\frac{dm \log\left(-\frac{4i f}{d}\right) - 4i de + 4i cf}{d}\right)}}{96}$$

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/96*(18*(-I*d*m - I*d)*e^(-(d*m*log(-2*I*f/d) - 2*I*d*e + 2*I*c*f)/d)*gamma(m + 1, -2*(I*d*f*x + I*c*f)/d) + 9*(I*d*m + I*d)*e^(-(d*m*log(-4*I*f/d) - 4*I*d*e + 4*I*c*f)/d)*gamma(m + 1, -4*(I*d*f*x + I*c*f)/d) + 2*(-I*d*m - I*d)*e^(-(d*m*log(-6*I*f/d) - 6*I*d*e + 6*I*c*f)/d)*gamma(m + 1, -6*(I*d*f*x + I*c*f)/d) - 12*(d*f*x + c*f)*(d*x + c)^m/(a^3*d*f*m + a^3*d*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c + dx)^m}{(a + ia \cot(e + fx))^3} dx = \text{Timed out}$$

```
[In] integrate((d*x+c)**m/(a+I*a*cot(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c + dx)^m}{(a + ia \cot(e + fx))^3} dx = \int \frac{(dx + c)^m}{(ia \cot(fx + e) + a)^3} dx$$

```
[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((d*m + d)*integrate((d*x + c)^m*cos(6*f*x + 6*e), x) - 3*(d*m + d)*integrate((d*x + c)^m*cos(4*f*x + 4*e), x) + 3*(d*m + d)*integrate((d*x + c)^m*cos(2*f*x + 2*e), x) + (I*d*m + I*d)*integrate((d*x + c)^m*sin(6*f*x + 6*e), x) - 3*(I*d*m + I*d)*integrate((d*x + c)^m*sin(4*f*x + 4*e), x) - 3*(-I*d*m - I*d)*integrate((d*x + c)^m*sin(2*f*x + 2*e), x) - e^(m*log(d*x + c) + log(d*x + c))/(a^3*d*m + a^3*d)
```

Giac [F]

$$\int \frac{(c + dx)^m}{(a + ia \cot(e + fx))^3} dx = \int \frac{(dx + c)^m}{(ia \cot(fx + e) + a)^3} dx$$

[In] integrate((d*x+c)^m/(a+I*a*cot(f*x+e))^3,x, algorithm="giac")

[Out] integrate((d*x + c)^m/(I*a*cot(f*x + e) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx)^m}{(a + ia \cot(e + fx))^3} dx = \int \frac{(c + dx)^m}{(a + a \cot(e + fx) 1i)^3} dx$$

[In] int((c + d*x)^m/(a + a*cot(e + f*x)*1i)^3,x)

[Out] int((c + d*x)^m/(a + a*cot(e + f*x)*1i)^3, x)

3.37 $\int (c + dx)^3 (a + b \cot(e + fx)) dx$

Optimal result	248
Rubi [A] (verified)	248
Mathematica [B] (verified)	251
Maple [B] (verified)	252
Fricas [B] (verification not implemented)	252
Sympy [F]	253
Maxima [B] (verification not implemented)	253
Giac [F]	254
Mupad [F(-1)]	254

Optimal result

Integrand size = 18, antiderivative size = 147

$$\int (c + dx)^3 (a + b \cot(e + fx)) dx = \frac{a(c + dx)^4}{4d} - \frac{ib(c + dx)^4}{4d} + \frac{b(c + dx)^3 \log(1 - e^{2i(e+fx)})}{f} - \frac{3ibd(c + dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} + \frac{3bd^2(c + dx) \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} + \frac{3ibd^3 \text{PolyLog}(4, e^{2i(e+fx)})}{4f^4}$$

[Out] $\frac{1}{4}a*(d*x+c)^4/d - \frac{1}{4}I*b*(d*x+c)^4/d + b*(d*x+c)^3*\ln(1-\exp(2*I*(f*x+e)))/f - \frac{3}{2}I*b*d*(d*x+c)^2*\text{polylog}(2, \exp(2*I*(f*x+e)))/f^2 + \frac{3}{2}*b*d^2*(d*x+c)*\text{polylog}(3, \exp(2*I*(f*x+e)))/f^3 + \frac{3}{4}I*b*d^3*\text{polylog}(4, \exp(2*I*(f*x+e)))/f^4$

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3803, 3798, 2221, 2611, 6744, 2320, 6724}

$$\int (c + dx)^3 (a + b \cot(e + fx)) dx = \frac{a(c + dx)^4}{4d} + \frac{3bd^2(c + dx) \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} - \frac{3ibd(c + dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} + \frac{b(c + dx)^3 \log(1 - e^{2i(e+fx)})}{f} - \frac{ib(c + dx)^4}{4d} + \frac{3ibd^3 \text{PolyLog}(4, e^{2i(e+fx)})}{4f^4}$$

[In] Int[(c + d*x)^3*(a + b*Cot[e + f*x]),x]

[Out] (a*(c + d*x)^4)/(4*d) - ((I/4)*b*(c + d*x)^4)/d + (b*(c + d*x)^3*Log[1 - E^((2*I)*(e + f*x))])/f - (((3*I)/2)*b*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + (3*b*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(e + f*x))])/(2*f^3) + (((3*I)/4)*b*d^3*PolyLog[4, E^((2*I)*(e + f*x))])/f^4

Rule 2221

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2611

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*(f_) + (g_)*(x_)^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 3798

Int[(((c_) + (d_)*(x_))^(m_))*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 3803

Int[(((c_) + (d_)*(x_))^(m_))*((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 6724

Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p/(e*p), x] /; FreeQ[{a, b, c, d

, e, n, p}, x] && EqQ[b*d, a*e]

Rule 6744

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_.)))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^(
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a(c + dx)^3 + b(c + dx)^3 \cot(e + fx)) dx \\
 &= \frac{a(c + dx)^4}{4d} + b \int (c + dx)^3 \cot(e + fx) dx \\
 &= \frac{a(c + dx)^4}{4d} - \frac{ib(c + dx)^4}{4d} - (2ib) \int \frac{e^{2i(e+fx)}(c + dx)^3}{1 - e^{2i(e+fx)}} dx \\
 &= \frac{a(c + dx)^4}{4d} - \frac{ib(c + dx)^4}{4d} + \frac{b(c + dx)^3 \log(1 - e^{2i(e+fx)})}{f} \\
 &\quad - \frac{(3bd) \int (c + dx)^2 \log(1 - e^{2i(e+fx)}) dx}{f} \\
 &= \frac{a(c + dx)^4}{4d} - \frac{ib(c + dx)^4}{4d} + \frac{b(c + dx)^3 \log(1 - e^{2i(e+fx)})}{f} \\
 &\quad - \frac{3ibd(c + dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} + \frac{(3ibd^2) \int (c + dx) \text{PolyLog}(2, e^{2i(e+fx)}) dx}{f^2} \\
 &= \frac{a(c + dx)^4}{4d} - \frac{ib(c + dx)^4}{4d} + \frac{b(c + dx)^3 \log(1 - e^{2i(e+fx)})}{f} \\
 &\quad - \frac{3ibd(c + dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} \\
 &\quad + \frac{3bd^2(c + dx) \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} - \frac{(3bd^3) \int \text{PolyLog}(3, e^{2i(e+fx)}) dx}{2f^3} \\
 &= \frac{a(c + dx)^4}{4d} - \frac{ib(c + dx)^4}{4d} + \frac{b(c + dx)^3 \log(1 - e^{2i(e+fx)})}{f} \\
 &\quad - \frac{3ibd(c + dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} + \frac{3bd^2(c + dx) \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} \\
 &\quad + \frac{(3ibd^3) \text{Subst}\left(\int \frac{\text{PolyLog}(3, x)}{x} dx, x, e^{2i(e+fx)}\right)}{4f^4}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{a(c+dx)^4}{4d} - \frac{ib(c+dx)^4}{4d} + \frac{b(c+dx)^3 \log(1 - e^{2i(e+fx)})}{f} \\
&\quad - \frac{3ibd(c+dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} \\
&\quad + \frac{3bd^2(c+dx) \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} + \frac{3ibd^3 \text{PolyLog}(4, e^{2i(e+fx)})}{4f^4}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 632 vs. $2(147) = 294$.

Time = 5.20 (sec) , antiderivative size = 632, normalized size of antiderivative = 4.30

$$\int (c+dx)^3(a+b \cot(e+fx)) dx$$

$$= \frac{4ac^3 f^4 x + 6ibc^2 d f^3 \pi x + 6ac^2 d f^4 x^2 + 4acd^2 f^4 x^3 + 4ibcd^2 f^4 x^3 + ad^3 f^4 x^4 + ibd^3 f^4 x^4 - 12ibc^2 d f^3 x \arctan}{}$$

[In] Integrate[(c + d*x)^3*(a + b*Cot[e + f*x]),x]

[Out] (4*a*c^3*f^4*x + (6*I)*b*c^2*d*f^3*Pi*x + 6*a*c^2*d*f^4*x^2 + 4*a*c*d^2*f^4*x^3 + (4*I)*b*c*d^2*f^4*x^3 + a*d^3*f^4*x^4 + I*b*d^3*f^4*x^4 - (12*I)*b*c^2*d*f^3*x*ArcTan[Tan[e]] + 6*b*c^2*d*f^4*x^2*Cot[e] + 6*b*c^2*d*f^2*Pi*Log[1 + E^((-2*I)*f*x)] + 12*b*c*d^2*f^3*x^2*Log[1 - E^((-I)*(e + f*x))] + 4*b*d^3*f^3*x^3*Log[1 - E^((-I)*(e + f*x))] + 12*b*c*d^2*f^3*x^2*Log[1 + E^((-I)*(e + f*x))] + 4*b*d^3*f^3*x^3*Log[1 + E^((-I)*(e + f*x))] + 12*b*c^2*d*f^3*x*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]])] + 12*b*c^2*d*f^2*ArcTan[Tan[e]]*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]])] - 6*b*c^2*d*f^2*Pi*Log[Cos[f*x]] + 4*b*c^3*f^3*Log[Sin[e + f*x]] - 12*b*c^2*d*f^2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]] + (12*I)*b*d^2*f^2*x*(2*c + d*x)*PolyLog[2, -E^((-I)*(e + f*x))] + (12*I)*b*d^2*f^2*x*(2*c + d*x)*PolyLog[2, E^((-I)*(e + f*x))] - (6*I)*b*c^2*d*f^2*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]])] + 24*b*c*d^2*f*PolyLog[3, -E^((-I)*(e + f*x))] + 24*b*d^3*f*x*PolyLog[3, -E^((-I)*(e + f*x))] + 24*b*c*d^2*f*PolyLog[3, E^((-I)*(e + f*x))] + 24*b*d^3*f*x*PolyLog[3, E^((-I)*(e + f*x))] - (24*I)*b*d^3*PolyLog[4, -E^((-I)*(e + f*x))] - (24*I)*b*d^3*PolyLog[4, E^((-I)*(e + f*x))] - 6*b*c^2*d*E^(I*ArcTan[Tan[e]])*f^4*x^2*Cot[e]*Sqrt[Sec[e]^2])/(4*f^4)

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(126) = 252$.

Time = 0.51 (sec) , antiderivative size = 876, normalized size of antiderivative = 5.96

method	result	size
risch	Expression too large to display	876

```
[In] int((d*x+c)^3*(a+b*cot(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/f^2*b*d*c^2*ln(1-exp(I*(f*x+e)))*e-6/f^3*b*e^2*c*d^2*ln(exp(I*(f*x+e)))+3/f^3*b*e^2*c*d^2*ln(exp(I*(f*x+e))-1)+6/f^2*b*e*c^2*d*ln(exp(I*(f*x+e)))-3/f^2*b*e*c^2*d*ln(exp(I*(f*x+e))-1)+3/f*b*c*d^2*ln(1-exp(I*(f*x+e)))*x^2+3/f*b*c*d^2*ln(exp(I*(f*x+e))+1)*x^2+3/f*b*d*c^2*ln(1-exp(I*(f*x+e)))*x+3/f*b*d*c^2*ln(exp(I*(f*x+e))+1)*x-3*I/f^2*b*d^3*polylog(2,-exp(I*(f*x+e)))*x^2-3*I/f^2*b*d^3*polylog(2,exp(I*(f*x+e)))*x^2-3*I/f^2*b*d*c^2*polylog(2,-exp(I*(f*x+e)))-2*I/f^3*b*d^3*e^3*x+4*I/f^3*b*c*d^2*e^3-3*I/f^2*b*d*c^2*e^2-3*I/f^2*b*d*c^2*polylog(2,exp(I*(f*x+e)))+6/f^3*b*d^3*polylog(3,-exp(I*(f*x+e)))*x+6/f^3*b*c*d^2*polylog(3,exp(I*(f*x+e)))+6/f^3*b*c*d^2*polylog(3,-exp(I*(f*x+e)))+2/f^4*b*e^3*d^3*ln(exp(I*(f*x+e)))-1/f^4*b*e^3*d^3*ln(exp(I*(f*x+e))-1)+6*I/f^4*b*d^3*polylog(4,exp(I*(f*x+e)))+6*I/f^4*b*d^3*polylog(4,-exp(I*(f*x+e)))-3/2*I/f^4*b*d^3*e^4-I*d^2*b*c*x^3-3/2*I*d*b*c^2*x^2-6*I/f*b*d*c^2*e*x-6*I/f^2*b*c*d^2*polylog(2,exp(I*(f*x+e)))*x-6*I/f^2*b*c*d^2*polylog(2,-exp(I*(f*x+e)))*x+6*I/f^2*b*c*d^2*e^2*x+1/f*b*d^3*ln(exp(I*(f*x+e))+1)*x^3+1/f*b*d^3*ln(1-exp(I*(f*x+e)))*x^3+1/f^4*b*d^3*ln(1-exp(I*(f*x+e)))*e^3+6/f^3*b*d^3*polylog(3,exp(I*(f*x+e)))*x+1/4*d^3*a*x^4+1/4/d*a*c^4+1/f*b*c^3*ln(exp(I*(f*x+e))+1)-2/f*b*c^3*ln(exp(I*(f*x+e)))+1/f*b*c^3*ln(exp(I*(f*x+e))-1)-1/4*I*d^3*b*x^4+d^2*a*c*x^3+3/2*d*a*c^2*x^2+a*c^3*x+I*b*c^3*x+1/4*I/d*b*c^4-3/f^3*b*c*d^2*ln(1-exp(I*(f*x+e)))*e^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 628 vs. $2(122) = 244$.

Time = 0.29 (sec) , antiderivative size = 628, normalized size of antiderivative = 4.27

$$\int (c + dx)^3 (a + b \cot(e + fx)) dx$$

$$= \frac{2ad^3f^4x^4 + 8acd^2f^4x^3 + 12ac^2df^4x^2 + 8ac^3f^4x + 3ibd^3\text{polylog}(4, \cos(2fx + 2e) + i \sin(2fx + 2e)) -$$

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*d^3*f^4*x^4 + 8*a*c*d^2*f^4*x^3 + 12*a*c^2*d*f^4*x^2 + 8*a*c^3*f^4*x + 3*I*b*d^3*polylog(4, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) - 3*I*b*d^3*polylog(4, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) - 6*(I*b*d^3*f^2*x^2 + 2*I*b*c*d^2*f^2*x + I*b*c^2*d*f^2)*dilog(cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) - 6*(-I*b*d^3*f^2*x^2 - 2*I*b*c*d^2*f^2*x - I*b*c^2*d*f^2)*dilog(cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) - 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(-1/2*cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2) - 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1) + 6*(b*d^3*f*x + b*c*d^2*f)*polylog(3, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) + 6*(b*d^3*f*x + b*c*d^2*f)*polylog(3, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)))/f^4
```

Sympy [F]

$$\int (c + dx)^3 (a + b \cot(e + fx)) dx = \int (a + b \cot(e + fx)) (c + dx)^3 dx$$

```
[In] integrate((d*x+c)**3*(a+b*cot(f*x+e)),x)
```

```
[Out] Integral((a + b*cot(e + f*x))*(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 978 vs. $2(122) = 244$.

Time = 0.49 (sec) , antiderivative size = 978, normalized size of antiderivative = 6.65

$$\int (c + dx)^3 (a + b \cot(e + fx)) dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a*c^3 + (f*x + e)^4*a*d^3/f^3 - 4*(f*x + e)^3*a*d^3*e/f^3 + 6*(f*x + e)^2*a*d^3*e^2/f^3 - 4*(f*x + e)*a*d^3*e^3/f^3 + 4*(f*x + e)^3*a*c*d^2/f^2 - 12*(f*x + e)^2*a*c*d^2*e/f^2 + 12*(f*x + e)*a*c*d^2*e^2/f^2 + 6*(f*x + e)^2*a*c^2*d/f - 12*(f*x + e)*a*c^2*d*e/f + 4*b*c^3*log(sin(f*x + e)) - 4*b*d^3*e^3*log(sin(f*x + e))/f^3 + 12*b*c*d^2*e^2*log(sin(f*x + e))/f^2 - 12*b*c^2*d*e*log(sin(f*x + e))/f + (-I*(f*x + e)^4*b*d^3 + 24*I*b*d^3*polylog(4, -e^(I*f*x + I*e)) + 24*I*b*d^3*polylog(4, e^(I*f*x + I*e)) - 4*
```

```
(-I*b*d^3*e + I*b*c*d^2*f)*(f*x + e)^3 - 6*(I*b*d^3*e^2 - 2*I*b*c*d^2*e*f +
I*b*c^2*d*f^2)*(f*x + e)^2 - 4*(-I*(f*x + e)^3*b*d^3 + 3*(I*b*d^3*e - I*b*
c*d^2*f)*(f*x + e)^2 + 3*(-I*b*d^3*e^2 + 2*I*b*c*d^2*e*f - I*b*c^2*d*f^2)*(
f*x + e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) - 4*(I*(f*x + e)^3*b*d^3
+ 3*(-I*b*d^3*e + I*b*c*d^2*f)*(f*x + e)^2 + 3*(I*b*d^3*e^2 - 2*I*b*c*d^2*
e*f + I*b*c^2*d*f^2)*(f*x + e))*arctan2(sin(f*x + e), -cos(f*x + e) + 1) - 1
2*(I*(f*x + e)^2*b*d^3 + I*b*d^3*e^2 - 2*I*b*c*d^2*e*f + I*b*c^2*d*f^2 + 2*
(-I*b*d^3*e + I*b*c*d^2*f)*(f*x + e))*dilog(-e^(I*f*x + I*e)) - 12*(I*(f*x
+ e)^2*b*d^3 + I*b*d^3*e^2 - 2*I*b*c*d^2*e*f + I*b*c^2*d*f^2 + 2*(-I*b*d^3*
e + I*b*c*d^2*f)*(f*x + e))*dilog(e^(I*f*x + I*e)) + 2*((f*x + e)^3*b*d^3 -
3*(b*d^3*e - b*c*d^2*f)*(f*x + e)^2 + 3*(b*d^3*e^2 - 2*b*c*d^2*e*f + b*c^2
*d*f^2)*(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1
) + 2*((f*x + e)^3*b*d^3 - 3*(b*d^3*e - b*c*d^2*f)*(f*x + e)^2 + 3*(b*d^3*
e^2 - 2*b*c*d^2*e*f + b*c^2*d*f^2)*(f*x + e))*log(cos(f*x + e)^2 + sin(f*x +
e)^2 - 2*cos(f*x + e) + 1) + 24*((f*x + e)*b*d^3 - b*d^3*e + b*c*d^2*f)*pol
ylog(3, -e^(I*f*x + I*e)) + 24*((f*x + e)*b*d^3 - b*d^3*e + b*c*d^2*f)*pol
ylog(3, e^(I*f*x + I*e)))/f^3)/f
```

Giac [F]

$$\int (c + dx)^3 (a + b \cot(e + fx)) dx = \int (dx + c)^3 (b \cot(fx + e) + a) dx$$

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*(b*cot(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c + dx)^3 (a + b \cot(e + fx)) dx = \int (a + b \cot(e + fx)) (c + dx)^3 dx$$

```
[In] int((a + b*cot(e + f*x))*(c + d*x)^3,x)
```

```
[Out] int((a + b*cot(e + f*x))*(c + d*x)^3, x)
```

3.38 $\int (c + dx)^2 (a + b \cot(e + fx)) dx$

Optimal result	255
Rubi [A] (verified)	255
Mathematica [B] (verified)	257
Maple [B] (verified)	258
Fricas [B] (verification not implemented)	259
Sympy [F]	259
Maxima [B] (verification not implemented)	259
Giac [F]	260
Mupad [F(-1)]	260

Optimal result

Integrand size = 18, antiderivative size = 112

$$\int (c + dx)^2 (a + b \cot(e + fx)) dx = \frac{a(c + dx)^3}{3d} - \frac{ib(c + dx)^3}{3d} + \frac{b(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f} - \frac{ibd(c + dx) \operatorname{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{bd^2 \operatorname{PolyLog}(3, e^{2i(e+fx)})}{2f^3}$$

[Out] $1/3*a*(d*x+c)^3/d-1/3*I*b*(d*x+c)^3/d+b*(d*x+c)^2*\ln(1-\exp(2*I*(f*x+e)))/f-I*b*d*(d*x+c)*\operatorname{polylog}(2,\exp(2*I*(f*x+e)))/f^2+1/2*b*d^2*\operatorname{polylog}(3,\exp(2*I*(f*x+e)))/f^3$

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3803, 3798, 2221, 2611, 2320, 6724}

$$\int (c + dx)^2 (a + b \cot(e + fx)) dx = \frac{a(c + dx)^3}{3d} - \frac{ibd(c + dx) \operatorname{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{b(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f} - \frac{ib(c + dx)^3}{3d} + \frac{bd^2 \operatorname{PolyLog}(3, e^{2i(e+fx)})}{2f^3}$$

[In] $\operatorname{Int}[(c + d*x)^2*(a + b*\operatorname{Cot}[e + f*x]),x]$

[Out] $(a*(c + d*x)^3)/(3*d) - ((I/3)*b*(c + d*x)^3)/d + (b*(c + d*x)^2*\text{Log}[1 - E^{((2*I)*(e + f*x))}])/f - (I*b*d*(c + d*x)*\text{PolyLog}[2, E^{((2*I)*(e + f*x))}])/f^2 + (b*d^2*\text{PolyLog}[3, E^{((2*I)*(e + f*x))}])/(2*f^3)$

Rule 2221

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2611

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 3798

Int[(((c_) + (d_)*(x_))^(m_))*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 3803

Int[(((c_) + (d_)*(x_))^(m_))*((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 6724

Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (a(c + dx)^2 + b(c + dx)^2 \cot(e + fx)) dx \\
&= \frac{a(c + dx)^3}{3d} + b \int (c + dx)^2 \cot(e + fx) dx \\
&= \frac{a(c + dx)^3}{3d} - \frac{ib(c + dx)^3}{3d} - (2ib) \int \frac{e^{2i(e+fx)}(c + dx)^2}{1 - e^{2i(e+fx)}} dx \\
&= \frac{a(c + dx)^3}{3d} - \frac{ib(c + dx)^3}{3d} + \frac{b(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f} \\
&\quad - \frac{(2bd) \int (c + dx) \log(1 - e^{2i(e+fx)}) dx}{f} \\
&= \frac{a(c + dx)^3}{3d} - \frac{ib(c + dx)^3}{3d} + \frac{b(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f} \\
&\quad - \frac{ibd(c + dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{(ibd^2) \int \text{PolyLog}(2, e^{2i(e+fx)}) dx}{f^2} \\
&= \frac{a(c + dx)^3}{3d} - \frac{ib(c + dx)^3}{3d} + \frac{b(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f} \\
&\quad - \frac{ibd(c + dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{(bd^2) \text{Subst}\left(\int \frac{\text{PolyLog}(2, x)}{x} dx, x, e^{2i(e+fx)}\right)}{2f^3} \\
&= \frac{a(c + dx)^3}{3d} - \frac{ib(c + dx)^3}{3d} + \frac{b(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f} \\
&\quad - \frac{ibd(c + dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{bd^2 \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 406 vs. $2(112) = 224$.

Time = 2.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.62

$$\begin{aligned}
&\int (c + dx)^2(a + b \cot(e + fx)) dx \\
&= \frac{3ac^2 f^3 x + 3ibcdf^2 \pi x + 3acdf^3 x^2 + ad^2 f^3 x^3 + ibd^2 f^3 x^3 - 6ibcdf^2 x \arctan(\tan(e)) + 3bcdf^3 x^2 \cot(e) + 3b}
\end{aligned}$$

[In] Integrate[(c + d*x)^2*(a + b*Cot[e + f*x]),x]

```
[Out] (3*a*c^2*f^3*x + (3*I)*b*c*d*f^2*Pi*x + 3*a*c*d*f^3*x^2 + a*d^2*f^3*x^3 + I
*b*d^2*f^3*x^3 - (6*I)*b*c*d*f^2*x*ArcTan[Tan[e]] + 3*b*c*d*f^3*x^2*Cot[e]
+ 3*b*c*d*f*Pi*Log[1 + E^((-2*I)*f*x)] + 3*b*d^2*f^2*x^2*Log[1 - E^((-I)*(e
+ f*x))] + 3*b*d^2*f^2*x^2*Log[1 + E^((-I)*(e + f*x))] + 6*b*c*d*f^2*x*Log
[1 - E^((2*I)*(f*x + ArcTan[Tan[e]])] + 6*b*c*d*f*ArcTan[Tan[e]]*Log[1 - E
^((2*I)*(f*x + ArcTan[Tan[e]])] - 3*b*c*d*f*Pi*Log[Cos[f*x]] + 3*b*c^2*f^2
*Log[Sin[e + f*x]] - 6*b*c*d*f*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]
] + (6*I)*b*d^2*f*x*PolyLog[2, -E^((-I)*(e + f*x))] + (6*I)*b*d^2*f*x*PolyL
og[2, E^((-I)*(e + f*x))] - (3*I)*b*c*d*f*PolyLog[2, E^((2*I)*(f*x + ArcTan
[Tan[e]])] + 6*b*d^2*PolyLog[3, -E^((-I)*(e + f*x))] + 6*b*d^2*PolyLog[3,
E^((-I)*(e + f*x))] - 3*b*c*d*E^(I*ArcTan[Tan[e]])*f^3*x^2*Cot[e]*Sqrt[Sec[
e]^2)]/(3*f^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(98) = 196$.

Time = 0.41 (sec) , antiderivative size = 535, normalized size of antiderivative = 4.78

method	result
risch	$\frac{2bd^2 \operatorname{polylog}(3, e^{i(fx+e)})}{f^3} + \frac{2bd^2 \operatorname{polylog}(3, -e^{i(fx+e)})}{f^3} + \frac{bc^2 \ln(e^{i(fx+e)}+1)}{f} - \frac{2bc^2 \ln(e^{i(fx+e)})}{f} + \frac{bc^2 \ln(e^{i(fx+e)}-1)}{f} - i$

```
[In] int((d*x+c)^2*(a+b*cot(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*b*d*c*ln(1-exp(I*(f*x+e)))*x+2/f*b*d*c*ln(exp(I*(f*x+e))+1)*x+4/f^2*b*e
*c*d*ln(exp(I*(f*x+e))-2/f^2*b*e*c*d*ln(exp(I*(f*x+e))-1)+2/f^2*b*d*c*ln(1
-exp(I*(f*x+e)))*e-2*I/f^2*b*d*c*e^2-2*I/f^2*b*d*c*polylog(2,exp(I*(f*x+e)
))-2*I/f^2*b*d*c*polylog(2,-exp(I*(f*x+e)))+2*I/f^2*b*d^2*e^2*x-2*I/f^2*b*d^
2*polylog(2,exp(I*(f*x+e)))*x-2*I/f^2*b*d^2*polylog(2,-exp(I*(f*x+e)))*x+2/
f^3*b*d^2*polylog(3,exp(I*(f*x+e)))+2/f^3*b*d^2*polylog(3,-exp(I*(f*x+e)))+
1/f*b*c^2*ln(exp(I*(f*x+e))+1)-2/f*b*c^2*ln(exp(I*(f*x+e)))+1/f*b*c^2*ln(ex
p(I*(f*x+e))-1)-1/3*I*d^2*b*x^3+1/3*d^2*a*x^3+1/3/d*a*c^3-I*d*b*c*x^2+d*a*c
*x^2+a*c^2*x+1/f*b*d^2*ln(1-exp(I*(f*x+e)))*x^2+1/f*b*d^2*ln(exp(I*(f*x+e)
+1))*x^2-2/f^3*b*e^2*d^2*ln(exp(I*(f*x+e)))+1/f^3*b*e^2*d^2*ln(exp(I*(f*x+e)
)-1)-1/f^3*b*d^2*ln(1-exp(I*(f*x+e)))*e^2+4/3*I/f^3*b*d^2*e^3+I*b*c^2*x+1/3
*I/d*b*c^3-4*I/f*b*d*c*e*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(95) = 190.

Time = 0.29 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.62

$$\int (c + dx)^2 (a + b \cot(e + fx)) dx$$

$$= \frac{4ad^2 f^3 x^3 + 12acdf^3 x^2 + 12ac^2 f^3 x + 3bd^2 \text{polylog}(3, \cos(2fx + 2e) + i \sin(2fx + 2e)) + 3bd^2 \text{polylog}(3, \cos(2fx + 2e) - i \sin(2fx + 2e)) - 6(I*b*d^2*f*x + I*b*c*d*f)*\text{dilog}(\cos(2*f*x + 2*e) + I*\sin(2*f*x + 2*e)) - 6*(-I*b*d^2*f*x - I*b*c*d*f)*\text{dilog}(\cos(2*f*x + 2*e) - I*\sin(2*f*x + 2*e)) + 6*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\log(-1/2*\cos(2*f*x + 2*e) + 1/2*I*\sin(2*f*x + 2*e) + 1/2) + 6*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\log(-1/2*\cos(2*f*x + 2*e) - 1/2*I*\sin(2*f*x + 2*e) + 1/2) + 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*\log(-\cos(2*f*x + 2*e) + I*\sin(2*f*x + 2*e) + 1) + 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*\log(-\cos(2*f*x + 2*e) - I*\sin(2*f*x + 2*e) + 1)}/f^3$$

[In] integrate((d*x+c)^2*(a+b*cot(f*x+e)),x, algorithm="fricas")

[Out] 1/12*(4*a*d^2*f^3*x^3 + 12*a*c*d*f^3*x^2 + 12*a*c^2*f^3*x + 3*b*d^2*polylog(3, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) + 3*b*d^2*polylog(3, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) - 6*(I*b*d^2*f*x + I*b*c*d*f)*dilog(cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) - 6*(-I*b*d^2*f*x - I*b*c*d*f)*dilog(cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) + 6*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(-1/2*cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2) + 6*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2) + 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1) + 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1))/f^3

Sympy [F]

$$\int (c + dx)^2 (a + b \cot(e + fx)) dx = \int (a + b \cot(e + fx)) (c + dx)^2 dx$$

[In] integrate((d*x+c)**2*(a+b*cot(f*x+e)),x)

[Out] Integral((a + b*cot(e + f*x))*(c + d*x)**2, x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(95) = 190.

Time = 0.44 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.71

$$\int (c + dx)^2 (a + b \cot(e + fx)) dx$$

$$= \frac{6(fx + e)ac^2 + \frac{2(fx+e)^3 ad^2}{f^2} - \frac{6(fx+e)^2 ad^2 e}{f^2} + \frac{6(fx+e) ad^2 e^2}{f^2} + \frac{6(fx+e)^2 acd}{f} - \frac{12(fx+e) acde}{f} + 6bc^2 \log(\sin(fx + e))}{f^3}$$

[In] integrate((d*x+c)^2*(a+b*cot(f*x+e)),x, algorithm="maxima")

[Out] $\frac{1}{6}*(6*(f*x + e)*a*c^2 + 2*(f*x + e)^3*a*d^2/f^2 - 6*(f*x + e)^2*a*d^2*e/f^2 + 6*(f*x + e)*a*d^2*e^2/f^2 + 6*(f*x + e)^2*a*c*d/f - 12*(f*x + e)*a*c*d*e/f + 6*b*c^2*\log(\sin(f*x + e)) + 6*b*d^2*e^2*\log(\sin(f*x + e))/f^2 - 12*b*c*d*e*\log(\sin(f*x + e))/f + (-2*I*(f*x + e)^3*b*d^2 + 12*b*d^2*\text{polylog}(3, -e^{(I*f*x + I*e)}) + 12*b*d^2*\text{polylog}(3, e^{(I*f*x + I*e)}) - 6*(-I*b*d^2*e + I*b*c*d*f)*(f*x + e)^2 - 6*(-I*(f*x + e)^2*b*d^2 + 2*(I*b*d^2*e - I*b*c*d*f)*(f*x + e))*\arctan2(\sin(f*x + e), \cos(f*x + e) + 1) - 6*(I*(f*x + e)^2*b*d^2 + 2*(-I*b*d^2*e + I*b*c*d*f)*(f*x + e))*\arctan2(\sin(f*x + e), -\cos(f*x + e) + 1) - 12*(I*(f*x + e)*b*d^2 - I*b*d^2*e + I*b*c*d*f)*\text{dilog}(-e^{(I*f*x + I*e)}) - 12*(I*(f*x + e)*b*d^2 - I*b*d^2*e + I*b*c*d*f)*\text{dilog}(e^{(I*f*x + I*e)}) + 3*((f*x + e)^2*b*d^2 - 2*(b*d^2*e - b*c*d*f)*(f*x + e))*\log(\cos(f*x + e)^2 + \sin(f*x + e)^2 + 2*\cos(f*x + e) + 1) + 3*((f*x + e)^2*b*d^2 - 2*(b*d^2*e - b*c*d*f)*(f*x + e))*\log(\cos(f*x + e)^2 + \sin(f*x + e)^2 - 2*\cos(f*x + e) + 1))/f^2)/f$

Giac [F]

$$\int (c + dx)^2 (a + b \cot(e + fx)) dx = \int (dx + c)^2 (b \cot(fx + e) + a) dx$$

[In] integrate((d*x+c)^2*(a+b*cot(f*x+e)),x, algorithm="giac")

[Out] integrate((d*x + c)^2*(b*cot(f*x + e) + a), x)

Mupad [F(-1)]

Timed out.

$$\int (c + dx)^2 (a + b \cot(e + fx)) dx = \int (a + b \cot(e + fx)) (c + dx)^2 dx$$

[In] int((a + b*cot(e + f*x))*(c + d*x)^2,x)

[Out] int((a + b*cot(e + f*x))*(c + d*x)^2, x)

3.39 $\int (c + dx)(a + b \cot(e + fx)) dx$

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Optimal result

Integrand size = 16, antiderivative size = 83

$$\int (c + dx)(a + b \cot(e + fx)) dx = \frac{a(c + dx)^2}{2d} - \frac{ib(c + dx)^2}{2d} + \frac{b(c + dx) \log(1 - e^{2i(e+fx)})}{f} - \frac{ibd \operatorname{PolyLog}(2, e^{2i(e+fx)})}{2f^2}$$

[Out] $1/2*a*(d*x+c)^2/d-1/2*I*b*(d*x+c)^2/d+b*(d*x+c)*\ln(1-\exp(2*I*(f*x+e)))/f-1/2*I*b*d*\operatorname{polylog}(2,\exp(2*I*(f*x+e)))/f^2$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3803, 3798, 2221, 2317, 2438}

$$\int (c + dx)(a + b \cot(e + fx)) dx = \frac{a(c + dx)^2}{2d} + \frac{b(c + dx) \log(1 - e^{2i(e+fx)})}{f} - \frac{ib(c + dx)^2}{2d} - \frac{ibd \operatorname{PolyLog}(2, e^{2i(e+fx)})}{2f^2}$$

[In] $\operatorname{Int}[(c + d*x)*(a + b*\operatorname{Cot}[e + f*x]),x]$

[Out] $(a*(c + d*x)^2)/(2*d) - ((I/2)*b*(c + d*x)^2)/d + (b*(c + d*x)*\operatorname{Log}[1 - E^{((2*I)*(e + f*x))}])/f - ((I/2)*b*d*\operatorname{PolyLog}[2, E^{((2*I)*(e + f*x))}])/f^2$

Rule 2221

$\operatorname{Int}[(((F_)^{((g_)*(e_) + (f_)*(x_))})^{(n_)*((c_) + (d_)*(x_))^{(m_))})/((a_) + (b_)*((F_)^{((g_)*(e_) + (f_)*(x_))})^{(n_)}), x_Symbol] \rightarrow \operatorname{Simp}$

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol]
:> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3803

```
Int[((c_) + (d_)*(x_))^(m_)*((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol]
:> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (a(c + dx) + b(c + dx) \cot(e + fx)) dx \\
&= \frac{a(c + dx)^2}{2d} + b \int (c + dx) \cot(e + fx) dx \\
&= \frac{a(c + dx)^2}{2d} - \frac{ib(c + dx)^2}{2d} - (2ib) \int \frac{e^{2i(e+fx)}(c + dx)}{1 - e^{2i(e+fx)}} dx \\
&= \frac{a(c + dx)^2}{2d} - \frac{ib(c + dx)^2}{2d} + \frac{b(c + dx) \log(1 - e^{2i(e+fx)})}{f} - \frac{(bd) \int \log(1 - e^{2i(e+fx)}) dx}{f} \\
&= \frac{a(c + dx)^2}{2d} - \frac{ib(c + dx)^2}{2d} + \frac{b(c + dx) \log(1 - e^{2i(e+fx)})}{f} \\
&\quad + \frac{(ibd) \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2i(e+fx)}\right)}{2f^2}
\end{aligned}$$

$$= \frac{a(c+dx)^2}{2d} - \frac{ib(c+dx)^2}{2d} + \frac{b(c+dx) \log(1 - e^{2i(e+fx)})}{f} - \frac{ibd \operatorname{PolyLog}(2, e^{2i(e+fx)})}{2f^2}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 209 vs. $2(83) = 166$.

Time = 5.75 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.52

$$\int (c+dx)(a+b \cot(e+fx)) dx$$

$$= acx + \frac{1}{2}adx^2 + \frac{1}{2}bdx^2 \cot(e) + \frac{bc \log(\cos(e+fx))}{f} + \frac{bc \log(\tan(e+fx))}{f}$$

$$- \frac{bd \csc(e) \sec(e) \left(e^{i \arctan(\tan(e))} f^2 x^2 + \frac{(ifx(-\pi+2 \arctan(\tan(e))) - \pi \log(1+e^{-2ifx}) - 2(fx + \arctan(\tan(e)))) \log(1-e^{2i(fx + \arctan(\tan(e)))})}{2f^2 \sqrt{\sec^2(e) (\cos^2(e) + \dots} \right)}{2f^2 \sqrt{\sec^2(e) (\cos^2(e) + \dots} \right)}$$

[In] Integrate[(c + d*x)*(a + b*Cot[e + f*x]),x]

[Out] a*c*x + (a*d*x^2)/2 + (b*d*x^2*Cot[e])/2 + (b*c*Log[Cos[e + f*x]])/f + (b*c*Log[Tan[e + f*x]])/f - (b*d*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x)] - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]])]) + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]]) + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]])])]*Tan[e])/Sqrt[1 + Tan[e]^2]))/(2*f^2*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)])

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(71) = 142$.

Time = 0.37 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.89

method	result
risch	$-\frac{ibde^2}{f^2} + \frac{adx^2}{2} - \frac{ibd \operatorname{polylog}(2, -e^{i(fx+e)})}{f^2} + acx + \frac{bc \ln(e^{i(fx+e)} + 1)}{f} - \frac{2bc \ln(e^{i(fx+e)})}{f} + \frac{bc \ln(e^{i(fx+e)} - 1)}{f} - \frac{2ibd}{f}$

[In] int((d*x+c)*(a+b*cot(f*x+e)),x,method=_RETURNVERBOSE)

[Out] -I/f^2*b*d*e^2+1/2*a*d*x^2-I/f^2*b*d*polylog(2,-exp(I*(f*x+e)))+a*c*x+1/f*b*c*ln(exp(I*(f*x+e))+1)-2/f*b*c*ln(exp(I*(f*x+e)))+1/f*b*c*ln(exp(I*(f*x+e))-1)-2*I/f*b*d*e*x-I/f^2*b*d*polylog(2,exp(I*(f*x+e)))-1/2*I*b*d*x^2+1/f*b*d*ln(1-exp(I*(f*x+e)))*x+1/f^2*b*d*ln(1-exp(I*(f*x+e)))*e+I*b*c*x+1/f*b*d*ln(exp(I*(f*x+e))+1)*x+2/f^2*b*d*e*ln(exp(I*(f*x+e)))-1/f^2*b*d*e*ln(exp(I*(f*x+e))-1)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(68) = 136$.

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.70

$$\int (c + dx)(a + b \cot(e + fx)) dx$$

$$= \frac{2adf^2x^2 + 4acf^2x - i bd\text{Li}_2(\cos(2fx + 2e) + i \sin(2fx + 2e)) + i bd\text{Li}_2(\cos(2fx + 2e) - i \sin(2fx + 2e))}{f^2}$$

[In] integrate((d*x+c)*(a+b*cot(f*x+e)),x, algorithm="fricas")

[Out] $\frac{1}{4}*(2*a*d*f^2*x^2 + 4*a*c*f^2*x - I*b*d*dilog(\cos(2*f*x + 2*e) + I*\sin(2*f*x + 2*e)) + I*b*d*dilog(\cos(2*f*x + 2*e) - I*\sin(2*f*x + 2*e)) - 2*(b*d*e - b*c*f)*\log(-1/2*\cos(2*f*x + 2*e) + 1/2*I*\sin(2*f*x + 2*e) + 1/2) - 2*(b*d*e - b*c*f)*\log(-1/2*\cos(2*f*x + 2*e) - 1/2*I*\sin(2*f*x + 2*e) + 1/2) + 2*(b*d*f*x + b*d*e)*\log(-\cos(2*f*x + 2*e) + I*\sin(2*f*x + 2*e) + 1) + 2*(b*d*f*x + b*d*e)*\log(-\cos(2*f*x + 2*e) - I*\sin(2*f*x + 2*e) + 1))/f^2$

Sympy [F]

$$\int (c + dx)(a + b \cot(e + fx)) dx = \int (a + b \cot(e + fx))(c + dx) dx$$

[In] integrate((d*x+c)*(a+b*cot(f*x+e)),x)

[Out] Integral((a + b*cot(e + f*x))*(c + d*x), x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(68) = 136$.

Time = 0.42 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.52

$$\int (c + dx)(a + b \cot(e + fx)) dx$$

$$= \frac{(a - ib)df^2x^2 + 2(a - ib)cf^2x - 2ibdfx \arctan(\sin(fx + e), -\cos(fx + e) + 1) + 2ibcf \arctan(\sin(fx + e), \cos(fx + e) - 1)}{f^2}$$

[In] integrate((d*x+c)*(a+b*cot(f*x+e)),x, algorithm="maxima")

[Out] $\frac{1}{2}*((a - I*b)*d*f^2*x^2 + 2*(a - I*b)*c*f^2*x - 2*I*b*d*f*x*\arctan2(\sin(f*x + e), -\cos(f*x + e) + 1) + 2*I*b*c*f*\arctan2(\sin(f*x + e), \cos(f*x + e) - 1))$

1) - 2*I*b*d*dilog(-e^(I*f*x + I*e)) - 2*I*b*d*dilog(e^(I*f*x + I*e)) - 2*(-I*b*d*f*x - I*b*c*f)*arctan2(sin(f*x + e), cos(f*x + e) + 1) + (b*d*f*x + b*c*f)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + (b*d*f*x + b*c*f)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1))/f^2

Giac [F]

$$\int (c + dx)(a + b \cot(e + fx)) dx = \int (dx + c)(b \cot(fx + e) + a) dx$$

[In] integrate((d*x+c)*(a+b*cot(f*x+e)),x, algorithm="giac")

[Out] integrate((d*x + c)*(b*cot(f*x + e) + a), x)

Mupad [F(-1)]

Timed out.

$$\int (c + dx)(a + b \cot(e + fx)) dx = \int (a + b \cot(e + fx)) (c + dx) dx$$

[In] int((a + b*cot(e + f*x))*(c + d*x),x)

[Out] int((a + b*cot(e + f*x))*(c + d*x), x)

3.40 $\int \frac{a+b \cot(e+fx)}{c+dx} dx$

Optimal result	266
Rubi [N/A]	266
Mathematica [N/A]	267
Maple [N/A] (verified)	267
Fricas [N/A]	267
Sympy [N/A]	267
Maxima [N/A]	268
Giac [N/A]	268
Mupad [N/A]	268

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a + b \cot(e + fx)}{c + dx} dx = \text{Int}\left(\frac{a + b \cot(e + fx)}{c + dx}, x\right)$$

[Out] Unintegrable((a+b*cot(f*x+e))/(d*x+c),x)

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a + b \cot(e + fx)}{c + dx} dx = \int \frac{a + b \cot(e + fx)}{c + dx} dx$$

[In] Int[(a + b*Cot[e + f*x])/(c + d*x),x]

[Out] Defer[Int] [(a + b*Cot[e + f*x])/(c + d*x), x]

Rubi steps

$$\text{integral} = \int \frac{a + b \cot(e + fx)}{c + dx} dx$$

Mathematica [N/A]

Not integrable

Time = 3.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \cot(e + fx)}{c + dx} dx = \int \frac{a + b \cot(e + fx)}{c + dx} dx$$

[In] Integrate[(a + b*Cot[e + f*x])/(c + d*x), x]

[Out] Integrate[(a + b*Cot[e + f*x])/(c + d*x), x]

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b \cot(fx + e)}{dx + c} dx$$

[In] int((a+b*cot(f*x+e))/(d*x+c), x)

[Out] int((a+b*cot(f*x+e))/(d*x+c), x)

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \cot(e + fx)}{c + dx} dx = \int \frac{b \cot(fx + e) + a}{dx + c} dx$$

[In] integrate((a+b*cot(f*x+e))/(d*x+c), x, algorithm="fricas")

[Out] integral((b*cot(f*x + e) + a)/(d*x + c), x)

Sympy [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{a + b \cot(e + fx)}{c + dx} dx = \int \frac{a + b \cot(e + fx)}{c + dx} dx$$

[In] integrate((a+b*cot(f*x+e))/(d*x+c), x)

[Out] Integral((a + b*cot(e + f*x))/(c + d*x), x)

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 136, normalized size of antiderivative = 7.56

$$\int \frac{a + b \cot(e + fx)}{c + dx} dx = \int \frac{b \cot(fx + e) + a}{dx + c} dx$$

[In] integrate((a+b*cot(f*x+e))/(d*x+c),x, algorithm="maxima")

[Out] -(b*d*integrate(sin(f*x + e)/((d*x + c)*cos(f*x + e)^2 + (d*x + c)*sin(f*x + e)^2 + d*x + 2*(d*x + c)*cos(f*x + e) + c), x) - b*d*integrate(sin(f*x + e)/((d*x + c)*cos(f*x + e)^2 + (d*x + c)*sin(f*x + e)^2 + d*x - 2*(d*x + c)*cos(f*x + e) + c), x) - a*log(d*x + c))/d

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \cot(e + fx)}{c + dx} dx = \int \frac{b \cot(fx + e) + a}{dx + c} dx$$

[In] integrate((a+b*cot(f*x+e))/(d*x+c),x, algorithm="giac")

[Out] integrate((b*cot(f*x + e) + a)/(d*x + c), x)

Mupad [N/A]

Not integrable

Time = 12.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \cot(e + fx)}{c + dx} dx = \int \frac{a + b \cot(e + fx)}{c + dx} dx$$

[In] int((a + b*cot(e + f*x))/(c + d*x),x)

[Out] int((a + b*cot(e + f*x))/(c + d*x), x)

3.41 $\int \frac{a+b \cot(e+fx)}{(c+dx)^2} dx$

Optimal result	269
Rubi [N/A]	269
Mathematica [N/A]	270
Maple [N/A] (verified)	270
Fricas [N/A]	270
Sympy [N/A]	270
Maxima [N/A]	271
Giac [N/A]	271
Mupad [N/A]	271

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a + b \cot(e + fx)}{(c + dx)^2} dx = \text{Int}\left(\frac{a + b \cot(e + fx)}{(c + dx)^2}, x\right)$$

[Out] Unintegrable((a+b*cot(f*x+e))/(d*x+c)^2,x)

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a + b \cot(e + fx)}{(c + dx)^2} dx = \int \frac{a + b \cot(e + fx)}{(c + dx)^2} dx$$

[In] Int[(a + b*Cot[e + f*x])/(c + d*x)^2,x]

[Out] Defer[Int] [(a + b*Cot[e + f*x])/(c + d*x)^2, x]

Rubi steps

$$\text{integral} = \int \frac{a + b \cot(e + fx)}{(c + dx)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 9.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \cot(e + fx)}{(c + dx)^2} dx = \int \frac{a + b \cot(e + fx)}{(c + dx)^2} dx$$

[In] Integrate[(a + b*Cot[e + f*x])/(c + d*x)^2,x]

[Out] Integrate[(a + b*Cot[e + f*x])/(c + d*x)^2, x]

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b \cot(fx + e)}{(dx + c)^2} dx$$

[In] int((a+b*cot(f*x+e))/(d*x+c)^2,x)

[Out] int((a+b*cot(f*x+e))/(d*x+c)^2,x)

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{a + b \cot(e + fx)}{(c + dx)^2} dx = \int \frac{b \cot(fx + e) + a}{(dx + c)^2} dx$$

[In] integrate((a+b*cot(f*x+e))/(d*x+c)^2,x, algorithm="fricas")

[Out] integral((b*cot(f*x + e) + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a + b \cot(e + fx)}{(c + dx)^2} dx = \int \frac{a + b \cot(e + fx)}{(c + dx)^2} dx$$

[In] integrate((a+b*cot(f*x+e))/(d*x+c)**2,x)

[Out] Integral((a + b*cot(e + f*x))/(c + d*x)**2, x)

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 242, normalized size of antiderivative = 13.44

$$\int \frac{a + b \cot(e + fx)}{(c + dx)^2} dx = \int \frac{b \cot(fx + e) + a}{(dx + c)^2} dx$$

[In] integrate((a+b*cot(f*x+e))/(d*x+c)^2,x, algorithm="maxima")

[Out] -((b*d^2*x + b*c*d)*integrate(sin(f*x + e)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(f*x + e)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(f*x + e)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(f*x + e)), x) - (b*d^2*x + b*c*d)*integrate(sin(f*x + e)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(f*x + e)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(f*x + e)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(f*x + e)), x) + a)/(d^2*x + c*d)

Giac [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \cot(e + fx)}{(c + dx)^2} dx = \int \frac{b \cot(fx + e) + a}{(dx + c)^2} dx$$

[In] integrate((a+b*cot(f*x+e))/(d*x+c)^2,x, algorithm="giac")

[Out] integrate((b*cot(f*x + e) + a)/(d*x + c)^2, x)

Mupad [N/A]

Not integrable

Time = 12.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \cot(e + fx)}{(c + dx)^2} dx = \int \frac{a + b \cot(e + fx)}{(c + dx)^2} dx$$

[In] int((a + b*cot(e + f*x))/(c + d*x)^2,x)

[Out] int((a + b*cot(e + f*x))/(c + d*x)^2, x)

3.42 $\int (c + dx)^3 (a + b \cot(e + fx))^2 dx$

Optimal result	272
Rubi [A] (verified)	273
Mathematica [B] (warning: unable to verify)	277
Maple [B] (verified)	278
Fricas [B] (verification not implemented)	279
Sympy [F]	280
Maxima [B] (verification not implemented)	280
Giac [F]	283
Mupad [F(-1)]	283

Optimal result

Integrand size = 20, antiderivative size = 295

$$\begin{aligned}
 \int (c + dx)^3 (a + b \cot(e + fx))^2 dx = & -\frac{ib^2(c + dx)^3}{f} + \frac{a^2(c + dx)^4}{4d} - \frac{iab(c + dx)^4}{2d} \\
 & - \frac{b^2(c + dx)^4}{4d} - \frac{b^2(c + dx)^3 \cot(e + fx)}{f} \\
 & + \frac{3b^2d(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f^2} \\
 & + \frac{2ab(c + dx)^3 \log(1 - e^{2i(e+fx)})}{f} \\
 & - \frac{3ib^2d^2(c + dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^3} \\
 & - \frac{3iabd(c + dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} \\
 & + \frac{3b^2d^3 \text{PolyLog}(3, e^{2i(e+fx)})}{2f^4} \\
 & + \frac{3abd^2(c + dx) \text{PolyLog}(3, e^{2i(e+fx)})}{f^3} \\
 & + \frac{3iabd^3 \text{PolyLog}(4, e^{2i(e+fx)})}{2f^4}
 \end{aligned}$$

```
[Out] -I*b^2*(d*x+c)^3/f+1/4*a^2*(d*x+c)^4/d-1/2*I*a*b*(d*x+c)^4/d-1/4*b^2*(d*x+c)^4/d-b^2*(d*x+c)^3*cot(f*x+e)/f+3*b^2*d*(d*x+c)^2*ln(1-exp(2*I*(f*x+e)))/f^2+2*a*b*(d*x+c)^3*ln(1-exp(2*I*(f*x+e)))/f-3*I*b^2*d^2*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/f^3-3*I*a*b*d*(d*x+c)^2*polylog(2,exp(2*I*(f*x+e)))/f^2+3/2*b^2*d^3*polylog(3,exp(2*I*(f*x+e)))/f^4+3*a*b*d^2*(d*x+c)*polylog(3,exp(2*I*(f*x+e)))/f^3+3/2*I*a*b*d^3*polylog(4,exp(2*I*(f*x+e)))/f^4
```


Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3803, 3798, 2221, 2611, 6744, 2320, 6724, 3801, 32}

$$\int (c + dx)^3 (a + b \cot(e + fx))^2 dx = \frac{a^2(c + dx)^4}{4d} + \frac{3abd^2(c + dx) \text{PolyLog}(3, e^{2i(e+fx)})}{f^3} - \frac{3iabd(c + dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{2ab(c + dx)^3 \log(1 - e^{2i(e+fx)})}{f} - \frac{iab(c + dx)^4}{2d} + \frac{3iabd^3 \text{PolyLog}(4, e^{2i(e+fx)})}{2f^4} - \frac{3ib^2d^2(c + dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^3} + \frac{3b^2d(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f^2} - \frac{b^2(c + dx)^3 \cot(e + fx)}{f} - \frac{ib^2(c + dx)^3}{f} - \frac{b^2(c + dx)^4}{4d} + \frac{3b^2d^3 \text{PolyLog}(3, e^{2i(e+fx)})}{2f^4}$$

[In] Int[(c + d*x)^3*(a + b*Cot[e + f*x])^2,x]

[Out] ((-I)*b^2*(c + d*x)^3)/f + (a^2*(c + d*x)^4)/(4*d) - ((I/2)*a*b*(c + d*x)^4)/d - (b^2*(c + d*x)^4)/(4*d) - (b^2*(c + d*x)^3*Cot[e + f*x])/f + (3*b^2*d*(c + d*x)^2*Log[1 - E^((2*I)*(e + f*x))])/f^2 + (2*a*b*(c + d*x)^3*Log[1 - E^((2*I)*(e + f*x))])/f - ((3*I)*b^2*d^2*(c + d*x)*PolyLog[2, E^((2*I)*(e + f*x))])/f^3 - ((3*I)*a*b*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + (3*b^2*d^3*PolyLog[3, E^((2*I)*(e + f*x))])/(2*f^4) + (3*a*b*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(e + f*x))])/f^3 + (((3*I)/2)*a*b*d^3*PolyLog[4, E^((2*I)*(e + f*x))])/f^4

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2221

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp[(((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x) - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a]], x)

)^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2320

Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2611

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :=> Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 3798

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] :=> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 3801

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Dist[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3803

Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :=> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 6724

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :=> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 6744

```

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_)))^(p_.)], x_Symbol] := Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (a^2(c + dx)^3 + 2ab(c + dx)^3 \cot(e + fx) + b^2(c + dx)^3 \cot^2(e + fx)) dx \\
&= \frac{a^2(c + dx)^4}{4d} + (2ab) \int (c + dx)^3 \cot(e + fx) dx + b^2 \int (c + dx)^3 \cot^2(e + fx) dx \\
&= \frac{a^2(c + dx)^4}{4d} - \frac{iab(c + dx)^4}{2d} - \frac{b^2(c + dx)^3 \cot(e + fx)}{f} \\
&\quad - (4iab) \int \frac{e^{2i(e+fx)}(c + dx)^3}{1 - e^{2i(e+fx)}} dx - b^2 \int (c + dx)^3 dx \\
&\quad + \frac{(3b^2d) \int (c + dx)^2 \cot(e + fx) dx}{f} \\
&= -\frac{ib^2(c + dx)^3}{f} + \frac{a^2(c + dx)^4}{4d} - \frac{iab(c + dx)^4}{2d} - \frac{b^2(c + dx)^4}{4d} \\
&\quad - \frac{b^2(c + dx)^3 \cot(e + fx)}{f} + \frac{2ab(c + dx)^3 \log(1 - e^{2i(e+fx)})}{f} \\
&\quad - \frac{(6abd) \int (c + dx)^2 \log(1 - e^{2i(e+fx)}) dx}{f} - \frac{(6ib^2d) \int \frac{e^{2i(e+fx)}(c+dx)^2}{1 - e^{2i(e+fx)}} dx}{f} \\
&= -\frac{ib^2(c + dx)^3}{f} + \frac{a^2(c + dx)^4}{4d} - \frac{iab(c + dx)^4}{2d} - \frac{b^2(c + dx)^4}{4d} \\
&\quad - \frac{b^2(c + dx)^3 \cot(e + fx)}{f} + \frac{3b^2d(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f^2} \\
&\quad + \frac{2ab(c + dx)^3 \log(1 - e^{2i(e+fx)})}{f} - \frac{3iabd(c + dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} \\
&\quad + \frac{(6iabd^2) \int (c + dx) \text{PolyLog}(2, e^{2i(e+fx)}) dx}{f^2} \\
&\quad - \frac{(6b^2d^2) \int (c + dx) \log(1 - e^{2i(e+fx)}) dx}{f^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{ib^2(c+dx)^3}{f} + \frac{a^2(c+dx)^4}{4d} - \frac{iab(c+dx)^4}{2d} - \frac{b^2(c+dx)^4}{4d} \\
&\quad - \frac{b^2(c+dx)^3 \cot(e+fx)}{f} + \frac{3b^2d(c+dx)^2 \log(1-e^{2i(e+fx)})}{f^2} \\
&\quad + \frac{2ab(c+dx)^3 \log(1-e^{2i(e+fx)})}{f} - \frac{3ib^2d^2(c+dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^3} \\
&\quad - \frac{3iabd(c+dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{3abd^2(c+dx) \text{PolyLog}(3, e^{2i(e+fx)})}{f^3} \\
&\quad - \frac{(3abd^3) \int \text{PolyLog}(3, e^{2i(e+fx)}) dx}{f^3} + \frac{(3ib^2d^3) \int \text{PolyLog}(2, e^{2i(e+fx)}) dx}{f^3} \\
&= -\frac{ib^2(c+dx)^3}{f} + \frac{a^2(c+dx)^4}{4d} - \frac{iab(c+dx)^4}{2d} - \frac{b^2(c+dx)^4}{4d} \\
&\quad - \frac{b^2(c+dx)^3 \cot(e+fx)}{f} + \frac{3b^2d(c+dx)^2 \log(1-e^{2i(e+fx)})}{f^2} \\
&\quad + \frac{2ab(c+dx)^3 \log(1-e^{2i(e+fx)})}{f} - \frac{3ib^2d^2(c+dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^3} \\
&\quad - \frac{3iabd(c+dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{3abd^2(c+dx) \text{PolyLog}(3, e^{2i(e+fx)})}{f^3} \\
&\quad + \frac{(3iabd^3) \text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x} dx, x, e^{2i(e+fx)}\right)}{2f^4} \\
&\quad + \frac{(3b^2d^3) \text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x} dx, x, e^{2i(e+fx)}\right)}{2f^4} \\
&= -\frac{ib^2(c+dx)^3}{f} + \frac{a^2(c+dx)^4}{4d} - \frac{iab(c+dx)^4}{2d} - \frac{b^2(c+dx)^4}{4d} \\
&\quad - \frac{b^2(c+dx)^3 \cot(e+fx)}{f} + \frac{3b^2d(c+dx)^2 \log(1-e^{2i(e+fx)})}{f^2} \\
&\quad + \frac{2ab(c+dx)^3 \log(1-e^{2i(e+fx)})}{f} - \frac{3ib^2d^2(c+dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^3} \\
&\quad - \frac{3iabd(c+dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{3b^2d^3 \text{PolyLog}(3, e^{2i(e+fx)})}{2f^4} \\
&\quad + \frac{3abd^2(c+dx) \text{PolyLog}(3, e^{2i(e+fx)})}{f^3} + \frac{3iabd^3 \text{PolyLog}(4, e^{2i(e+fx)})}{2f^4}
\end{aligned}$$


```

)*(e + f*x))] + (12*I)*(1 - E^((-2*I)*e))*f*x*PolyLog[3, -E^((-I)*(e + f*x)
)] + (12*I)*(1 - E^((-2*I)*e))*f*x*PolyLog[3, E^((-I)*(e + f*x))] + 12*(1 -
E^((-2*I)*e))*PolyLog[4, -E^((-I)*(e + f*x))] + 12*(1 - E^((-2*I)*e))*Poly
Log[4, E^((-I)*(e + f*x)))]/(2*f^4) + (3*b^2*c^2*d*Csc[e]*(-(f*x*Cos[e]) +
Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e]))/(f^2*(Cos[e]^2 + Sin[e]^2)
) + (2*a*b*c^3*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x
]]*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2)) + (Csc[e]*Csc[e + f*x]*(4*a^2*c^3*f*x
*Cos[f*x] - 4*b^2*c^3*f*x*Cos[f*x] + 6*a^2*c^2*d*f*x^2*Cos[f*x] - 6*b^2*c^2
*d*f*x^2*Cos[f*x] + 4*a^2*c*d^2*f*x^3*Cos[f*x] - 4*b^2*c*d^2*f*x^3*Cos[f*x]
+ a^2*d^3*f*x^4*Cos[f*x] - b^2*d^3*f*x^4*Cos[f*x] - 4*a^2*c^3*f*x*Cos[2*e
+ f*x] + 4*b^2*c^3*f*x*Cos[2*e + f*x] - 6*a^2*c^2*d*f*x^2*Cos[2*e + f*x] +
6*b^2*c^2*d*f*x^2*Cos[2*e + f*x] - 4*a^2*c*d^2*f*x^3*Cos[2*e + f*x] + 4*b^2
*c*d^2*f*x^3*Cos[2*e + f*x] - a^2*d^3*f*x^4*Cos[2*e + f*x] + b^2*d^3*f*x^4*
Cos[2*e + f*x] + 8*b^2*c^3*Ssin[f*x] + 24*b^2*c^2*d*x*Ssin[f*x] + 8*a*b*c^3*f
*x*Ssin[f*x] + 24*b^2*c*d^2*x^2*Ssin[f*x] + 12*a*b*c^2*d*f*x^2*Ssin[f*x] + 8*b
^2*d^3*x^3*Ssin[f*x] + 8*a*b*c*d^2*f*x^3*Ssin[f*x] + 2*a*b*d^3*f*x^4*Ssin[f*x]
+ 8*a*b*c^3*f*x*Ssin[2*e + f*x] + 12*a*b*c^2*d*f*x^2*Ssin[2*e + f*x] + 8*a*b
*c*d^2*f*x^3*Ssin[2*e + f*x] + 2*a*b*d^3*f*x^4*Ssin[2*e + f*x]))/(8*f) - (3*b
^2*c*d^2*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi + 2*Arc
Tan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x]) - 2*(f*x + ArcTan[Tan[e]])*Log[1
- E^((2*I)*(f*x + ArcTan[Tan[e]])]) + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*L
og[Sin[f*x + ArcTan[Tan[e]]]]) + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]
)])]*Tan[e])/Sqrt[1 + Tan[e]^2]))/(f^3*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2)
]) - (3*a*b*c^2*d*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]])*f^2*x^2 + ((I*f*x*(-Pi
+ 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x]) - 2*(f*x + ArcTan[Tan[e]]
)*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]])]) + Pi*Log[Cos[f*x]] + 2*ArcTan[T
an[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]]) + I*PolyLog[2, E^((2*I)*(f*x + ArcTan
[Tan[e])])]*Tan[e])/Sqrt[1 + Tan[e]^2]))/(f^2*Sqrt[Sec[e]^2*(Cos[e]^2 + Si
n[e]^2)))

```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(266) = 532$.

Time = 0.85 (sec) , antiderivative size = 1604, normalized size of antiderivative = 5.44

method	result	size
risch	Expression too large to display	1604

```
[In] int((d*x+c)^3*(a+b*cot(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/f^4*b*e^3*d^3*a*ln(exp(I*(f*x+e)))-2/f^4*b*e^3*d^3*a*ln(exp(I*(f*x+e))-1)
+12*I/f^4*b*d^3*a*polylog(4,-exp(I*(f*x+e)))-3*I/f^4*b*d^3*a*e^4-6*I/f^3*b^
2*d^3*polylog(2,exp(I*(f*x+e)))*x-6*I/f^3*b^2*d^3*polylog(2,-exp(I*(f*x+e)
))*x+6*I/f^3*b^2*e^2*d^3*x-6*I/f*b^2*d^2*c*x^2-6*I/f^3*b^2*d^2*c*e^2-6*I/f^3
```

```

*b^2*d^2*c*polylog(2,exp(I*(f*x+e)))-6*I/f^3*b^2*d^2*c*polylog(2,-exp(I*(f*
x+e)))+12*I/f^4*b*d^3*a*polylog(4,exp(I*(f*x+e)))-2*I*d^2*a*b*c*x^3-3*I*d*a
*b*c^2*x^2+d^2*a^2*c*x^3+3/2*d*a^2*c^2*x^2+a^2*c^3*x-d^2*b^2*c*x^3-3/2*d*b^
2*c^2*x^2+2*I*a*b*c^3*x+1/2*I/d*a*b*c^4-1/2*I*d^3*a*b*x^4+12/f^2*b*e*a*c^2*
d*ln(exp(I*(f*x+e)))+12/f^3*b^2*e*c*d^2*ln(exp(I*(f*x+e)))-6/f^3*b^2*e*c*d^
2*ln(exp(I*(f*x+e))-1)+6/f^3*b^2*d^2*c*ln(1-exp(I*(f*x+e)))*e+6/f^2*b^2*d^2
*c*ln(1-exp(I*(f*x+e)))*x+6/f^2*b^2*d^2*c*ln(exp(I*(f*x+e))+1)*x+12/f^3*b*d
^3*a*polylog(3,-exp(I*(f*x+e)))*x+12/f^3*b*d^3*a*polylog(3,exp(I*(f*x+e)))*
x+2/f*b*d^3*a*ln(exp(I*(f*x+e))+1)*x^3+2/f^4*b*d^3*a*ln(1-exp(I*(f*x+e)))*e
^3+2/f*b*d^3*a*ln(1-exp(I*(f*x+e)))*x^3+12/f^3*b*a*c*d^2*polylog(3,exp(I*(f
*x+e)))+12/f^3*b*a*c*d^2*polylog(3,-exp(I*(f*x+e)))+3/f^2*b^2*d^3*ln(exp(I*(
f*x+e))+1)*x^2+2/f*b*a*c^3*ln(exp(I*(f*x+e))+1)-4/f*b*a*c^3*ln(exp(I*(f*x+
e)))+2/f*b*a*c^3*ln(exp(I*(f*x+e))-1)+3/f^2*b^2*c^2*d*ln(exp(I*(f*x+e))+1)-
6/f^2*b^2*c^2*d*ln(exp(I*(f*x+e)))+3/f^2*b^2*c^2*d*ln(exp(I*(f*x+e))-1)-6/f
^4*b^2*e^2*d^3*ln(exp(I*(f*x+e)))+3/f^4*b^2*e^2*d^3*ln(exp(I*(f*x+e))-1)-3/
f^4*b^2*e^2*d^3*ln(1-exp(I*(f*x+e)))+3/f^2*b^2*d^3*ln(1-exp(I*(f*x+e)))*x^2
-2*I/f*b^2*d^3*x^3+4*I/f^4*b^2*e^3*d^3-6/f^2*b*e*a*c^2*d*ln(exp(I*(f*x+e))-
1)+6/f*b*d*c^2*a*ln(1-exp(I*(f*x+e)))*x+6/f*b*d*c^2*a*ln(exp(I*(f*x+e))+1)*
x+6/f*b*a*c*d^2*ln(1-exp(I*(f*x+e)))*x^2-6/f^3*b*a*c*d^2*ln(1-exp(I*(f*x+e)
))*e^2+6/f*b*a*c*d^2*ln(exp(I*(f*x+e))+1)*x^2-12/f^3*b*e^2*a*c*d^2*ln(exp(I
*(f*x+e)))+6/f^3*b*e^2*a*c*d^2*ln(exp(I*(f*x+e))-1)+6/f^2*b*d*c^2*a*ln(1-ex
p(I*(f*x+e)))*e-12*I/f^2*b^2*d^2*c*e*x+8*I/f^3*b*a*c*d^2*e^3-6*I/f^2*b*d^3*
a*polylog(2,-exp(I*(f*x+e)))*x^2-6*I/f^2*b*d^3*a*polylog(2,exp(I*(f*x+e)))*
x^2-4*I/f^3*b*d^3*a*e^3*x-6*I/f^2*b*d*c^2*a*e^2-6*I/f^2*b*d*c^2*a*polylog(2
,exp(I*(f*x+e)))-6*I/f^2*b*d*c^2*a*polylog(2,-exp(I*(f*x+e)))-2*I*b^2*(d^3*
x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/(exp(2*I*(f*x+e))-1)+6/f^4*b^2*d^3*polylog
(3,exp(I*(f*x+e)))+6/f^4*b^2*d^3*polylog(3,-exp(I*(f*x+e)))+1/4*d^3*a^2*x^4
+1/4/d*a^2*c^4-1/4*d^3*b^2*x^4-b^2*c^3*x-1/4/d*b^2*c^4-12*I/f*b*d*c^2*a*e*x
+12*I/f^2*b*a*c*d^2*e^2*x-12*I/f^2*b*a*c*d^2*polylog(2,exp(I*(f*x+e)))*x-12
*I/f^2*b*a*c*d^2*polylog(2,-exp(I*(f*x+e)))*x

```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1154 vs. $2(259) = 518$.

Time = 0.35 (sec) , antiderivative size = 1154, normalized size of antiderivative = 3.91

$$\int (c + dx)^3 (a + b \cot(e + fx))^2 dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x + 4*b^2
*c^3*f^3 - 3*I*a*b*d^3*polylog(4, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e))*si
n(2*f*x + 2*e) + 3*I*a*b*d^3*polylog(4, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*

```

```
e))*sin(2*f*x + 2*e) + 6*(I*a*b*d^3*f^2*x^2 + I*a*b*c^2*d*f^2 + I*b^2*c*d^2
*f + I*(2*a*b*c*d^2*f^2 + b^2*d^3*f)*x)*dilog(cos(2*f*x + 2*e) + I*sin(2*f*x
x + 2*e))*sin(2*f*x + 2*e) + 6*(-I*a*b*d^3*f^2*x^2 - I*a*b*c^2*d*f^2 - I*b^
2*c*d^2*f - I*(2*a*b*c*d^2*f^2 + b^2*d^3*f)*x)*dilog(cos(2*f*x + 2*e) - I*s
in(2*f*x + 2*e))*sin(2*f*x + 2*e) + 2*(2*a*b*d^3*e^3 - 2*a*b*c^3*f^3 - 3*b^
2*d^3*e^2 + 3*(2*a*b*c^2*d*e - b^2*c^2*d)*f^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^
2*e)*f)*log(-1/2*cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2)*sin(2*f*x
+ 2*e) + 2*(2*a*b*d^3*e^3 - 2*a*b*c^3*f^3 - 3*b^2*d^3*e^2 + 3*(2*a*b*c^2*d
*e - b^2*c^2*d)*f^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f)*log(-1/2*cos(2*f*x
+ 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2)*sin(2*f*x + 2*e) - 2*(2*a*b*d^3*f^3
*x^3 + 2*a*b*d^3*e^3 + 6*a*b*c^2*d*e*f^2 - 3*b^2*d^3*e^2 + 3*(2*a*b*c*d^2*f
^3 + b^2*d^3*f^2)*x^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f + 6*(a*b*c^2*d*f^
3 + b^2*c*d^2*f^2)*x)*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1)*sin(2
*f*x + 2*e) - 2*(2*a*b*d^3*f^3*x^3 + 2*a*b*d^3*e^3 + 6*a*b*c^2*d*e*f^2 - 3*
b^2*d^3*e^2 + 3*(2*a*b*c*d^2*f^3 + b^2*d^3*f^2)*x^2 - 6*(a*b*c*d^2*e^2 - b^
2*c*d^2*e)*f + 6*(a*b*c^2*d*f^3 + b^2*c*d^2*f^2)*x)*log(-cos(2*f*x + 2*e) -
I*sin(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e) - 3*(2*a*b*d^3*f*x + 2*a*b*c*d^2*
f + b^2*d^3)*polylog(3, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e))*sin(2*f*x +
2*e) - 3*(2*a*b*d^3*f*x + 2*a*b*c*d^2*f + b^2*d^3)*polylog(3, cos(2*f*x + 2
*e) - I*sin(2*f*x + 2*e))*sin(2*f*x + 2*e) + 4*(b^2*d^3*f^3*x^3 + 3*b^2*c*d
^2*f^3*x^2 + 3*b^2*c^2*d*f^3*x + b^2*c^3*f^3)*cos(2*f*x + 2*e) - ((a^2 - b^
2)*d^3*f^4*x^4 + 4*(a^2 - b^2)*c*d^2*f^4*x^3 + 6*(a^2 - b^2)*c^2*d*f^4*x^2
+ 4*(a^2 - b^2)*c^3*f^4*x)*sin(2*f*x + 2*e))/(f^4*sin(2*f*x + 2*e))
```

Sympy [F]

$$\int (c + dx)^3 (a + b \cot(e + fx))^2 dx = \int (a + b \cot(e + fx))^2 (c + dx)^3 dx$$

```
[In] integrate((d*x+c)**3*(a+b*cot(f*x+e))**2,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**2*(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4024 vs. $2(259) = 518$.

Time = 1.51 (sec) , antiderivative size = 4024, normalized size of antiderivative = 13.64

$$\int (c + dx)^3 (a + b \cot(e + fx))^2 dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```



```
[Out] 1/4*(4*(f*x + e)*a^2*c^3 + (f*x + e)^4*a^2*d^3/f^3 - 4*(f*x + e)^3*a^2*d^3*
e/f^3 + 6*(f*x + e)^2*a^2*d^3*e^2/f^3 - 4*(f*x + e)*a^2*d^3*e^3/f^3 + 4*(f*
x + e)^3*a^2*c*d^2/f^2 - 12*(f*x + e)^2*a^2*c*d^2*e/f^2 + 12*(f*x + e)*a^2*
c*d^2*e^2/f^2 + 6*(f*x + e)^2*a^2*c^2*d/f - 12*(f*x + e)*a^2*c^2*d*e/f + 8*
a*b*c^3*log(sin(f*x + e)) - 8*a*b*d^3*e^3*log(sin(f*x + e))/f^3 + 24*a*b*c*
d^2*e^2*log(sin(f*x + e))/f^2 - 24*a*b*c^2*d*e*log(sin(f*x + e))/f + 4*((2*
a*b - I*b^2)*(f*x + e)^4*d^3 + 8*b^2*d^3*e^3 - 24*b^2*c*d^2*e^2*f + 24*b^2*
c^2*d*e*f^2 - 8*b^2*c^3*f^3 - 4*((2*a*b - I*b^2)*d^3*e - (2*a*b - I*b^2)*c*
d^2*f)*(f*x + e)^3 + 6*((2*a*b - I*b^2)*d^3*e^2 - 2*(2*a*b - I*b^2)*c*d^2*e
*f + (2*a*b - I*b^2)*c^2*d*f^2)*(f*x + e)^2 - 4*(-I*b^2*d^3*e^3 + 3*I*b^2*c
*d^2*e^2*f - 3*I*b^2*c^2*d*e*f^2 + I*b^2*c^3*f^3)*(f*x + e) - 4*(2*(f*x + e
)^3*a*b*d^3 + 3*b^2*d^3*e^2 - 6*b^2*c*d^2*e*f + 3*b^2*c^2*d*f^2 - 3*(2*a*b*
d^3*e - 2*a*b*c*d^2*f - b^2*d^3)*(f*x + e)^2 + 6*(a*b*d^3*e^2 + a*b*c^2*d*f
^2 - b^2*d^3*e - (2*a*b*c*d^2*e - b^2*c*d^2)*f)*(f*x + e) - (2*(f*x + e)^3*
a*b*d^3 + 3*b^2*d^3*e^2 - 6*b^2*c*d^2*e*f + 3*b^2*c^2*d*f^2 - 3*(2*a*b*d^3*
e - 2*a*b*c*d^2*f - b^2*d^3)*(f*x + e)^2 + 6*(a*b*d^3*e^2 + a*b*c^2*d*f^2 -
b^2*d^3*e - (2*a*b*c*d^2*e - b^2*c*d^2)*f)*(f*x + e))*cos(2*f*x + 2*e) + (
-2*I*(f*x + e)^3*a*b*d^3 - 3*I*b^2*d^3*e^2 + 6*I*b^2*c*d^2*e*f - 3*I*b^2*c^
2*d*f^2 + 3*(2*I*a*b*d^3*e - 2*I*a*b*c*d^2*f - I*b^2*d^3)*(f*x + e)^2 + 6*(
-I*a*b*d^3*e^2 - I*a*b*c^2*d*f^2 + I*b^2*d^3*e + (2*I*a*b*c*d^2*e - I*b^2*c
*d^2)*f)*(f*x + e))*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*x + e) +
1) - 12*(b^2*d^3*e^2 - 2*b^2*c*d^2*e*f + b^2*c^2*d*f^2 - (b^2*d^3*e^2 - 2*b
^2*c*d^2*e*f + b^2*c^2*d*f^2)*cos(2*f*x + 2*e) + (-I*b^2*d^3*e^2 + 2*I*b^2*
c*d^2*e*f - I*b^2*c^2*d*f^2)*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*
x + e) - 1) + 4*(2*(f*x + e)^3*a*b*d^3 - 3*(2*a*b*d^3*e - 2*a*b*c*d^2*f - b
^2*d^3)*(f*x + e)^2 + 6*(a*b*d^3*e^2 + a*b*c^2*d*f^2 - b^2*d^3*e - (2*a*b*c
*d^2*e - b^2*c*d^2)*f)*(f*x + e) - (2*(f*x + e)^3*a*b*d^3 - 3*(2*a*b*d^3*e
- 2*a*b*c*d^2*f - b^2*d^3)*(f*x + e)^2 + 6*(a*b*d^3*e^2 + a*b*c^2*d*f^2 - b
^2*d^3*e - (2*a*b*c*d^2*e - b^2*c*d^2)*f)*(f*x + e))*cos(2*f*x + 2*e) - (2*
I*(f*x + e)^3*a*b*d^3 + 3*(-2*I*a*b*d^3*e + 2*I*a*b*c*d^2*f + I*b^2*d^3)*(f
*x + e)^2 + 6*(I*a*b*d^3*e^2 + I*a*b*c^2*d*f^2 - I*b^2*d^3*e + (-2*I*a*b*c*
d^2*e + I*b^2*c*d^2)*f)*(f*x + e))*sin(2*f*x + 2*e))*arctan2(sin(f*x + e),
-cos(f*x + e) + 1) - ((2*a*b - I*b^2)*(f*x + e)^4*d^3 + 4*(2*b^2*d^3 - (2*a
*b - I*b^2)*d^3*e + (2*a*b - I*b^2)*c*d^2*f)*(f*x + e)^3 - 6*(4*b^2*d^3*e -
(2*a*b - I*b^2)*d^3*e^2 - (2*a*b - I*b^2)*c^2*d*f^2 - 2*(2*b^2*c*d^2 - (2*
a*b - I*b^2)*c*d^2*e)*f)*(f*x + e)^2 + 4*(I*b^2*d^3*e^3 - I*b^2*c^3*f^3 + 6
*b^2*d^3*e^2 + 3*(I*b^2*c^2*d*e + 2*b^2*c^2*d)*f^2 + 3*(-I*b^2*c*d^2*e^2 -
4*b^2*c*d^2*e)*f)*(f*x + e))*cos(2*f*x + 2*e) + 24*((f*x + e)^2*a*b*d^3 + a
*b*d^3*e^2 + a*b*c^2*d*f^2 - b^2*d^3*e - (2*a*b*d^3*e - 2*a*b*c*d^2*f - b^2
*d^3)*(f*x + e) - (2*a*b*c*d^2*e - b^2*c*d^2)*f - ((f*x + e)^2*a*b*d^3 + a*
b*d^3*e^2 + a*b*c^2*d*f^2 - b^2*d^3*e - (2*a*b*d^3*e - 2*a*b*c*d^2*f - b^2*
d^3)*(f*x + e) - (2*a*b*c*d^2*e - b^2*c*d^2)*f)*cos(2*f*x + 2*e) - (I*(f*x
+ e)^2*a*b*d^3 + I*a*b*d^3*e^2 + I*a*b*c^2*d*f^2 - I*b^2*d^3*e + (-2*I*a*b*
d^3*e + 2*I*a*b*c*d^2*f + I*b^2*d^3)*(f*x + e) + (-2*I*a*b*c*d^2*e + I*b^2*
c*d^2)*f)*sin(2*f*x + 2*e))*dilog(-e^(I*f*x + I*e)) + 24*((f*x + e)^2*a*b*d
```

$$\begin{aligned}
&^3 + a*b*d^3*e^2 + a*b*c^2*d*f^2 - b^2*d^3*e - (2*a*b*d^3*e - 2*a*b*c*d^2*f \\
&- b^2*d^3)*(f*x + e) - (2*a*b*c*d^2*e - b^2*c*d^2)*f - ((f*x + e)^2*a*b*d^3 \\
&+ a*b*d^3*e^2 + a*b*c^2*d*f^2 - b^2*d^3*e - (2*a*b*d^3*e - 2*a*b*c*d^2*f \\
&- b^2*d^3)*(f*x + e) - (2*a*b*c*d^2*e - b^2*c*d^2)*f)*\cos(2*f*x + 2*e) - (I \\
&*(f*x + e)^2*a*b*d^3 + I*a*b*d^3*e^2 + I*a*b*c^2*d*f^2 - I*b^2*d^3*e + (-2* \\
&I*a*b*d^3*e + 2*I*a*b*c*d^2*f + I*b^2*d^3)*(f*x + e) + (-2*I*a*b*c*d^2*e + \\
&I*b^2*c*d^2)*f)*\sin(2*f*x + 2*e))*\operatorname{dilog}(e^{(I*f*x + I*e)}) - 2*(-2*I*(f*x + e) \\
&)^3*a*b*d^3 - 3*I*b^2*d^3*e^2 + 6*I*b^2*c*d^2*e*f - 3*I*b^2*c^2*d*f^2 + 3*(\\
&2*I*a*b*d^3*e - 2*I*a*b*c*d^2*f - I*b^2*d^3)*(f*x + e)^2 + 6*(-I*a*b*d^3*e^2 \\
&- I*a*b*c^2*d*f^2 + I*b^2*d^3*e + (2*I*a*b*c*d^2*e - I*b^2*c*d^2)*f)*(f*x \\
&+ e) + (2*I*(f*x + e)^3*a*b*d^3 + 3*I*b^2*d^3*e^2 - 6*I*b^2*c*d^2*e*f + 3* \\
&I*b^2*c^2*d*f^2 + 3*(-2*I*a*b*d^3*e + 2*I*a*b*c*d^2*f + I*b^2*d^3)*(f*x + e) \\
&)^2 + 6*(I*a*b*d^3*e^2 + I*a*b*c^2*d*f^2 - I*b^2*d^3*e + (-2*I*a*b*c*d^2*e \\
&+ I*b^2*c*d^2)*f)*(f*x + e))*\cos(2*f*x + 2*e) - (2*(f*x + e)^3*a*b*d^3 + 3* \\
&b^2*d^3*e^2 - 6*b^2*c*d^2*e*f + 3*b^2*c^2*d*f^2 - 3*(2*a*b*d^3*e - 2*a*b*c* \\
&d^2*f - b^2*d^3)*(f*x + e)^2 + 6*(a*b*d^3*e^2 + a*b*c^2*d*f^2 - b^2*d^3*e - \\
&(2*a*b*c*d^2*e - b^2*c*d^2)*f)*(f*x + e))*\sin(2*f*x + 2*e))*\log(\cos(f*x + \\
&e)^2 + \sin(f*x + e)^2 + 2*\cos(f*x + e) + 1) - 2*(-2*I*(f*x + e)^3*a*b*d^3 - \\
&3*I*b^2*d^3*e^2 + 6*I*b^2*c*d^2*e*f - 3*I*b^2*c^2*d*f^2 + 3*(2*I*a*b*d^3*e \\
&- 2*I*a*b*c*d^2*f - I*b^2*d^3)*(f*x + e)^2 + 6*(-I*a*b*d^3*e^2 - I*a*b*c^2 \\
&*d*f^2 + I*b^2*d^3*e + (2*I*a*b*c*d^2*e - I*b^2*c*d^2)*f)*(f*x + e) + (2*I* \\
&(f*x + e)^3*a*b*d^3 + 3*I*b^2*d^3*e^2 - 6*I*b^2*c*d^2*e*f + 3*I*b^2*c^2*d*f \\
&^2 + 3*(-2*I*a*b*d^3*e + 2*I*a*b*c*d^2*f + I*b^2*d^3)*(f*x + e)^2 + 6*(I*a* \\
&b*d^3*e^2 + I*a*b*c^2*d*f^2 - I*b^2*d^3*e + (-2*I*a*b*c*d^2*e + I*b^2*c*d^2 \\
&)*f)*(f*x + e))*\cos(2*f*x + 2*e) - (2*(f*x + e)^3*a*b*d^3 + 3*b^2*d^3*e^2 - \\
&6*b^2*c*d^2*e*f + 3*b^2*c^2*d*f^2 - 3*(2*a*b*d^3*e - 2*a*b*c*d^2*f - b^2*d \\
&^3)*(f*x + e)^2 + 6*(a*b*d^3*e^2 + a*b*c^2*d*f^2 - b^2*d^3*e - (2*a*b*c*d^2 \\
&*e - b^2*c*d^2)*f)*(f*x + e))*\sin(2*f*x + 2*e))*\log(\cos(f*x + e)^2 + \sin(f* \\
&x + e)^2 - 2*\cos(f*x + e) + 1) + 48*(a*b*d^3*\cos(2*f*x + 2*e) + I*a*b*d^3*s \\
&\sin(2*f*x + 2*e) - a*b*d^3)*\operatorname{polylog}(4, -e^{(I*f*x + I*e)}) + 48*(a*b*d^3*\cos(2 \\
&*f*x + 2*e) + I*a*b*d^3*\sin(2*f*x + 2*e) - a*b*d^3)*\operatorname{polylog}(4, e^{(I*f*x + I \\
&e)}) - 24*(-2*I*(f*x + e)*a*b*d^3 + 2*I*a*b*d^3*e - 2*I*a*b*c*d^2*f - I*b^2 \\
&*d^3 + (2*I*(f*x + e)*a*b*d^3 - 2*I*a*b*d^3*e + 2*I*a*b*c*d^2*f + I*b^2*d^3 \\
&)*\cos(2*f*x + 2*e) - (2*(f*x + e)*a*b*d^3 - 2*a*b*d^3*e + 2*a*b*c*d^2*f + b \\
&^2*d^3)*\sin(2*f*x + 2*e))*\operatorname{polylog}(3, -e^{(I*f*x + I*e)}) - 24*(-2*I*(f*x + e) \\
&*a*b*d^3 + 2*I*a*b*d^3*e - 2*I*a*b*c*d^2*f - I*b^2*d^3 + (2*I*(f*x + e)*a*b \\
&*d^3 - 2*I*a*b*d^3*e + 2*I*a*b*c*d^2*f + I*b^2*d^3)*\cos(2*f*x + 2*e) - (2*(\\
&f*x + e)*a*b*d^3 - 2*a*b*d^3*e + 2*a*b*c*d^2*f + b^2*d^3)*\sin(2*f*x + 2*e)) \\
&*\operatorname{polylog}(3, e^{(I*f*x + I*e)}) + ((-2*I*a*b - b^2)*(f*x + e)^4*d^3 - 4*(2*I*b \\
&^2*d^3 + (-2*I*a*b - b^2)*d^3*e + (2*I*a*b + b^2)*c*d^2*f)*(f*x + e)^3 - 6* \\
&(-4*I*b^2*d^3*e + (2*I*a*b + b^2)*d^3*e^2 + (2*I*a*b + b^2)*c^2*d*f^2 + 2*(\\
&2*I*b^2*c*d^2 + (-2*I*a*b - b^2)*c*d^2*e)*f)*(f*x + e)^2 + 4*(b^2*d^3*e^3 - \\
&b^2*c^3*f^3 - 6*I*b^2*d^3*e^2 + 3*(b^2*c^2*d*e - 2*I*b^2*c^2*d)*f^2 - 3*(b \\
&^2*c*d^2*e^2 - 4*I*b^2*c*d^2*e)*f)*(f*x + e))*\sin(2*f*x + 2*e))/(-4*I*f^3*c \\
&\os(2*f*x + 2*e) + 4*f^3*\sin(2*f*x + 2*e) + 4*I*f^3))/f
\end{aligned}$$

Giac [F]

$$\int (c + dx)^3 (a + b \cot(e + fx))^2 dx = \int (dx + c)^3 (b \cot(fx + e) + a)^2 dx$$

[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^2,x, algorithm="giac")

[Out] integrate((d*x + c)^3*(b*cot(f*x + e) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (c + dx)^3 (a + b \cot(e + fx))^2 dx = \int (a + b \cot(e + fx))^2 (c + dx)^3 dx$$

[In] int((a + b*cot(e + f*x))^2*(c + d*x)^3,x)

[Out] int((a + b*cot(e + f*x))^2*(c + d*x)^3, x)

3.43 $\int (c + dx)^2 (a + b \cot(e + fx))^2 dx$

Optimal result	284
Rubi [A] (verified)	285
Mathematica [B] (warning: unable to verify)	288
Maple [B] (verified)	290
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Optimal result

Integrand size = 20, antiderivative size = 227

$$\int (c + dx)^2 (a + b \cot(e + fx))^2 dx = -\frac{ib^2(c + dx)^2}{f} + \frac{a^2(c + dx)^3}{3d} - \frac{2iab(c + dx)^3}{3d} - \frac{b^2(c + dx)^3}{3d} - \frac{b^2(c + dx)^2 \cot(e + fx)}{f} + \frac{2b^2d(c + dx) \log(1 - e^{2i(e+fx)})}{f^2} + \frac{2ab(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f} - \frac{ib^2d^2 \text{PolyLog}(2, e^{2i(e+fx)})}{f^3} - \frac{2iabd(c + dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{abd^2 \text{PolyLog}(3, e^{2i(e+fx)})}{f^3}$$

```
[Out] -I*b^2*(d*x+c)^2/f+1/3*a^2*(d*x+c)^3/d-2/3*I*a*b*(d*x+c)^3/d-1/3*b^2*(d*x+c)^3/d-b^2*(d*x+c)^2*cot(f*x+e)/f+2*b^2*d*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f^2+2*a*b*(d*x+c)^2*ln(1-exp(2*I*(f*x+e)))/f-I*b^2*d^2*polylog(2,exp(2*I*(f*x+e)))/f^3-2*I*a*b*d*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/f^2+a*b*d^2*polylog(3,exp(2*I*(f*x+e)))/f^3
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3803, 3798, 2221, 2611, 2320, 6724, 3801, 2317, 2438, 32}

$$\int (c + dx)^2 (a + b \cot(e + fx))^2 dx = \frac{a^2(c + dx)^3}{3d} - \frac{2iabd(c + dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{2ab(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f} - \frac{2iab(c + dx)^3}{3d} + \frac{abd^2 \text{PolyLog}(3, e^{2i(e+fx)})}{f^3} + \frac{2b^2d(c + dx) \log(1 - e^{2i(e+fx)})}{f^2} - \frac{b^2(c + dx)^2 \cot(e + fx)}{f} - \frac{ib^2(c + dx)^2}{f} - \frac{b^2(c + dx)^3}{3d} - \frac{ib^2d^2 \text{PolyLog}(2, e^{2i(e+fx)})}{f^3}$$

[In] Int[(c + d*x)^2*(a + b*Cot[e + f*x])^2,x]

[Out] ((-I)*b^2*(c + d*x)^2)/f + (a^2*(c + d*x)^3)/(3*d) - (((2*I)/3)*a*b*(c + d*x)^3)/d - (b^2*(c + d*x)^3)/(3*d) - (b^2*(c + d*x)^2*Cot[e + f*x])/f + (2*b^2*d*(c + d*x)*Log[1 - E^((2*I)*(e + f*x))])/f^2 + (2*a*b*(c + d*x)^2*Log[1 - E^((2*I)*(e + f*x))])/f - (I*b^2*d^2*PolyLog[2, E^((2*I)*(e + f*x))])/f^3 - ((2*I)*a*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + (a*b*d^2*PolyLog[3, E^((2*I)*(e + f*x))])/f^3

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2221

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.), x_Symbol] :> Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2317

Int[Log[(a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))]

)ⁿ], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2438

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 2611

Int[Log[1 + (e_)*((F_)^(c_)*((a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)*(x_)^(m_)), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 3798

Int[((c_) + (d_)*(x_)^(m_))*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 3801

Int[((c_) + (d_)*(x_)^(m_))*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Dist[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3803

Int[((c_) + (d_)*(x_)^(m_))*((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (a^2(c + dx)^2 + 2ab(c + dx)^2 \cot(e + fx) + b^2(c + dx)^2 \cot^2(e + fx)) dx \\
&= \frac{a^2(c + dx)^3}{3d} + (2ab) \int (c + dx)^2 \cot(e + fx) dx + b^2 \int (c + dx)^2 \cot^2(e + fx) dx \\
&= \frac{a^2(c + dx)^3}{3d} - \frac{2iab(c + dx)^3}{3d} - \frac{b^2(c + dx)^2 \cot(e + fx)}{f} \\
&\quad - (4iab) \int \frac{e^{2i(e+fx)}(c + dx)^2}{1 - e^{2i(e+fx)}} dx - b^2 \int (c + dx)^2 dx \\
&\quad + \frac{(2b^2d) \int (c + dx) \cot(e + fx) dx}{f} \\
&= -\frac{ib^2(c + dx)^2}{f} + \frac{a^2(c + dx)^3}{3d} - \frac{2iab(c + dx)^3}{3d} - \frac{b^2(c + dx)^3}{3d} \\
&\quad - \frac{b^2(c + dx)^2 \cot(e + fx)}{f} + \frac{2ab(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f} \\
&\quad - \frac{(4abd) \int (c + dx) \log(1 - e^{2i(e+fx)}) dx}{f} - \frac{(4ib^2d) \int \frac{e^{2i(e+fx)}(c+dx)}{1 - e^{2i(e+fx)}} dx}{f} \\
&= -\frac{ib^2(c + dx)^2}{f} + \frac{a^2(c + dx)^3}{3d} - \frac{2iab(c + dx)^3}{3d} - \frac{b^2(c + dx)^3}{3d} \\
&\quad - \frac{b^2(c + dx)^2 \cot(e + fx)}{f} + \frac{2b^2d(c + dx) \log(1 - e^{2i(e+fx)})}{f^2} \\
&\quad + \frac{2ab(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f} - \frac{2iabd(c + dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} \\
&\quad + \frac{(2iabd^2) \int \text{PolyLog}(2, e^{2i(e+fx)}) dx}{f^2} - \frac{(2b^2d^2) \int \log(1 - e^{2i(e+fx)}) dx}{f^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{ib^2(c+dx)^2}{f} + \frac{a^2(c+dx)^3}{3d} - \frac{2iab(c+dx)^3}{3d} - \frac{b^2(c+dx)^3}{3d} \\
&\quad - \frac{b^2(c+dx)^2 \cot(e+fx)}{f} + \frac{2b^2d(c+dx) \log(1-e^{2i(e+fx)})}{f^2} \\
&\quad + \frac{2ab(c+dx)^2 \log(1-e^{2i(e+fx)})}{f} - \frac{2iabd(c+dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} \\
&\quad + \frac{(abd^2) \text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x} dx, x, e^{2i(e+fx)}\right)}{f^3} \\
&\quad + \frac{(ib^2d^2) \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2i(e+fx)}\right)}{f^3} \\
&= -\frac{ib^2(c+dx)^2}{f} + \frac{a^2(c+dx)^3}{3d} - \frac{2iab(c+dx)^3}{3d} - \frac{b^2(c+dx)^3}{3d} \\
&\quad - \frac{b^2(c+dx)^2 \cot(e+fx)}{f} + \frac{2b^2d(c+dx) \log(1-e^{2i(e+fx)})}{f^2} \\
&\quad + \frac{2ab(c+dx)^2 \log(1-e^{2i(e+fx)})}{f} - \frac{ib^2d^2 \text{PolyLog}(2, e^{2i(e+fx)})}{f^3} \\
&\quad - \frac{2iabd(c+dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{abd^2 \text{PolyLog}(3, e^{2i(e+fx)})}{f^3}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 737 vs. $2(227) = 454$.

Time = 7.15 (sec) , antiderivative size = 737, normalized size of antiderivative = 3.25

$$\int (c + dx)^2 (a + b \cot(e + fx))^2 dx =$$

$$\frac{abd^2 e^{ie} \csc(e) (2e^{-2ie} f^3 x^3 + 3i(1 - e^{-2ie}) f^2 x^2 \log(1 - e^{-i(e+fx)}) + 3i(1 - e^{-2ie}) f^2 x^2 \log(1 + e^{-i(e+fx)}))}{1} + \frac{1}{3} x (3c^2 + 3cdx + d^2 x^2) \csc(e) (2ab \cos(e) + a^2 \sin(e) - b^2 \sin(e))$$

$$+ \frac{2b^2 cd \csc(e) (-fx \cos(e) + \log(\cos(fx) \sin(e) + \cos(e) \sin(fx))) \sin(e)}{f^2 (\cos^2(e) + \sin^2(e))}$$

$$+ \frac{2abc^2 \csc(e) (-fx \cos(e) + \log(\cos(fx) \sin(e) + \cos(e) \sin(fx))) \sin(e)}{f (\cos^2(e) + \sin^2(e))}$$

$$+ \frac{\csc(e) \csc(e + fx) (b^2 c^2 \sin(fx) + 2b^2 cdx \sin(fx) + b^2 d^2 x^2 \sin(fx))}{f}$$

$$+ \frac{b^2 d^2 \csc(e) \sec(e) \left(e^{i \arctan(\tan(e))} f^2 x^2 + \frac{(ifx(-\pi + 2 \arctan(\tan(e))) - \pi \log(1 + e^{-2ifx}) - 2(fx + \arctan(\tan(e))) \log(1 - e^{2i(fx + \arctan(\tan(e))))}{f^3 \sqrt{\sec^2(e) (\cos^2(e) + \sin^2(e))}} \right)}{f^3 \sqrt{\sec^2(e) (\cos^2(e) + \sin^2(e))}}$$

$$+ \frac{2abcd \csc(e) \sec(e) \left(e^{i \arctan(\tan(e))} f^2 x^2 + \frac{(ifx(-\pi + 2 \arctan(\tan(e))) - \pi \log(1 + e^{-2ifx}) - 2(fx + \arctan(\tan(e))) \log(1 - e^{2i(fx + \arctan(\tan(e))))}{f^2 \sqrt{\sec^2(e) (\cos^2(e) + \sin^2(e))}} \right)}{f^2 \sqrt{\sec^2(e) (\cos^2(e) + \sin^2(e))}}$$

[In] Integrate[(c + d*x)^2*(a + b*Cot[e + f*x])^2,x]

[Out] -1/3*(a*b*d^2*E^(I*e)*Csc[e]*((2*f^3*x^3)/E^((2*I)*e) + (3*I)*(1 - E^((-2*I)*e)))*f^2*x^2*Log[1 - E^((-I)*(e + f*x))] + (3*I)*(1 - E^((-2*I)*e))*f^2*x^2*Log[1 + E^((-I)*(e + f*x))] - 6*(1 - E^((-2*I)*e))*f*x*PolyLog[2, -E^((-I)*(e + f*x))] - 6*(1 - E^((-2*I)*e))*f*x*PolyLog[2, E^((-I)*(e + f*x))] + (6*I)*(1 - E^((-2*I)*e))*PolyLog[3, -E^((-I)*(e + f*x))] + (6*I)*(1 - E^((-2*I)*e))*PolyLog[3, E^((-I)*(e + f*x))]/f^3 + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Csc[e]*(2*a*b*Cos[e] + a^2*Sin[e] - b^2*Sin[e])/3 + (2*b^2*c*d*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x])*Sin[e]))/(f^2*(Cos[e]^2 + Sin[e]^2)) + (2*a*b*c^2*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x])*Sin[e]))/(f*(Cos[e]^2 + Sin[e]^2)) + (Csc[e]*Csc[e + f*x]*(b^2*c^2*Sin[f*x] + 2*b^2*c*d*x*Sin[f*x] + b^2*d^2*x^2*Sin[f*x]))/f - (b^2*d^2*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]]))*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x]) - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]])])) + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]]) + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]])]))*Tan[e])/Sqrt[1 + Tan[e]^2]]/(f^3*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2))] - (2*a*b*c*d*Csc[e]*Sec[e]*(E^(I*ArcTan[Tan[e]]))*f^2*x^2 + ((I*f*x*(-Pi + 2*ArcTan[Tan[e]]) - Pi*Log[1 + E^((-2*I)*f*x]) - 2*(f*x + ArcTan[Tan[e]])*Log[1 - E^((2*I)*(f*x + ArcTan[Tan[e]])])) + Pi*Log[Cos[f*x]] + 2*ArcTan[Tan[e]]*Log[Sin[f*x + ArcTan[Tan[e]]]]) + I*PolyLog[2, E^((2*I)*(f*x + ArcTan[Tan[e]])]))*Tan[e])/Sqrt[1 + Tan[e]^2]]/(f^2*Sqrt[Sec[e]^2*(Cos[e]^2 + Sin[e]^2))])

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(207) = 414$.

Time = 0.80 (sec) , antiderivative size = 932, normalized size of antiderivative = 4.11

method	result	size
risch	Expression too large to display	932

[In] `int((d*x+c)^2*(a+b*cot(f*x+e))^2,x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & d^2 a^2 c x^2 + a^2 c^2 x - d b^2 c x^2 + 4/f^3 b^2 e d^2 \ln(\exp(I(f*x+e))) - 2/f^3 b^2 e d^2 \ln(\exp(I(f*x+e))-1) \\ & + 2/f b^2 a c^2 \ln(\exp(I(f*x+e))+1) - 4/f b^2 a c^2 \ln(\exp(I(f*x+e))) + 2/f b^2 a c^2 \ln(\exp(I(f*x+e))-1) \\ & + 2/f^2 b^2 c d \ln(\exp(I(f*x+e))+1) - 4/f^2 b^2 c d \ln(\exp(I(f*x+e))) + 2/f^2 b^2 c d \ln(\exp(I(f*x+e))-1) \\ & + 4/f^3 b^2 a d^2 \operatorname{polylog}(3, \exp(I(f*x+e))) + 4/f^3 b^2 a d^2 \operatorname{polylog}(3, -\exp(I(f*x+e))) \\ & + 2/f^3 b^2 d^2 \ln(1 - \exp(I(f*x+e))) e + 2/f^2 b^2 d^2 \ln(1 - \exp(I(f*x+e))) x + 2/f^2 b^2 d^2 \ln(\exp(I(f*x+e))+1) x \\ & - 2I/f b^2 d^2 x^2 - 2I/f^3 b^2 d^2 e^2 - 2I/f^3 b^2 d^2 \operatorname{polylog}(2, \exp(I(f*x+e))) - 2I/f^3 b^2 d^2 \operatorname{polylog}(2, -\exp(I(f*x+e))) \\ & - 2/3 I d^2 a b x^3 - 2I b^2 (d^2 x^2 + 2 c d x + c^2) / f / (\exp(2 I (f*x+e)) - 1) - 2/f^3 b^2 a d^2 \ln(1 - \exp(I(f*x+e))) e^2 + 2/f b^2 a d^2 \ln(\exp(I(f*x+e))+1) x^2 \\ & + 2/f b^2 a d^2 \ln(1 - \exp(I(f*x+e))) x^2 - 4/f^3 b^2 e^2 a d^2 \ln(\exp(I(f*x+e))) + 2/f^3 b^2 e^2 a d^2 \ln(\exp(I(f*x+e))-1) \\ & + 8/3 I / f^3 b^2 a d^2 e^3 - 4 I / f^2 b^2 d^2 e x - 2 I d^2 a b c x^2 - 8 I / f b^2 d^2 c a e x - 4 I / f^2 b^2 a d^2 \operatorname{polylog}(2, -\exp(I(f*x+e))) x \\ & + 4 I / f^2 b^2 a d^2 e^2 x - 4 I / f^2 b^2 a d^2 \operatorname{polylog}(2, \exp(I(f*x+e))) x - 4 I / f^2 b^2 d^2 c a e^2 + 8 / f^2 b^2 e a c d \ln(\exp(I(f*x+e))) - 4 / f^2 b^2 e a c d \ln(\exp(I(f*x+e))-1) \\ & + 4 / f b^2 d^2 c a \ln(1 - \exp(I(f*x+e))) x + 4 / f b^2 d^2 c a \ln(\exp(I(f*x+e))+1) x + 4 / f^2 b^2 d^2 c a \ln(1 - \exp(I(f*x+e))) e - 4 I / f^2 b^2 d^2 c a \operatorname{polylog}(2, \exp(I(f*x+e))) \\ & - 4 I / f^2 b^2 d^2 c a \operatorname{polylog}(2, -\exp(I(f*x+e))) + 2 I a b c^2 x + 2/3 I / d a b c^3 + 1/3 d^2 a^2 x^3 + 1/3 / d a^2 c^3 - 1/3 d^2 b^2 x^3 - b^2 c^2 x - 1/3 / d b^2 c^3 \end{aligned}$$

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(201) = 402$.

Time = 0.29 (sec) , antiderivative size = 715, normalized size of antiderivative = 3.15

$$\int (c + dx)^2 (a + b \cot(e + fx))^2 dx = \frac{6 b^2 d^2 f^2 x^2 + 12 b^2 c d f^2 x + 6 b^2 c^2 f^2 - 3 a b d^2 \operatorname{polylog}(3, \cos(2fx + 2e) + i \sin(2fx + 2e)) \sin(2fx + 2e)}{\dots}$$

[In] `integrate((d*x+c)^2*(a+b*cot(f*x+e))^2,x, algorithm="fricas")`

```
[Out] -1/6*(6*b^2*d^2*f^2*x^2 + 12*b^2*c*d*f^2*x + 6*b^2*c^2*f^2 - 3*a*b*d^2*poly
log(3, cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e))*sin(2*f*x + 2*e) - 3*a*b*d^2*
polylog(3, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e))*sin(2*f*x + 2*e) + 3*(2*I
*a*b*d^2*f*x + 2*I*a*b*c*d*f + I*b^2*d^2)*dilog(cos(2*f*x + 2*e) + I*sin(2*
f*x + 2*e))*sin(2*f*x + 2*e) + 3*(-2*I*a*b*d^2*f*x - 2*I*a*b*c*d*f - I*b^2*
d^2)*dilog(cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e))*sin(2*f*x + 2*e) - 6*(a*b
*d^2*e^2 + a*b*c^2*f^2 - b^2*d^2*e - (2*a*b*c*d*e - b^2*c*d)*f)*log(-1/2*co
s(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2)*sin(2*f*x + 2*e) - 6*(a*b*d^
2*e^2 + a*b*c^2*f^2 - b^2*d^2*e - (2*a*b*c*d*e - b^2*c*d)*f)*log(-1/2*cos(2
*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e) + 1/2)*sin(2*f*x + 2*e) - 6*(a*b*d^2*f
^2*x^2 - a*b*d^2*e^2 + 2*a*b*c*d*e*f + b^2*d^2*e + (2*a*b*c*d*f^2 + b^2*d^2
*f)*x)*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e) - 6
*(a*b*d^2*f^2*x^2 - a*b*d^2*e^2 + 2*a*b*c*d*e*f + b^2*d^2*e + (2*a*b*c*d*f^
2 + b^2*d^2*f)*x)*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1)*sin(2*f*x
+ 2*e) + 6*(b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*cos(2*f*x + 2
*e) - 2*((a^2 - b^2)*d^2*f^3*x^3 + 3*(a^2 - b^2)*c*d*f^3*x^2 + 3*(a^2 - b^2
)*c^2*f^3*x)*sin(2*f*x + 2*e))/(f^3*sin(2*f*x + 2*e))
```

Sympy [F]

$$\int (c + dx)^2 (a + b \cot(e + fx))^2 dx = \int (a + b \cot(e + fx))^2 (c + dx)^2 dx$$

```
[In] integrate((d*x+c)**2*(a+b*cot(f*x+e))**2,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**2*(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1948 vs. $2(201) = 402$.

Time = 0.59 (sec) , antiderivative size = 1948, normalized size of antiderivative = 8.58

$$\int (c + dx)^2 (a + b \cot(e + fx))^2 dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a^2*c^2 + (f*x + e)^3*a^2*d^2/f^2 - 3*(f*x + e)^2*a^2*d^2*
e/f^2 + 3*(f*x + e)*a^2*d^2*e^2/f^2 + 3*(f*x + e)^2*a^2*c*d/f - 6*(f*x + e)
*a^2*c*d*e/f + 6*a*b*c^2*log(sin(f*x + e)) + 6*a*b*d^2*e^2*log(sin(f*x + e)
)/f^2 - 12*a*b*c*d*e*log(sin(f*x + e))/f + 3*((2*a*b - I*b^2)*(f*x + e)^3*d
^2 - 6*b^2*d^2*e^2 + 12*b^2*c*d*e*f - 6*b^2*c^2*f^2 - 3*((2*a*b - I*b^2)*d^
```

$$\begin{aligned}
& 2*e - (2*a*b - I*b^2)*c*d*f*(f*x + e)^2 - 3*(I*b^2*d^2*e^2 - 2*I*b^2*c*d*e \\
& *f + I*b^2*c^2*f^2)*(f*x + e) - 6*((f*x + e)^2*a*b*d^2 - b^2*d^2*e + b^2*c* \\
& d*f - (2*a*b*d^2*e - 2*a*b*c*d*f - b^2*d^2)*(f*x + e) - ((f*x + e)^2*a*b*d^ \\
& 2 - b^2*d^2*e + b^2*c*d*f - (2*a*b*d^2*e - 2*a*b*c*d*f - b^2*d^2)*(f*x + e) \\
&)*\cos(2*f*x + 2*e) + (-I*(f*x + e)^2*a*b*d^2 + I*b^2*d^2*e - I*b^2*c*d*f + \\
& (2*I*a*b*d^2*e - 2*I*a*b*c*d*f - I*b^2*d^2)*(f*x + e))*\sin(2*f*x + 2*e))*\ar \\
& \tan^2(\sin(f*x + e), \cos(f*x + e) + 1) + 6*(b^2*d^2*e - b^2*c*d*f - (b^2*d^2 \\
& *e - b^2*c*d*f)*\cos(2*f*x + 2*e) - (I*b^2*d^2*e - I*b^2*c*d*f)*\sin(2*f*x + \\
& 2*e))*\arctan^2(\sin(f*x + e), \cos(f*x + e) - 1) + 6*((f*x + e)^2*a*b*d^2 - (2 \\
& *a*b*d^2*e - 2*a*b*c*d*f - b^2*d^2)*(f*x + e) - ((f*x + e)^2*a*b*d^2 - (2*a \\
& *b*d^2*e - 2*a*b*c*d*f - b^2*d^2)*(f*x + e))*\cos(2*f*x + 2*e) - (I*(f*x + e \\
&)^2*a*b*d^2 + (-2*I*a*b*d^2*e + 2*I*a*b*c*d*f + I*b^2*d^2)*(f*x + e))*\sin(2 \\
& *f*x + 2*e))*\arctan^2(\sin(f*x + e), -\cos(f*x + e) + 1) - ((2*a*b - I*b^2)*(f \\
& *x + e)^3*d^2 + 3*(2*b^2*d^2 - (2*a*b - I*b^2)*d^2*e + (2*a*b - I*b^2)*c*d* \\
& f)*(f*x + e)^2 + 3*(-I*b^2*d^2*e^2 - I*b^2*c^2*f^2 - 4*b^2*d^2*e + 2*(I*b^2 \\
& *c*d*e + 2*b^2*c*d)*f)*(f*x + e))*\cos(2*f*x + 2*e) + 6*(2*(f*x + e)*a*b*d^2 \\
& - 2*a*b*d^2*e + 2*a*b*c*d*f + b^2*d^2 - (2*(f*x + e)*a*b*d^2 - 2*a*b*d^2*e \\
& + 2*a*b*c*d*f + b^2*d^2)*\cos(2*f*x + 2*e) - (2*I*(f*x + e)*a*b*d^2 - 2*I*a \\
& *b*d^2*e + 2*I*a*b*c*d*f + I*b^2*d^2)*\sin(2*f*x + 2*e))*\operatorname{dilog}(-e^{(I*f*x + I \\
& *e)}) + 6*(2*(f*x + e)*a*b*d^2 - 2*a*b*d^2*e + 2*a*b*c*d*f + b^2*d^2 - (2*(f \\
& *x + e)*a*b*d^2 - 2*a*b*d^2*e + 2*a*b*c*d*f + b^2*d^2)*\cos(2*f*x + 2*e) - (\\
& 2*I*(f*x + e)*a*b*d^2 - 2*I*a*b*d^2*e + 2*I*a*b*c*d*f + I*b^2*d^2)*\sin(2*f* \\
& x + 2*e))*\operatorname{dilog}(e^{(I*f*x + I*e)}) - 3*(-I*(f*x + e)^2*a*b*d^2 + I*b^2*d^2*e \\
& - I*b^2*c*d*f + (2*I*a*b*d^2*e - 2*I*a*b*c*d*f - I*b^2*d^2)*(f*x + e) + (I \\
& (f*x + e)^2*a*b*d^2 - I*b^2*d^2*e + I*b^2*c*d*f + (-2*I*a*b*d^2*e + 2*I*a*b \\
& *c*d*f + I*b^2*d^2)*(f*x + e))*\cos(2*f*x + 2*e) - ((f*x + e)^2*a*b*d^2 - b^ \\
& 2*d^2*e + b^2*c*d*f - (2*a*b*d^2*e - 2*a*b*c*d*f - b^2*d^2)*(f*x + e))*\sin(\\
& 2*f*x + 2*e))*\log(\cos(f*x + e)^2 + \sin(f*x + e)^2 + 2*\cos(f*x + e) + 1) - 3 \\
& *(-I*(f*x + e)^2*a*b*d^2 + I*b^2*d^2*e - I*b^2*c*d*f + (2*I*a*b*d^2*e - 2*I \\
& *a*b*c*d*f - I*b^2*d^2)*(f*x + e) + (I*(f*x + e)^2*a*b*d^2 - I*b^2*d^2*e + \\
& I*b^2*c*d*f + (-2*I*a*b*d^2*e + 2*I*a*b*c*d*f + I*b^2*d^2)*(f*x + e))*\cos(2 \\
& *f*x + 2*e) - ((f*x + e)^2*a*b*d^2 - b^2*d^2*e + b^2*c*d*f - (2*a*b*d^2*e - \\
& 2*a*b*c*d*f - b^2*d^2)*(f*x + e))*\sin(2*f*x + 2*e))*\log(\cos(f*x + e)^2 + s \\
& \sin(f*x + e)^2 - 2*\cos(f*x + e) + 1) - 12*(I*a*b*d^2*\cos(2*f*x + 2*e) - a*b* \\
& d^2*\sin(2*f*x + 2*e) - I*a*b*d^2)*\operatorname{polylog}(3, -e^{(I*f*x + I*e)}) - 12*(I*a*b* \\
& d^2*\cos(2*f*x + 2*e) - a*b*d^2*\sin(2*f*x + 2*e) - I*a*b*d^2)*\operatorname{polylog}(3, e^{(\\
& I*f*x + I*e)}) + ((-2*I*a*b - b^2)*(f*x + e)^3*d^2 - 3*(2*I*b^2*d^2 + (-2*I* \\
& a*b - b^2)*d^2*e + (2*I*a*b + b^2)*c*d*f)*(f*x + e)^2 - 3*(b^2*d^2*e^2 + b^ \\
& 2*c^2*f^2 - 4*I*b^2*d^2*e - 2*(b^2*c*d*e - 2*I*b^2*c*d)*f)*(f*x + e))*\sin(2 \\
& *f*x + 2*e))/(-3*I*f^2*\cos(2*f*x + 2*e) + 3*f^2*\sin(2*f*x + 2*e) + 3*I*f^2) \\
&)/f
\end{aligned}$$

Giac [F]

$$\int (c + dx)^2 (a + b \cot(e + fx))^2 dx = \int (dx + c)^2 (b \cot(fx + e) + a)^2 dx$$

[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2*(b*cot(f*x + e) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (c + dx)^2 (a + b \cot(e + fx))^2 dx = \int (a + b \cot(e + fx))^2 (c + dx)^2 dx$$

[In] int((a + b*cot(e + f*x))^2*(c + d*x)^2,x)

[Out] int((a + b*cot(e + f*x))^2*(c + d*x)^2, x)

3.44 $\int (c + dx)(a + b \cot(e + fx))^2 dx$

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Rubi [A] (verified)	294
Mathematica [A] (verified)	297
Maple [B] (verified)	297
Fricas [B] (verification not implemented)	298
Sympy [F]	298
Maxima [B] (verification not implemented)	298
Giac [F]	299
Mupad [F(-1)]	299

Optimal result

Integrand size = 18, antiderivative size = 137

$$\int (c + dx)(a + b \cot(e + fx))^2 dx = -b^2 cx - \frac{1}{2} b^2 dx^2 + \frac{a^2(c + dx)^2}{2d} - \frac{iab(c + dx)^2}{d} - \frac{b^2(c + dx) \cot(e + fx)}{f} + \frac{2ab(c + dx) \log(1 - e^{2i(e+fx)})}{f} + \frac{b^2 d \log(\sin(e + fx))}{f^2} - \frac{iabd \operatorname{PolyLog}(2, e^{2i(e+fx)})}{f^2}$$

[Out] $-b^2*c*x - 1/2*b^2*d*x^2 + 1/2*a^2*(d*x+c)^2/d - I*a*b*(d*x+c)^2/d - b^2*(d*x+c)*\cot(f*x+e)/f + 2*a*b*(d*x+c)*\ln(1-\exp(2*I*(f*x+e)))/f + b^2*d*\ln(\sin(f*x+e))/f^2 - I*a*b*d*\operatorname{polylog}(2, \exp(2*I*(f*x+e)))/f^2$

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3803, 3798, 2221, 2317, 2438, 3801, 3556}

$$\int (c + dx)(a + b \cot(e + fx))^2 dx = \frac{a^2(c + dx)^2}{2d} + \frac{2ab(c + dx) \log(1 - e^{2i(e+fx)})}{f} - \frac{iab(c + dx)^2}{d} - \frac{iabd \operatorname{PolyLog}(2, e^{2i(e+fx)})}{f^2} - \frac{b^2(c + dx) \cot(e + fx)}{f} - b^2 cx + \frac{b^2 d \log(\sin(e + fx))}{f^2} - \frac{1}{2} b^2 dx^2$$

[In] Int[(c + d*x)*(a + b*Cot[e + f*x])^2,x]

[Out] $-(b^2*c*x) - (b^2*d*x^2)/2 + (a^2*(c + d*x)^2)/(2*d) - (I*a*b*(c + d*x)^2)/d - (b^2*(c + d*x)*\text{Cot}[e + f*x])/f + (2*a*b*(c + d*x)*\text{Log}[1 - E^{((2*I)*(e + f*x)})])/f + (b^2*d*\text{Log}[\text{Sin}[e + f*x]])/f^2 - (I*a*b*d*\text{PolyLog}[2, E^{((2*I)*(e + f*x)})])/f^2$

Rule 2221

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[(c + d*x)^m/(b*f*g*n*Log[F])*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2317

Int[Log[(a_) + (b_)*((F_)^((e_)*(c_) + (d_)*(x_)))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2438

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 3556

Int[tan[(c_) + (d_)*(x_)], x_Symbol] := Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3798

Int[(((c_) + (d_)*(x_))^(m_))*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 3801

Int[(((c_) + (d_)*(x_))^(m_))*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Dist[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (a^2(c + dx) + 2ab(c + dx) \cot(e + fx) + b^2(c + dx) \cot^2(e + fx)) dx \\
&= \frac{a^2(c + dx)^2}{2d} + (2ab) \int (c + dx) \cot(e + fx) dx + b^2 \int (c + dx) \cot^2(e + fx) dx \\
&= \frac{a^2(c + dx)^2}{2d} - \frac{iab(c + dx)^2}{d} - \frac{b^2(c + dx) \cot(e + fx)}{f} \\
&\quad - (4iab) \int \frac{e^{2i(e+fx)}(c + dx)}{1 - e^{2i(e+fx)}} dx - b^2 \int (c + dx) dx + \frac{(b^2d) \int \cot(e + fx) dx}{f} \\
&= -b^2cx - \frac{1}{2}b^2dx^2 + \frac{a^2(c + dx)^2}{2d} - \frac{iab(c + dx)^2}{d} \\
&\quad - \frac{b^2(c + dx) \cot(e + fx)}{f} + \frac{2ab(c + dx) \log(1 - e^{2i(e+fx)})}{f} \\
&\quad + \frac{b^2d \log(\sin(e + fx))}{f^2} - \frac{(2abd) \int \log(1 - e^{2i(e+fx)}) dx}{f} \\
&= -b^2cx - \frac{1}{2}b^2dx^2 + \frac{a^2(c + dx)^2}{2d} - \frac{iab(c + dx)^2}{d} \\
&\quad - \frac{b^2(c + dx) \cot(e + fx)}{f} + \frac{2ab(c + dx) \log(1 - e^{2i(e+fx)})}{f} \\
&\quad + \frac{b^2d \log(\sin(e + fx))}{f^2} + \frac{(iabd) \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2i(e+fx)}\right)}{f^2} \\
&= -b^2cx - \frac{1}{2}b^2dx^2 + \frac{a^2(c + dx)^2}{2d} - \frac{iab(c + dx)^2}{d} \\
&\quad - \frac{b^2(c + dx) \cot(e + fx)}{f} + \frac{2ab(c + dx) \log(1 - e^{2i(e+fx)})}{f} \\
&\quad + \frac{b^2d \log(\sin(e + fx))}{f^2} - \frac{iabd \text{PolyLog}(2, e^{2i(e+fx)})}{f^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 8.52 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.74

$$\int (c + dx)(a + b \cot(e + fx))^2 dx$$

$$= \frac{(a + b \cot(e + fx))^2 \sin(e + fx) (-2b^2 f(c + dx) \cos(e + fx) - (a - b)(a + b)(e + fx)(-2cf + d(e - fx)))}{2f^2(b \cos(e + fx) + a \sin(e + fx))^2}$$

[In] Integrate[(c + d*x)*(a + b*Cot[e + f*x])^2,x]

[Out] ((a + b*Cot[e + f*x])^2*Sin[e + f*x]*(-2*b^2*f*(c + d*x)*Cos[e + f*x] - (a - b)*(a + b)*(e + f*x)*(-2*c*f + d*(e - f*x))*Sin[e + f*x] + 2*b^2*d*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]])*Sin[e + f*x] - 4*a*b*d*e*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]])*Sin[e + f*x] + 4*a*b*c*f*(Log[Cos[e + f*x]] + Log[Tan[e + f*x]])*Sin[e + f*x] + 4*a*b*d*((e + f*x)*Log[1 - E^((2*I)*(e + f*x))] - (I/2)*((e + f*x)^2 + PolyLog[2, E^((2*I)*(e + f*x))]))*Sin[e + f*x]))/(2*f^2*(b*Cos[e + f*x] + a*Sin[e + f*x])^2)

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(127) = 254.

Time = 0.72 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.66

method	result
risch	$-\frac{b^2 dx^2}{2} - \frac{2ib^2(dx+c)}{f(e^{2i(fx+e)}-1)} + \frac{a^2 dx^2}{2} - b^2 cx - \frac{2ibad \operatorname{polylog}(2, e^{i(fx+e)})}{f^2} + a^2 cx - \frac{4ibadex}{f} + \frac{b^2 d \ln(e^{i(fx+e)}+1)}{f^2} -$

[In] int((d*x+c)*(a+b*cot(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] $-1/2*b^2*d*x^2-2*I*b^2*(d*x+c)/f/(exp(2*I*(f*x+e))-1)+1/2*a^2*d*x^2-b^2*c*x-2*I/f^2*b*a*d*polylog(2,exp(I*(f*x+e)))+a^2*c*x-4*I/f*b*a*d*e*x+1/f^2*b^2*d*\ln(exp(I*(f*x+e))+1)-2/f^2*b^2*d*\ln(exp(I*(f*x+e)))+1/f^2*b^2*d*\ln(exp(I*(f*x+e))-1)+2/f*b*a*c*\ln(exp(I*(f*x+e))+1)-4/f*b*a*c*\ln(exp(I*(f*x+e)))+2/f*b*a*c*\ln(exp(I*(f*x+e))-1)+4/f^2*b*e*a*d*\ln(exp(I*(f*x+e)))-2/f^2*b*e*a*d*\ln(exp(I*(f*x+e))-1)-I*a*b*d*x^2+2*I*a*b*c*x-2*I/f^2*b*a*d*polylog(2,-exp(I*(f*x+e)))+2/f*b*a*d*\ln(1-exp(I*(f*x+e)))*x+2/f^2*b*a*d*\ln(1-exp(I*(f*x+e)))*e-2*I/f^2*b*a*d*e^2+2/f*b*a*d*\ln(exp(I*(f*x+e))+1)*x$

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(124) = 248$.

Time = 0.28 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.76

$$\int (c + dx)(a + b \cot(e + fx))^2 dx = \frac{2b^2dfx + iabd\text{Li}_2(\cos(2fx + 2e) + i \sin(2fx + 2e)) \sin(2fx + 2e) - iabd\text{Li}_2(\cos(2fx + 2e) - i \sin(2fx + 2e)) \sin(2fx + 2e) + \dots}{f^2 \sin(2fx + 2e)}$$

[In] integrate((d*x+c)*(a+b*cot(f*x+e))^2,x, algorithm="fricas")

[Out]
$$\frac{-1/2*(2*b^2*d*f*x + I*a*b*d*\text{dilog}(\cos(2*f*x + 2*e) + I*\sin(2*f*x + 2*e))*\sin(2*f*x + 2*e) - I*a*b*d*\text{dilog}(\cos(2*f*x + 2*e) - I*\sin(2*f*x + 2*e))*\sin(2*f*x + 2*e) + 2*b^2*c*f + (2*a*b*d*e - 2*a*b*c*f - b^2*d)*\log(-1/2*\cos(2*f*x + 2*e) + 1/2*I*\sin(2*f*x + 2*e) + 1/2)*\sin(2*f*x + 2*e) + (2*a*b*d*e - 2*a*b*c*f - b^2*d)*\log(-1/2*\cos(2*f*x + 2*e) - 1/2*I*\sin(2*f*x + 2*e) + 1/2)*\sin(2*f*x + 2*e) - 2*(a*b*d*f*x + a*b*d*e)*\log(-\cos(2*f*x + 2*e) + I*\sin(2*f*x + 2*e) + 1)*\sin(2*f*x + 2*e) - 2*(a*b*d*f*x + a*b*d*e)*\log(-\cos(2*f*x + 2*e) - I*\sin(2*f*x + 2*e) + 1)*\sin(2*f*x + 2*e) + 2*(b^2*d*f*x + b^2*c*f)*\cos(2*f*x + 2*e) - ((a^2 - b^2)*d*f^2*x^2 + 2*(a^2 - b^2)*c*f^2*x)*\sin(2*f*x + 2*e)}{f^2*\sin(2*f*x + 2*e)}$$

Sympy [F]

$$\int (c + dx)(a + b \cot(e + fx))^2 dx = \int (a + b \cot(e + fx))^2 (c + dx) dx$$

[In] integrate((d*x+c)*(a+b*cot(f*x+e))**2,x)

[Out] Integral((a + b*cot(e + f*x))**2*(c + d*x), x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(124) = 248$.

Time = 0.34 (sec) , antiderivative size = 774, normalized size of antiderivative = 5.65

$$\int (c + dx)(a + b \cot(e + fx))^2 dx = \frac{2(fx + e)a^2c + \frac{(fx+e)^2a^2d}{f} - \frac{2(fx+e)a^2de}{f} + 4abc \log(\sin(fx + e)) - \frac{4abde \log(\sin(fx+e))}{f} + \frac{2((2ab-ib^2)(fx+e)^2d+4a^2c)}{f^2}}{f^2}$$

[In] integrate((d*x+c)*(a+b*cot(f*x+e))^2,x, algorithm="maxima")

[Out] $\frac{1}{2}*(2*(f*x + e)*a^2*c + (f*x + e)^2*a^2*d/f - 2*(f*x + e)*a^2*d*e/f + 4*a*b*c*\log(\sin(f*x + e)) - 4*a*b*d*e*\log(\sin(f*x + e)))/f + 2*((2*a*b - I*b^2)*(f*x + e)^2*d + 4*b^2*d*e - 4*b^2*c*f - 2*(-I*b^2*d*e + I*b^2*c*f)*(f*x + e) - 2*(2*(f*x + e)*a*b*d + b^2*d - (2*(f*x + e)*a*b*d + b^2*d)*\cos(2*f*x + 2*e) + (-2*I*(f*x + e)*a*b*d - I*b^2*d)*\sin(2*f*x + 2*e))*\arctan2(\sin(f*x + e), \cos(f*x + e) + 1) + 2*(b^2*d*\cos(2*f*x + 2*e) + I*b^2*d*\sin(2*f*x + 2*e) - b^2*d)*\arctan2(\sin(f*x + e), \cos(f*x + e) - 1) - 4*((f*x + e)*a*b*d*\cos(2*f*x + 2*e) + I*(f*x + e)*a*b*d*\sin(2*f*x + 2*e) - (f*x + e)*a*b*d)*\arctan2(\sin(f*x + e), -\cos(f*x + e) + 1) - ((2*a*b - I*b^2)*(f*x + e)^2*d + 2*(I*b^2*d*e - I*b^2*c*f + 2*b^2*d)*(f*x + e))*\cos(2*f*x + 2*e) - 4*(a*b*d*\cos(2*f*x + 2*e) + I*a*b*d*\sin(2*f*x + 2*e) - a*b*d)*\operatorname{dilog}(-e^{(I*f*x + I*e)}) - 4*(a*b*d*\cos(2*f*x + 2*e) + I*a*b*d*\sin(2*f*x + 2*e) - a*b*d)*\operatorname{dilog}(e^{(I*f*x + I*e)}) + (2*I*(f*x + e)*a*b*d + I*b^2*d + (-2*I*(f*x + e)*a*b*d - I*b^2*d)*\cos(2*f*x + 2*e) + (2*(f*x + e)*a*b*d + b^2*d)*\sin(2*f*x + 2*e))*\log(\cos(f*x + e)^2 + \sin(f*x + e)^2 + 2*\cos(f*x + e) + 1) + (2*I*(f*x + e)*a*b*d + I*b^2*d + (-2*I*(f*x + e)*a*b*d - I*b^2*d)*\cos(2*f*x + 2*e) + (2*(f*x + e)*a*b*d + b^2*d)*\sin(2*f*x + 2*e))*\log(\cos(f*x + e)^2 + \sin(f*x + e)^2 - 2*\cos(f*x + e) + 1) + ((-2*I*a*b - b^2)*(f*x + e)^2*d + 2*(b^2*d*e - b^2*c*f - 2*I*b^2*d)*(f*x + e))*\sin(2*f*x + 2*e))/(-2*I*f*\cos(2*f*x + 2*e) + 2*f*\sin(2*f*x + 2*e) + 2*I*f))/f$

Giac [F]

$$\int (c + dx)(a + b \cot(e + fx))^2 dx = \int (dx + c)(b \cot(fx + e) + a)^2 dx$$

[In] integrate((d*x+c)*(a+b*cot(f*x+e))^2,x, algorithm="giac")

[Out] integrate((d*x + c)*(b*cot(f*x + e) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (c + dx)(a + b \cot(e + fx))^2 dx = \int (a + b \cot(e + fx))^2 (c + dx) dx$$

[In] int((a + b*cot(e + f*x))^2*(c + d*x),x)

[Out] int((a + b*cot(e + f*x))^2*(c + d*x), x)

3.45 $\int \frac{(a+b \cot(e+fx))^2}{c+dx} dx$

Optimal result	300
Rubi [N/A]	300
Mathematica [N/A]	301
Maple [N/A] (verified)	301
Fricas [N/A]	301
Sympy [N/A]	301
Maxima [N/A]	302
Giac [N/A]	302
Mupad [N/A]	303

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a + b \cot(e + fx))^2}{c + dx} dx = \text{Int}\left(\frac{(a + b \cot(e + fx))^2}{c + dx}, x\right)$$

[Out] Unintegrable((a+b*cot(f*x+e))^2/(d*x+c),x)

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a + b \cot(e + fx))^2}{c + dx} dx = \int \frac{(a + b \cot(e + fx))^2}{c + dx} dx$$

[In] Int[(a + b*Cot[e + f*x])^2/(c + d*x),x]

[Out] Defer[Int][(a + b*Cot[e + f*x])^2/(c + d*x), x]

Rubi steps

$$\text{integral} = \int \frac{(a + b \cot(e + fx))^2}{c + dx} dx$$

Mathematica [N/A]

Not integrable

Time = 25.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \cot(e + fx))^2}{c + dx} dx = \int \frac{(a + b \cot(e + fx))^2}{c + dx} dx$$

[In] Integrate[(a + b*Cot[e + f*x])^2/(c + d*x), x]

[Out] Integrate[(a + b*Cot[e + f*x])^2/(c + d*x), x]

Maple [N/A] (verified)

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \cot(fx + e))^2}{dx + c} dx$$

[In] int((a+b*cot(f*x+e))^2/(d*x+c), x)

[Out] int((a+b*cot(f*x+e))^2/(d*x+c), x)

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{(a + b \cot(e + fx))^2}{c + dx} dx = \int \frac{(b \cot(fx + e) + a)^2}{dx + c} dx$$

[In] integrate((a+b*cot(f*x+e))^2/(d*x+c), x, algorithm="fricas")

[Out] integral((b^2*cot(f*x + e)^2 + 2*a*b*cot(f*x + e) + a^2)/(d*x + c), x)

Sympy [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(a + b \cot(e + fx))^2}{c + dx} dx = \int \frac{(a + b \cot(e + fx))^2}{c + dx} dx$$

[In] integrate((a+b*cot(f*x+e))**2/(d*x+c), x)

[Out] Integral((a + b*cot(e + f*x))**2/(c + d*x), x)

Maxima [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 697, normalized size of antiderivative = 34.85

$$\int \frac{(a + b \cot(e + fx))^2}{c + dx} dx = \int \frac{(b \cot(fx + e) + a)^2}{dx + c} dx$$

[In] integrate((a+b*cot(f*x+e))^2/(d*x+c),x, algorithm="maxima")

```
[Out] (((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*cos(2*f*x + 2*e)^2*log(d*x + c) - 2*
b^2*d*sin(2*f*x + 2*e) + ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(d*x + c)
*sin(2*f*x + 2*e)^2 - 2*((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*cos(2*f*x + 2
*e)*log(d*x + c) - (d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2
+ (d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 - 2*(d^2*f*x + c*d*f)*cos(2*f*x + 2*
e))*integrate((2*a*b*d*f*x + 2*a*b*c*f - b^2*d)*sin(f*x + e)/(d^2*f*x^2 + 2
*c*d*f*x + c^2*f + (d^2*f*x^2 + 2*c*d*f*x + c^2*f)*cos(f*x + e)^2 + (d^2*f*
x^2 + 2*c*d*f*x + c^2*f)*sin(f*x + e)^2 + 2*(d^2*f*x^2 + 2*c*d*f*x + c^2*f)
*cos(f*x + e)), x) + (d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^
2 + (d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 - 2*(d^2*f*x + c*d*f)*cos(2*f*x +
2*e))*integrate((2*a*b*d*f*x + 2*a*b*c*f - b^2*d)*sin(f*x + e)/(d^2*f*x^2 +
2*c*d*f*x + c^2*f + (d^2*f*x^2 + 2*c*d*f*x + c^2*f)*cos(f*x + e)^2 + (d^2*
f*x^2 + 2*c*d*f*x + c^2*f)*sin(f*x + e)^2 - 2*(d^2*f*x^2 + 2*c*d*f*x + c^2*
f)*cos(f*x + e)), x) + ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*log(d*x + c))/
(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 + (d^2*f*x + c*d*f)
*sin(2*f*x + 2*e)^2 - 2*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e))
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \cot(e + fx))^2}{c + dx} dx = \int \frac{(b \cot(fx + e) + a)^2}{dx + c} dx$$

[In] integrate((a+b*cot(f*x+e))^2/(d*x+c),x, algorithm="giac")

[Out] integrate((b*cot(f*x + e) + a)^2/(d*x + c), x)

Mupad [N/A]

Not integrable

Time = 12.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \cot(e + f x))^2}{c + d x} dx = \int \frac{(a + b \cot(e + f x))^2}{c + d x} dx$$

```
[In] int((a + b*cot(e + f*x))^2/(c + d*x),x)
```

```
[Out] int((a + b*cot(e + f*x))^2/(c + d*x), x)
```

$$3.46 \quad \int \frac{(a+b \cot(e+fx))^2}{(c+dx)^2} dx$$

Optimal result	304
Rubi [N/A]	304
Mathematica [N/A]	305
Maple [N/A] (verified)	305
Fricas [N/A]	305
Sympy [N/A]	306
Maxima [N/A]	306
Giac [N/A]	307
Mupad [N/A]	307

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b \cot(e+fx))^2}{(c+dx)^2} dx = \text{Int}\left(\frac{(a+b \cot(e+fx))^2}{(c+dx)^2}, x\right)$$

[Out] Unintegrable((a+b*cot(f*x+e))^2/(d*x+c)^2,x)

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b \cot(e+fx))^2}{(c+dx)^2} dx = \int \frac{(a+b \cot(e+fx))^2}{(c+dx)^2} dx$$

[In] Int[(a + b*Cot[e + f*x])^2/(c + d*x)^2,x]

[Out] Defer[Int] [(a + b*Cot[e + f*x])^2/(c + d*x)^2, x]

Rubi steps

$$\text{integral} = \int \frac{(a+b \cot(e+fx))^2}{(c+dx)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 20.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \cot(e + fx))^2}{(c + dx)^2} dx = \int \frac{(a + b \cot(e + fx))^2}{(c + dx)^2} dx$$

[In] Integrate[(a + b*Cot[e + f*x])^2/(c + d*x)^2,x]

[Out] Integrate[(a + b*Cot[e + f*x])^2/(c + d*x)^2, x]

Maple [N/A] (verified)

Not integrable

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \cot(fx + e))^2}{(dx + c)^2} dx$$

[In] int((a+b*cot(f*x+e))^2/(d*x+c)^2,x)

[Out] int((a+b*cot(f*x+e))^2/(d*x+c)^2,x)

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \frac{(a + b \cot(e + fx))^2}{(c + dx)^2} dx = \int \frac{(b \cot(fx + e) + a)^2}{(dx + c)^2} dx$$

[In] integrate((a+b*cot(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")

[Out] integral((b^2*cot(f*x + e)^2 + 2*a*b*cot(f*x + e) + a^2)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(a + b \cot(e + fx))^2}{(c + dx)^2} dx = \int \frac{(a + b \cot(e + fx))^2}{(c + dx)^2} dx$$

[In] integrate((a+b*cot(f*x+e))**2/(d*x+c)**2,x)

[Out] Integral((a + b*cot(e + f*x))**2/(c + d*x)**2, x)

Maxima [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 910, normalized size of antiderivative = 45.50

$$\int \frac{(a + b \cot(e + fx))^2}{(c + dx)^2} dx = \int \frac{(b \cot(fx + e) + a)^2}{(dx + c)^2} dx$$

[In] integrate((a+b*cot(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")

```
[Out] -((a^2 - b^2)*d*f*x + 2*b^2*d*sin(2*f*x + 2*e) + (a^2 - b^2)*c*f + ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*cos(2*f*x + 2*e)^2 + ((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*sin(2*f*x + 2*e)^2 - 2*((a^2 - b^2)*d*f*x + (a^2 - b^2)*c*f)*cos(2*f*x + 2*e) + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*x + 2*e)^2 + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*sin(2*f*x + 2*e)^2 - 2*(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*x + 2*e)) *integrate(2*(a*b*d*f*x + a*b*c*f - b^2*d)*sin(f*x + e)/(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f + (d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)*cos(f*x + e)^2 + (d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)*sin(f*x + e)^2 + 2*(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)*cos(f*x + e)), x) - (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*x + 2*e)^2 + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*sin(2*f*x + 2*e)^2 - 2*(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*x + 2*e)) *integrate(2*(a*b*d*f*x + a*b*c*f - b^2*d)*sin(f*x + e)/(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f + (d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)*cos(f*x + e)^2 + (d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)*sin(f*x + e)^2 - 2*(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f)*cos(f*x + e)), x)/(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*x + 2*e)^2 + (d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*sin(2*f*x + 2*e)^2 - 2*(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f)*cos(2*f*x + 2*e))
```

Giac [N/A]

Not integrable

Time = 2.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \cot(e + fx))^2}{(c + dx)^2} dx = \int \frac{(b \cot(fx + e) + a)^2}{(dx + c)^2} dx$$

[In] integrate((a+b*cot(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")

[Out] integrate((b*cot(f*x + e) + a)^2/(d*x + c)^2, x)

Mupad [N/A]

Not integrable

Time = 12.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \cot(e + fx))^2}{(c + dx)^2} dx = \int \frac{(a + b \cot(e + fx))^2}{(c + dx)^2} dx$$

[In] int((a + b*cot(e + f*x))^2/(c + d*x)^2,x)

[Out] int((a + b*cot(e + f*x))^2/(c + d*x)^2, x)

3.47 $\int (c + dx)^3 (a + b \cot(e + fx))^3 dx$

Optimal result	309
Rubi [A] (verified)	310
Mathematica [B] (warning: unable to verify)	318
Maple [B] (verified)	320
Fricas [B] (verification not implemented)	322
Sympy [F]	323
Maxima [B] (verification not implemented)	323
Giac [F]	329
Mupad [F(-1)]	329

Optimal result

Integrand size = 20, antiderivative size = 603

$$\begin{aligned}
 \int (c + dx)^3 (a + b \cot(e + fx))^3 dx = & -\frac{3ib^3d(c + dx)^2}{2f^2} - \frac{3iab^2(c + dx)^3}{f} - \frac{b^3(c + dx)^3}{2f} \\
 & + \frac{a^3(c + dx)^4}{4d} - \frac{3ia^2b(c + dx)^4}{4d} - \frac{3ab^2(c + dx)^4}{4d} \\
 & + \frac{ib^3(c + dx)^4}{4d} - \frac{3b^3d(c + dx)^2 \cot(e + fx)}{2f^2} \\
 & - \frac{3ab^2(c + dx)^3 \cot(e + fx)}{f} \\
 & - \frac{b^3(c + dx)^3 \cot^2(e + fx)}{2f} \\
 & + \frac{3b^3d^2(c + dx) \log(1 - e^{2i(e+fx)})}{f^3} \\
 & + \frac{9ab^2d(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f^2} \\
 & + \frac{3a^2b(c + dx)^3 \log(1 - e^{2i(e+fx)})}{f} \\
 & - \frac{b^3(c + dx)^3 \log(1 - e^{2i(e+fx)})}{f} \\
 & - \frac{3ib^3d^3 \operatorname{PolyLog}(2, e^{2i(e+fx)})}{2f^4} \\
 & - \frac{9iab^2d^2(c + dx) \operatorname{PolyLog}(2, e^{2i(e+fx)})}{f^3} \\
 & - \frac{9ia^2bd(c + dx)^2 \operatorname{PolyLog}(2, e^{2i(e+fx)})}{2f^2} \\
 & + \frac{3ib^3d(c + dx)^2 \operatorname{PolyLog}(2, e^{2i(e+fx)})}{2f^2} \\
 & + \frac{9ab^2d^3 \operatorname{PolyLog}(3, e^{2i(e+fx)})}{2f^4} \\
 & + \frac{9a^2bd^2(c + dx) \operatorname{PolyLog}(3, e^{2i(e+fx)})}{2f^3} \\
 & - \frac{3b^3d^2(c + dx) \operatorname{PolyLog}(3, e^{2i(e+fx)})}{2f^3} \\
 & + \frac{9ia^2bd^3 \operatorname{PolyLog}(4, e^{2i(e+fx)})}{4f^4} \\
 & - \frac{3ib^3d^3 \operatorname{PolyLog}(4, e^{2i(e+fx)})}{4f^4}
 \end{aligned}$$

```
[Out] -9/2*I*a^2*b*d*(d*x+c)^2*polylog(2,exp(2*I*(f*x+e)))/f^2+1/4*I*b^3*(d*x+c)^
4/d-1/2*b^3*(d*x+c)^3/f+1/4*a^3*(d*x+c)^4/d+3/2*I*b^3*d*(d*x+c)^2*polylog(2
,exp(2*I*(f*x+e)))/f^2-3/4*a*b^2*(d*x+c)^4/d-3/4*I*a^2*b*(d*x+c)^4/d-3/2*b^
3*d*(d*x+c)^2*cot(f*x+e)/f^2-3*a*b^2*(d*x+c)^3*cot(f*x+e)/f-1/2*b^3*(d*x+c)
^3*cot(f*x+e)^2/f+3*b^3*d^2*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f^3+9*a*b^2*d*(d
*x+c)^2*ln(1-exp(2*I*(f*x+e)))/f^2+3*a^2*b*(d*x+c)^3*ln(1-exp(2*I*(f*x+e)))
/f-b^3*(d*x+c)^3*ln(1-exp(2*I*(f*x+e)))/f-3/4*I*b^3*d^3*polylog(4,exp(2*I*(
f*x+e)))/f^4-3/2*I*b^3*d^3*polylog(2,exp(2*I*(f*x+e)))/f^4-3*I*a*b^2*(d*x+c)
^3/f-9*I*a*b^2*d^2*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/f^3+9/2*a*b^2*d^3*p
olylog(3,exp(2*I*(f*x+e)))/f^4+9/2*a^2*b*d^2*(d*x+c)*polylog(3,exp(2*I*(f*x
+e)))/f^3-3/2*b^3*d^2*(d*x+c)*polylog(3,exp(2*I*(f*x+e)))/f^3+9/4*I*a^2*b*d
^3*polylog(4,exp(2*I*(f*x+e)))/f^4-3/2*I*b^3*d*(d*x+c)^2/f^2
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.00,
 number of steps used = 28, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules

used = {3803, 3798, 2221, 2611, 6744, 2320, 6724, 3801, 32, 2317, 2438}

$$\begin{aligned}
 \int (c + dx)^3 (a + b \cot(e + fx))^3 dx = & \frac{a^3(c + dx)^4}{4d} + \frac{9a^2bd^2(c + dx) \operatorname{PolyLog}(3, e^{2i(e+fx)})}{2f^3} \\
 & - \frac{9ia^2bd(c + dx)^2 \operatorname{PolyLog}(2, e^{2i(e+fx)})}{2f^2} \\
 & + \frac{3a^2b(c + dx)^3 \log(1 - e^{2i(e+fx)})}{f} \\
 & - \frac{3ia^2b(c + dx)^4}{4d} + \frac{9ia^2bd^3 \operatorname{PolyLog}(4, e^{2i(e+fx)})}{4f^4} \\
 & - \frac{9iab^2d^2(c + dx) \operatorname{PolyLog}(2, e^{2i(e+fx)})}{f^3} \\
 & + \frac{9ab^2d(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f^2} \\
 & - \frac{3ab^2(c + dx)^3 \cot(e + fx)}{f} - \frac{3iab^2(c + dx)^3}{f} \\
 & - \frac{3ab^2(c + dx)^4}{4d} + \frac{9ab^2d^3 \operatorname{PolyLog}(3, e^{2i(e+fx)})}{2f^4} \\
 & - \frac{3b^3d^2(c + dx) \operatorname{PolyLog}(3, e^{2i(e+fx)})}{2f^3} \\
 & + \frac{3b^3d^2(c + dx) \log(1 - e^{2i(e+fx)})}{f^3} \\
 & + \frac{3ib^3d(c + dx)^2 \operatorname{PolyLog}(2, e^{2i(e+fx)})}{2f^2} \\
 & - \frac{3b^3d(c + dx)^2 \cot(e + fx)}{2f^2} \\
 & - \frac{b^3(c + dx)^3 \log(1 - e^{2i(e+fx)})}{f} \\
 & - \frac{b^3(c + dx)^3 \cot^2(e + fx)}{2f} - \frac{3ib^3d(c + dx)^2}{2f^2} \\
 & - \frac{b^3(c + dx)^3}{2f} + \frac{ib^3(c + dx)^4}{4d} \\
 & - \frac{3ib^3d^3 \operatorname{PolyLog}(2, e^{2i(e+fx)})}{2f^4} \\
 & - \frac{3ib^3d^3 \operatorname{PolyLog}(4, e^{2i(e+fx)})}{4f^4}
 \end{aligned}$$

[In] Int[(c + d*x)^3*(a + b*Cot[e + f*x])^3,x]

[Out] (((-3*I)/2)*b^3*d*(c + d*x)^2)/f^2 - ((3*I)*a*b^2*(c + d*x)^3)/f - (b^3*(c + d*x)^3)/(2*f) + (a^3*(c + d*x)^4)/(4*d) - (((3*I)/4)*a^2*b*(c + d*x)^4)/d

$$\begin{aligned}
& - (3*a*b^2*(c + d*x)^4)/(4*d) + ((I/4)*b^3*(c + d*x)^4)/d - (3*b^3*d*(c + d*x)^2*\text{Cot}[e + f*x])/(2*f^2) - (3*a*b^2*(c + d*x)^3*\text{Cot}[e + f*x])/f - (b^3*(c + d*x)^3*\text{Cot}[e + f*x]^2)/(2*f) + (3*b^3*d^2*(c + d*x)*\text{Log}[1 - E^((2*I)*(e + f*x))])/f^3 + (9*a*b^2*d*(c + d*x)^2*\text{Log}[1 - E^((2*I)*(e + f*x))])/f^2 + (3*a^2*b*(c + d*x)^3*\text{Log}[1 - E^((2*I)*(e + f*x))])/f - (b^3*(c + d*x)^3*\text{Log}[1 - E^((2*I)*(e + f*x))])/f - (((3*I)/2)*b^3*d^3*\text{PolyLog}[2, E^((2*I)*(e + f*x))])/f^4 - ((9*I)*a*b^2*d^2*(c + d*x)*\text{PolyLog}[2, E^((2*I)*(e + f*x))])/f^3 - (((9*I)/2)*a^2*b*d*(c + d*x)^2*\text{PolyLog}[2, E^((2*I)*(e + f*x))])/f^2 + (((3*I)/2)*b^3*d*(c + d*x)^2*\text{PolyLog}[2, E^((2*I)*(e + f*x))])/f^2 + (9*a*b^2*d^3*\text{PolyLog}[3, E^((2*I)*(e + f*x))])/(2*f^4) + (9*a^2*b*d^2*(c + d*x)*\text{PolyLog}[3, E^((2*I)*(e + f*x))])/(2*f^3) - (3*b^3*d^2*(c + d*x)*\text{PolyLog}[3, E^((2*I)*(e + f*x))])/(2*f^3) + (((9*I)/4)*a^2*b*d^3*\text{PolyLog}[4, E^((2*I)*(e + f*x))])/f^4 - (((3*I)/4)*b^3*d^3*\text{PolyLog}[4, E^((2*I)*(e + f*x))])/f^4
\end{aligned}$$

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611


```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Simp[I*(c + d*x)^(m + 1)/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] := Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int (a^3(c + dx)^3 + 3a^2b(c + dx)^3 \cot(e + fx) + 3ab^2(c + dx)^3 \cot^2(e + fx) + b^3(c + dx)^3 \cot^3(e + fx)) dx$$

$$\begin{aligned}
&= \frac{a^3(c+dx)^4}{4d} + (3a^2b) \int (c+dx)^3 \cot(e+fx) dx \\
&\quad + (3ab^2) \int (c+dx)^3 \cot^2(e+fx) dx + b^3 \int (c+dx)^3 \cot^3(e+fx) dx \\
&= \frac{a^3(c+dx)^4}{4d} - \frac{3ia^2b(c+dx)^4}{4d} - \frac{3ab^2(c+dx)^3 \cot(e+fx)}{f} \\
&\quad - \frac{b^3(c+dx)^3 \cot^2(e+fx)}{2f} - (6ia^2b) \int \frac{e^{2i(e+fx)}(c+dx)^3}{1-e^{2i(e+fx)}} dx \\
&\quad - (3ab^2) \int (c+dx)^3 dx - b^3 \int (c+dx)^3 \cot(e+fx) dx \\
&\quad + \frac{(9ab^2d) \int (c+dx)^2 \cot(e+fx) dx}{f} + \frac{(3b^3d) \int (c+dx)^2 \cot^2(e+fx) dx}{2f} \\
&= -\frac{3iab^2(c+dx)^3}{f} + \frac{a^3(c+dx)^4}{4d} - \frac{3ia^2b(c+dx)^4}{4d} \\
&\quad - \frac{3ab^2(c+dx)^4}{4d} + \frac{ib^3(c+dx)^4}{4d} - \frac{3b^3d(c+dx)^2 \cot(e+fx)}{2f^2} \\
&\quad - \frac{3ab^2(c+dx)^3 \cot(e+fx)}{f} - \frac{b^3(c+dx)^3 \cot^2(e+fx)}{2f} \\
&\quad + \frac{3a^2b(c+dx)^3 \log(1-e^{2i(e+fx)})}{f} + (2ib^3) \int \frac{e^{2i(e+fx)}(c+dx)^3}{1-e^{2i(e+fx)}} dx \\
&\quad + \frac{(3b^3d^2) \int (c+dx) \cot(e+fx) dx}{f^2} - \frac{(9a^2bd) \int (c+dx)^2 \log(1-e^{2i(e+fx)}) dx}{f} \\
&\quad - \frac{(18iab^2d) \int \frac{e^{2i(e+fx)}(c+dx)^2}{1-e^{2i(e+fx)}} dx}{f} - \frac{(3b^3d) \int (c+dx)^2 dx}{2f} \\
&= -\frac{3ib^3d(c+dx)^2}{2f^2} - \frac{3iab^2(c+dx)^3}{f} - \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} \\
&\quad - \frac{3ia^2b(c+dx)^4}{4d} - \frac{3ab^2(c+dx)^4}{4d} + \frac{ib^3(c+dx)^4}{4d} - \frac{3b^3d(c+dx)^2 \cot(e+fx)}{2f^2} \\
&\quad - \frac{3ab^2(c+dx)^3 \cot(e+fx)}{f} - \frac{b^3(c+dx)^3 \cot^2(e+fx)}{2f} \\
&\quad + \frac{9ab^2d(c+dx)^2 \log(1-e^{2i(e+fx)})}{f^2} + \frac{3a^2b(c+dx)^3 \log(1-e^{2i(e+fx)})}{f} \\
&\quad - \frac{b^3(c+dx)^3 \log(1-e^{2i(e+fx)})}{f} - \frac{9ia^2bd(c+dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} \\
&\quad + \frac{(9ia^2bd^2) \int (c+dx) \text{PolyLog}(2, e^{2i(e+fx)}) dx}{f^2} \\
&\quad - \frac{(18ab^2d^2) \int (c+dx) \log(1-e^{2i(e+fx)}) dx}{f^2} \\
&\quad - \frac{(6ib^3d^2) \int \frac{e^{2i(e+fx)}(c+dx)}{1-e^{2i(e+fx)}} dx}{f^2} + \frac{(3b^3d) \int (c+dx)^2 \log(1-e^{2i(e+fx)}) dx}{f}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{3ib^3d(c+dx)^2}{2f^2} - \frac{3iab^2(c+dx)^3}{f} - \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} - \frac{3ia^2b(c+dx)^4}{4d} \\
&\quad - \frac{3ab^2(c+dx)^4}{4d} + \frac{ib^3(c+dx)^4}{4d} - \frac{3b^3d(c+dx)^2 \cot(e+fx)}{2f^2} \\
&\quad - \frac{3ab^2(c+dx)^3 \cot(e+fx)}{f} - \frac{b^3(c+dx)^3 \cot^2(e+fx)}{2f} \\
&\quad + \frac{3b^3d^2(c+dx) \log(1-e^{2i(e+fx)})}{f^3} + \frac{9ab^2d(c+dx)^2 \log(1-e^{2i(e+fx)})}{f^2} \\
&\quad + \frac{3a^2b(c+dx)^3 \log(1-e^{2i(e+fx)})}{f} - \frac{b^3(c+dx)^3 \log(1-e^{2i(e+fx)})}{f} \\
&\quad - \frac{9iab^2d^2(c+dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^3} - \frac{9ia^2bd(c+dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} \\
&\quad + \frac{3ib^3d(c+dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} + \frac{9a^2bd^2(c+dx) \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} \\
&\quad - \frac{(9a^2bd^3) \int \text{PolyLog}(3, e^{2i(e+fx)}) dx}{2f^3} + \frac{(9iab^2d^3) \int \text{PolyLog}(2, e^{2i(e+fx)}) dx}{f^3} \\
&\quad - \frac{(3b^3d^3) \int \log(1-e^{2i(e+fx)}) dx}{f^3} - \frac{(3ib^3d^2) \int (c+dx) \text{PolyLog}(2, e^{2i(e+fx)}) dx}{f^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{3ib^3d(c+dx)^2}{2f^2} - \frac{3iab^2(c+dx)^3}{f} - \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} \\
&\quad - \frac{3ia^2b(c+dx)^4}{4d} - \frac{3ab^2(c+dx)^4}{4d} + \frac{ib^3(c+dx)^4}{4d} \\
&\quad - \frac{3b^3d(c+dx)^2 \cot(e+fx)}{2f^2} - \frac{3ab^2(c+dx)^3 \cot(e+fx)}{f} \\
&\quad - \frac{b^3(c+dx)^3 \cot^2(e+fx)}{2f} + \frac{3b^3d^2(c+dx) \log(1-e^{2i(e+fx)})}{f^3} \\
&\quad + \frac{9ab^2d(c+dx)^2 \log(1-e^{2i(e+fx)})}{f^2} + \frac{3a^2b(c+dx)^3 \log(1-e^{2i(e+fx)})}{f} \\
&\quad - \frac{b^3(c+dx)^3 \log(1-e^{2i(e+fx)})}{f} - \frac{9iab^2d^2(c+dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^3} \\
&\quad - \frac{9ia^2bd(c+dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} + \frac{3ib^3d(c+dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} \\
&\quad + \frac{9a^2bd^2(c+dx) \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} - \frac{3b^3d^2(c+dx) \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} \\
&\quad + \frac{(9ia^2bd^3) \text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x} dx, x, e^{2i(e+fx)}\right)}{4f^4} \\
&\quad + \frac{(9ab^2d^3) \text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x} dx, x, e^{2i(e+fx)}\right)}{2f^4} \\
&\quad + \frac{(3ib^3d^3) \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2i(e+fx)}\right)}{2f^4} + \frac{(3b^3d^3) \int \text{PolyLog}(3, e^{2i(e+fx)}) dx}{2f^3}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{3ib^3d(c+dx)^2}{2f^2} - \frac{3iab^2(c+dx)^3}{f} - \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} \\
&\quad - \frac{3ia^2b(c+dx)^4}{4d} - \frac{3ab^2(c+dx)^4}{4d} + \frac{ib^3(c+dx)^4}{4d} \\
&\quad - \frac{3b^3d(c+dx)^2 \cot(e+fx)}{2f^2} - \frac{3ab^2(c+dx)^3 \cot(e+fx)}{f} \\
&\quad - \frac{b^3(c+dx)^3 \cot^2(e+fx)}{2f} + \frac{3b^3d^2(c+dx) \log(1-e^{2i(e+fx)})}{f^3} \\
&\quad + \frac{9ab^2d(c+dx)^2 \log(1-e^{2i(e+fx)})}{f^2} + \frac{3a^2b(c+dx)^3 \log(1-e^{2i(e+fx)})}{f} \\
&\quad - \frac{b^3(c+dx)^3 \log(1-e^{2i(e+fx)})}{f} - \frac{3ib^3d^3 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^4} \\
&\quad - \frac{9iab^2d^2(c+dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^3} - \frac{9ia^2bd(c+dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} \\
&\quad + \frac{3ib^3d(c+dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} + \frac{9ab^2d^3 \text{PolyLog}(3, e^{2i(e+fx)})}{2f^4} \\
&\quad + \frac{9a^2bd^2(c+dx) \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} - \frac{3b^3d^2(c+dx) \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} \\
&\quad + \frac{9ia^2bd^3 \text{PolyLog}(4, e^{2i(e+fx)})}{4f^4} - \frac{(3ib^3d^3) \text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x} dx, x, e^{2i(e+fx)}\right)}{4f^4} \\
&= -\frac{3ib^3d(c+dx)^2}{2f^2} - \frac{3iab^2(c+dx)^3}{f} - \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} \\
&\quad - \frac{3ia^2b(c+dx)^4}{4d} - \frac{3ab^2(c+dx)^4}{4d} + \frac{ib^3(c+dx)^4}{4d} \\
&\quad - \frac{3b^3d(c+dx)^2 \cot(e+fx)}{2f^2} - \frac{3ab^2(c+dx)^3 \cot(e+fx)}{f} \\
&\quad - \frac{b^3(c+dx)^3 \cot^2(e+fx)}{2f} + \frac{3b^3d^2(c+dx) \log(1-e^{2i(e+fx)})}{f^3} \\
&\quad + \frac{9ab^2d(c+dx)^2 \log(1-e^{2i(e+fx)})}{f^2} + \frac{3a^2b(c+dx)^3 \log(1-e^{2i(e+fx)})}{f} \\
&\quad - \frac{b^3(c+dx)^3 \log(1-e^{2i(e+fx)})}{f} - \frac{3ib^3d^3 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^4} \\
&\quad - \frac{9iab^2d^2(c+dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^3} - \frac{9ia^2bd(c+dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} \\
&\quad + \frac{3ib^3d(c+dx)^2 \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} + \frac{9ab^2d^3 \text{PolyLog}(3, e^{2i(e+fx)})}{2f^4} \\
&\quad + \frac{9a^2bd^2(c+dx) \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} - \frac{3b^3d^2(c+dx) \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} \\
&\quad + \frac{9ia^2bd^3 \text{PolyLog}(4, e^{2i(e+fx)})}{4f^4} - \frac{3ib^3d^3 \text{PolyLog}(4, e^{2i(e+fx)})}{4f^4}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3129 vs. $2(603) = 1206$.

Time = 8.22 (sec) , antiderivative size = 3129, normalized size of antiderivative = 5.19

$$\int (c + dx)^3 (a + b \cot(e + fx))^3 dx = \text{Result too large to show}$$

[In] Integrate[(c + d*x)^3*(a + b*Cot[e + f*x])^3,x]

[Out]
$$\begin{aligned} & ((-b^3c^3) - 3b^3c^2dx - 3b^3cd^2x^2 - b^3d^3x^3) \text{Csc}[e + fx]^{2/2} / (2f) - (3ab^2d^3E^{(Ie)} \text{Csc}[e] * ((2f^3x^3)/E^{(2I)e} + (3I)*(1 - E^{(-2I)e})) * f^2x^2 \text{Log}[1 - E^{(-I)(e + fx)}]) + (3I)*(1 - E^{(-2I)e}) * f^2x^2 \text{Log}[1 + E^{(-I)(e + fx)}]) - 6*(1 - E^{(-2I)e}) * f * x * \text{PolyLog}[2, -E^{(-I)(e + fx)}] - 6*(1 - E^{(-2I)e}) * f * x * \text{PolyLog}[2, E^{(-I)(e + fx)}] + (6I)*(1 - E^{(-2I)e}) * \text{PolyLog}[3, -E^{(-I)(e + fx)}] + (6I)*(1 - E^{(-2I)e}) * \text{PolyLog}[3, E^{(-I)(e + fx)}]) / (2f^4) - (3a^2b^2cd^2E^{(Ie)} \text{Csc}[e] * ((2f^3x^3)/E^{(2I)e} + (3I)*(1 - E^{(-2I)e})) * f^2x^2 \text{Log}[1 - E^{(-I)(e + fx)}] + (3I)*(1 - E^{(-2I)e}) * f^2x^2 \text{Log}[1 + E^{(-I)(e + fx)}] - 6*(1 - E^{(-2I)e}) * f * x * \text{PolyLog}[2, -E^{(-I)(e + fx)}] - 6*(1 - E^{(-2I)e}) * f * x * \text{PolyLog}[2, E^{(-I)(e + fx)}] + (6I)*(1 - E^{(-2I)e}) * \text{PolyLog}[3, -E^{(-I)(e + fx)}] + (6I)*(1 - E^{(-2I)e}) * \text{PolyLog}[3, E^{(-I)(e + fx)}]) / (2f^3) + (b^3cd^2E^{(Ie)} \text{Csc}[e] * ((2f^3x^3)/E^{(2I)e} + (3I)*(1 - E^{(-2I)e})) * f^2x^2 \text{Log}[1 - E^{(-I)(e + fx)}] + (3I)*(1 - E^{(-2I)e}) * f^2x^2 \text{Log}[1 + E^{(-I)(e + fx)}] - 6*(1 - E^{(-2I)e}) * f * x * \text{PolyLog}[2, -E^{(-I)(e + fx)}] - 6*(1 - E^{(-2I)e}) * f * x * \text{PolyLog}[2, E^{(-I)(e + fx)}] + (6I)*(1 - E^{(-2I)e}) * \text{PolyLog}[3, -E^{(-I)(e + fx)}] + (6I)*(1 - E^{(-2I)e}) * \text{PolyLog}[3, E^{(-I)(e + fx)}]) / (2f^3) - (3a^2bd^3E^{(Ie)} \text{Csc}[e] * ((f^4x^4)/E^{(2I)e} + (2I)*(1 - E^{(-2I)e})) * f^3x^3 \text{Log}[1 - E^{(-I)(e + fx)}] + (2I)*(1 - E^{(-2I)e}) * f^3x^3 \text{Log}[1 + E^{(-I)(e + fx)}] - 6*(1 - E^{(-2I)e}) * f^2x^2 \text{PolyLog}[2, -E^{(-I)(e + fx)}] - 6*(1 - E^{(-2I)e}) * f^2x^2 \text{PolyLog}[2, E^{(-I)(e + fx)}] + (12I)*(1 - E^{(-2I)e}) * f * x * \text{PolyLog}[3, -E^{(-I)(e + fx)}] + (12I)*(1 - E^{(-2I)e}) * f * x * \text{PolyLog}[3, E^{(-I)(e + fx)}] + 12*(1 - E^{(-2I)e}) * \text{PolyLog}[4, -E^{(-I)(e + fx)}] + 12*(1 - E^{(-2I)e}) * \text{PolyLog}[4, E^{(-I)(e + fx)}]) / (4f^4) + (b^3d^3E^{(Ie)} \text{Csc}[e] * ((f^4x^4)/E^{(2I)e} + (2I)*(1 - E^{(-2I)e})) * f^3x^3 \text{Log}[1 - E^{(-I)(e + fx)}] + (2I)*(1 - E^{(-2I)e}) * f^3x^3 \text{Log}[1 + E^{(-I)(e + fx)}] - 6*(1 - E^{(-2I)e}) * f^2x^2 \text{PolyLog}[2, -E^{(-I)(e + fx)}] - 6*(1 - E^{(-2I)e}) * f^2x^2 \text{PolyLog}[2, E^{(-I)(e + fx)}] + (12I)*(1 - E^{(-2I)e}) * f * x * \text{PolyLog}[3, -E^{(-I)(e + fx)}] + (12I)*(1 - E^{(-2I)e}) * f * x * \text{PolyLog}[3, E^{(-I)(e + fx)}] + 12*(1 - E^{(-2I)e}) * \text{PolyLog}[4, -E^{(-I)(e + fx)}] + 12*(1 - E^{(-2I)e}) * \text{PolyLog}[4, E^{(-I)(e + fx)}]) / (4f^4) + (3b^3cd^2 \text{Csc}[e] * (-f * x * \text{Cos}[e]) + \text{Log}[\text{Cos}[fx] * \text{Sin}[e] + \text{Cos}[e] * \text{Sin}[fx]] * \text{Sin}[e]) / (f^3 \end{aligned}$$

$$\frac{[\text{Tan}[e]] * \text{Log}[\text{Sin}[f*x + \text{ArcTan}[\text{Tan}[e]]]] + I * \text{PolyLog}[2, E^{((2*I)*(f*x + \text{ArcTan}[\text{Tan}[e]))}] * \text{Tan}[e]] / \text{Sqrt}[1 + \text{Tan}[e]^2]] / (2*f^2 * \text{Sqrt}[\text{Sec}[e]^2 * (\text{Cos}[e]^2 + \text{Sin}[e]^2)])}$$

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3160 vs. $2(535) = 1070$.

Time = 0.94 (sec) , antiderivative size = 3161, normalized size of antiderivative = 5.24

method	result	size
risch	Expression too large to display	3161

```
[In] int((d*x+c)^3*(a+b*cot(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*b^3*c^3*ln(exp(I*(f*x+e))+1)+2/f*b^3*c^3*ln(exp(I*(f*x+e)))-1/f*b^3*c^3*ln(exp(I*(f*x+e))-1)-3*I*d^2*a^2*b*c*x^3-9/2*I*d*a^2*b*c^2*x^2+18/f^3*b*d^3*a^2*polylog(3,exp(I*(f*x+e)))*x+18/f^3*b*d^3*a^2*polylog(3,-exp(I*(f*x+e)))*x-6/f^2*b^3*e*d*c^2*ln(exp(I*(f*x+e)))+3/f^2*b^3*e*d*c^2*ln(exp(I*(f*x+e))-1)+9/f^2*b^2*a*c^2*d*ln(exp(I*(f*x+e))+1)-18/f^2*b^2*a*c^2*d*ln(exp(I*(f*x+e)))+9/f^2*b^2*a*c^2*d*ln(exp(I*(f*x+e))-1)-18/f^4*b^2*e^2*a*d^3*ln(exp(I*(f*x+e)))+9/f^4*b^2*e^2*a*d^3*ln(exp(I*(f*x+e))-1)-3/f*b^3*c*d^2*ln(1-exp(I*(f*x+e)))*x^2-3/f*b^3*c*d^2*ln(exp(I*(f*x+e))+1)*x^2+3/f^3*b^3*c*d^2*ln(1-exp(I*(f*x+e)))*e^2-9/2*I/f^4*b*d^3*a^2*e^4+18*I/f^4*b*d^3*a^2*polylog(4,exp(I*(f*x+e)))+18*I/f^4*b*d^3*a^2*polylog(4,-exp(I*(f*x+e)))+3*I/f^2*b^3*d*c^2*e^2+3*I/f^2*b^3*d*c^2*polylog(2,exp(I*(f*x+e)))+3*I/f^2*b^3*d*c^2*polylog(2,-exp(I*(f*x+e)))+2*I/f^3*b^3*d^3*e^3*x-6*I/f^3*b^3*d^3*e*x+3*I/f^2*b^3*d^3*polylog(2,-exp(I*(f*x+e)))*x^2+3*I/f^2*b^3*d^3*polylog(2,exp(I*(f*x+e)))*x^2-4*I/f^3*b^3*c*d^2*e^3-6*I/f*b^2*a*d^3*x^3+12*I/f^4*b^2*a*d^3*e^3+6/f^4*b*e^3*d^3*a^2*ln(exp(I*(f*x+e)))-3/f^4*b*e^3*d^3*a^2*ln(exp(I*(f*x+e))-1)+6/f^3*b^3*e^2*c*d^2*ln(exp(I*(f*x+e)))-3/f^3*b^3*e^2*c*d^2*ln(exp(I*(f*x+e))-1)+3/f^4*b*d^3*a^2*ln(1-exp(I*(f*x+e)))*e^3+3/f*b*d^3*a^2*ln(1-exp(I*(f*x+e)))*x^3+3/f*b*d^3*a^2*ln(exp(I*(f*x+e))+1)*x^3-3/f*b^3*d*c^2*ln(1-exp(I*(f*x+e)))*x-3/f^2*b^3*d*c^2*ln(1-exp(I*(f*x+e)))*e-3/f*b^3*d*c^2*ln(exp(I*(f*x+e))+1)*x+9/f^2*b^2*a*d^3*ln(1-exp(I*(f*x+e)))*x^2-9/f^4*b^2*a*d^3*ln(1-exp(I*(f*x+e)))*e^2+9/f^2*b^2*a*d^3*ln(exp(I*(f*x+e))+1)*x^2+18/f^3*b*a^2*c*d^2*polylog(3,exp(I*(f*x+e)))+18/f^3*b*a^2*c*d^2*polylog(3,-exp(I*(f*x+e)))+b^2*(-6*I*b*c*d^2*x*exp(2*I*(f*x+e))+18*I*a*c^2*d*f*x+2*b*d^3*f*x^3*exp(2*I*(f*x+e))-3*I*b*d^3*x^2*exp(2*I*(f*x+e))-18*I*a*c^2*d*f*x*exp(2*I*(f*x+e))+3*I*b*d^3*x^2+6*b*c*d^2*f*x^2*exp(2*I*(f*x+e))+6*I*b*c*d^2*x+3*I*b*c^2*d-6*I*a*c^3*f*exp(2*I*(f*x+e))+6*b*c^2*d*f*x*exp(2*I*(f*x+e))-18*I*a*c*d^2*f*x^2*exp(2*I*(f*x+e))+18*I*a*c*d^2*f*x^2+6*I*a*c^3*f+2*b*c^3*f*exp(2*I*(f*x+e))-3*I*b*c^2*d*exp(2*I*(f*x+e))-6*I*a*d^3*f*x^3*exp(2*I*(f*x+e))+6*I*a*d^3*f*x^3)/f^2/(exp(2*I*(f*x+e))-1)^2-1/f^4*b^3*d^3*ln(1-exp(I*(f*x+e)))*e^3-1/f*b^3*d^3*ln(1-exp(I*(f*x+e)))*x^3+3/f*b*a^2*c^3*ln(exp(I*(f*x+e))+1)-6
```


$$\begin{aligned}
& /f*b*a^2*c^3*\ln(\exp(I*(f*x+e)))+3/f*b*a^2*c^3*\ln(\exp(I*(f*x+e))-1)-3*I/f^2* \\
& b^3*d^3*x^2-3*I/f^4*b^3*d^3*e^2-3*I/f^4*b^3*d^3*\text{polylog}(2,\exp(I*(f*x+e)))-3 \\
& *I/f^4*b^3*d^3*\text{polylog}(2,-\exp(I*(f*x+e)))+3/2*I/f^4*b^3*d^3*e^4-6*I/f^4*b^3 \\
& *d^3*\text{polylog}(4,\exp(I*(f*x+e)))-6*I/f^4*b^3*d^3*\text{polylog}(4,-\exp(I*(f*x+e)))+1 \\
& /4*d^3*a^3*x^4+1/4/d*c^4*a^3-3/4*d^3*a*b^2*x^4-I*b^3*c^3*x-3*a*b^2*c^3*x-3/ \\
& 4/d*a*b^2*c^4-1/4*I/d*b^3*c^4+9/f*b*d*c^2*a^2*\ln(1-\exp(I*(f*x+e)))*x+9/f^2* \\
& b*d*c^2*a^2*\ln(1-\exp(I*(f*x+e)))*e+9/f*b*d*c^2*a^2*\ln(\exp(I*(f*x+e))+1)*x+1 \\
& 8/f^2*b^2*d^2*c*a*\ln(1-\exp(I*(f*x+e)))*x+18/f^2*b^2*d^2*c*a*\ln(\exp(I*(f*x+e) \\
&))+1)*x+18*I/f^3*b^2*a*d^3*e^2*x-18*I/f^3*b^2*a*d^3*\text{polylog}(2,\exp(I*(f*x+e) \\
&))*x-18*I/f^3*b^2*a*d^3*\text{polylog}(2,-\exp(I*(f*x+e)))*x-9*I/f^2*b*d*c^2*a^2*e^ \\
& 2-9*I/f^2*b*d*c^2*a^2*\text{polylog}(2,\exp(I*(f*x+e)))-9*I/f^2*b*d*c^2*a^2*\text{polylog} \\
& (2,-\exp(I*(f*x+e)))-9*I/f^2*b*d^3*a^2*\text{polylog}(2,\exp(I*(f*x+e)))*x^2-9*I/f^2 \\
& *b*d^3*a^2*\text{polylog}(2,-\exp(I*(f*x+e)))*x^2-6*I/f^2*b^3*c*d^2*e^2*x+6*I/f^2*b \\
& ^3*c*d^2*\text{polylog}(2,\exp(I*(f*x+e)))*x+6*I/f^2*b^3*c*d^2*\text{polylog}(2,-\exp(I*(f \\
& x+e)))*x-6*I/f^3*b*d^3*a^2*e^3*x-18*I/f*b^2*d^2*c*a*x^2+12*I/f^3*b*e^3*d^2* \\
& c*a^2+6*I/f*b^3*d*c^2*e*x-18*I/f^3*b^2*d^2*c*a*e^2-18*I/f^3*b^2*d^2*c*a*\text{pol} \\
& \text{ylog}(2,\exp(I*(f*x+e)))-18*I/f^3*b^2*d^2*c*a*\text{polylog}(2,-\exp(I*(f*x+e)))+d^2* \\
& a^3*c*x^3+3/2*d*a^3*c^2*x^2+a^3*c^3*x+1/4*I*b^3*d^3*x^4+36/f^3*b^2*e*a*c*d^ \\
& 2*\ln(\exp(I*(f*x+e)))-18/f^3*b^2*e*a*c*d^2*\ln(\exp(I*(f*x+e))-1)+18/f^2*b*e*d \\
& *a^2*c^2*\ln(\exp(I*(f*x+e)))-9/f^2*b*e*d*a^2*c^2*\ln(\exp(I*(f*x+e))-1)+9/f*b \\
& a^2*c*d^2*\ln(1-\exp(I*(f*x+e)))*x^2+9/f*b*a^2*c*d^2*\ln(\exp(I*(f*x+e))+1)*x^2 \\
& +18/f^3*b^2*d^2*c*a*\ln(1-\exp(I*(f*x+e)))*e-18/f^3*b*e^2*a^2*c*d^2*\ln(\exp(I* \\
& (f*x+e)))+9/f^3*b*e^2*a^2*c*d^2*\ln(\exp(I*(f*x+e))-1)-9/f^3*b*e^2*d^2*c*a^2* \\
& \ln(1-\exp(I*(f*x+e)))-6/f^3*b^3*c*d^2*\text{polylog}(3,\exp(I*(f*x+e)))+I*b^3*c*d^2* \\
& x^3-6/f^3*b^3*c*d^2*\text{polylog}(3,-\exp(I*(f*x+e)))+3/f^3*b^3*c*d^2*\ln(\exp(I*(f \\
& x+e))+1)-6/f^3*b^3*c*d^2*\ln(\exp(I*(f*x+e)))+3/f^3*b^3*c*d^2*\ln(\exp(I*(f*x+e) \\
&))-1)-6/f^3*b^3*d^3*\text{polylog}(3,\exp(I*(f*x+e)))*x+3/f^3*b^3*d^3*\ln(1-\exp(I*(f \\
& *x+e)))*x+3/f^3*b^3*d^3*\ln(\exp(I*(f*x+e))+1)*x-6/f^3*b^3*d^3*\text{polylog}(3,-\exp \\
& (I*(f*x+e)))*x+6/f^4*b^3*e*d^3*\ln(\exp(I*(f*x+e)))-3/f^4*b^3*e*d^3*\ln(\exp(I* \\
& (f*x+e))-1)+18/f^4*b^2*a*d^3*\text{polylog}(3,\exp(I*(f*x+e)))+18/f^4*b^2*a*d^3*\text{pol} \\
& \text{ylog}(3,-\exp(I*(f*x+e)))-2/f^4*b^3*e^3*d^3*\ln(\exp(I*(f*x+e)))+1/f^4*b^3*e^3* \\
& d^3*\ln(\exp(I*(f*x+e))-1)-1/f*b^3*d^3*\ln(\exp(I*(f*x+e))+1)*x^3+3/f^4*b^3*d^3 \\
& *\ln(1-\exp(I*(f*x+e)))*e+3/2*I*b^3*d*c^2*x^2-3/4*I*d^3*a^2*b*x^4-3*d^2*a*b^2 \\
& *c*x^3-9/2*d*a*b^2*c^2*x^2+3*I*a^2*b*c^3*x+3/4*I/d*a^2*b*c^4-36*I/f^2*b^2*d \\
& ^2*c*a*e*x+18*I/f^2*b*e^2*d^2*c*a^2*x-18*I/f*b*d*c^2*a^2*e*x-18*I/f^2*b*a^2 \\
& *c*d^2*\text{polylog}(2,\exp(I*(f*x+e)))*x-18*I/f^2*b*a^2*c*d^2*\text{polylog}(2,-\exp(I*(f \\
& *x+e)))*x
\end{aligned}$$

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2749 vs. $2(521) = 1042$.

Time = 0.37 (sec) , antiderivative size = 2749, normalized size of antiderivative = 4.56

$$\int (c + dx)^3 (a + b \cot(e + fx))^3 dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(a^3 - 3*a*b^2)*d^3*f^4*x^4 - 8*b^3*c^3*f^3 - 8*(b^3*d^3*f^3 - (a^3
- 3*a*b^2)*c*d^2*f^4)*x^3 - 12*(2*b^3*c*d^2*f^3 - (a^3 - 3*a*b^2)*c^2*d*f^
4)*x^2 - 8*(3*b^3*c^2*d*f^3 - (a^3 - 3*a*b^2)*c^3*f^4)*x - 2*((a^3 - 3*a*b^
2)*d^3*f^4*x^4 + 4*(a^3 - 3*a*b^2)*c*d^2*f^4*x^3 + 6*(a^3 - 3*a*b^2)*c^2*d*
f^4*x^2 + 4*(a^3 - 3*a*b^2)*c^3*f^4*x)*cos(2*f*x + 2*e) + 6*(-I*(3*a^2*b -
b^3)*d^3*f^2*x^2 - 6*I*a*b^2*c*d^2*f - I*b^3*d^3 - I*(3*a^2*b - b^3)*c^2*d*
f^2 - 2*I*(3*a*b^2*d^3*f + (3*a^2*b - b^3)*c*d^2*f^2)*x + (I*(3*a^2*b - b^3
)*d^3*f^2*x^2 + 6*I*a*b^2*c*d^2*f + I*b^3*d^3 + I*(3*a^2*b - b^3)*c^2*d*f^2
+ 2*I*(3*a*b^2*d^3*f + (3*a^2*b - b^3)*c*d^2*f^2)*x)*cos(2*f*x + 2*e))*dilog(cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) + 6*(I*(3*a^2*b - b^3)*d^3*f^2*x^
2 + 6*I*a*b^2*c*d^2*f + I*b^3*d^3 + I*(3*a^2*b - b^3)*c^2*d*f^2 + 2*I*(3*a*
b^2*d^3*f + (3*a^2*b - b^3)*c*d^2*f^2)*x + (-I*(3*a^2*b - b^3)*d^3*f^2*x^2
- 6*I*a*b^2*c*d^2*f - I*b^3*d^3 - I*(3*a^2*b - b^3)*c^2*d*f^2 - 2*I*(3*a*b^
2*d^3*f + (3*a^2*b - b^3)*c*d^2*f^2)*x)*cos(2*f*x + 2*e))*dilog(cos(2*f*x +
2*e) - I*sin(2*f*x + 2*e)) + 4*(9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b -
b^3)*d^3*e^3 + (3*a^2*b - b^3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b - b^3
)*c^2*d*e)*f^2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b - b^3)*c*d^2*e^2
)*f - (9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b - b^3)*d^3*e^3 + (3*a^2*b -
b^3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b - b^3)*c^2*d*e)*f^2 - 3*(6*a*b^
2*c*d^2*e - b^3*c*d^2 - (3*a^2*b - b^3)*c*d^2*e^2)*f)*cos(2*f*x + 2*e))*log
(-1/2*cos(2*f*x + 2*e) + 1/2*I*sin(2*f*x + 2*e) + 1/2) + 4*(9*a*b^2*d^3*e^2
- 3*b^3*d^3*e - (3*a^2*b - b^3)*d^3*e^3 + (3*a^2*b - b^3)*c^3*f^3 + 3*(3*a
*b^2*c^2*d - (3*a^2*b - b^3)*c^2*d*e)*f^2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^2
- (3*a^2*b - b^3)*c*d^2*e^2)*f - (9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b
- b^3)*d^3*e^3 + (3*a^2*b - b^3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b - b^
3)*c^2*d*e)*f^2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b - b^3)*c*d^2*e^
2)*f)*cos(2*f*x + 2*e))*log(-1/2*cos(2*f*x + 2*e) - 1/2*I*sin(2*f*x + 2*e)
+ 1/2) + 4*((3*a^2*b - b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (
3*a^2*b - b^3)*d^3*e^3 + 3*(3*a^2*b - b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2
+ (3*a^2*b - b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b - b^3)*c*
d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b - b^3)*c^2*d*f^3)*
x - ((3*a^2*b - b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b
- b^3)*d^3*e^3 + 3*(3*a^2*b - b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a
^2*b - b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b - b^3)*c*d^2*e^2
```

```

)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b - b^3)*c^2*d*f^3)*x)*cos(
2*f*x + 2*e))*log(-cos(2*f*x + 2*e) + I*sin(2*f*x + 2*e) + 1) + 4*((3*a^2*b
- b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b - b^3)*d^3*e
^3 + 3*(3*a^2*b - b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b - b^3)*c
*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b - b^3)*c*d^2*e^2)*f + 3*(6*a*
b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b - b^3)*c^2*d*f^3)*x - ((3*a^2*b - b^3)
*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b - b^3)*d^3*e^3 + 3*
(3*a^2*b - b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b - b^3)*c*d^2*f^
3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b - b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d
^2*f^2 + b^3*d^3*f + (3*a^2*b - b^3)*c^2*d*f^3)*x)*cos(2*f*x + 2*e))*log(-c
os(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1) + 3*(-I*(3*a^2*b - b^3)*d^3*cos(2
*f*x + 2*e) + I*(3*a^2*b - b^3)*d^3)*polylog(4, cos(2*f*x + 2*e) + I*sin(2*
f*x + 2*e)) + 3*(I*(3*a^2*b - b^3)*d^3*cos(2*f*x + 2*e) - I*(3*a^2*b - b^3)
*d^3)*polylog(4, cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e)) + 6*(3*a*b^2*d^3 +
(3*a^2*b - b^3)*d^3*f*x + (3*a^2*b - b^3)*c*d^2*f - (3*a*b^2*d^3 + (3*a^2*b
- b^3)*d^3*f*x + (3*a^2*b - b^3)*c*d^2*f)*cos(2*f*x + 2*e))*polylog(3, cos
(2*f*x + 2*e) + I*sin(2*f*x + 2*e)) + 6*(3*a*b^2*d^3 + (3*a^2*b - b^3)*d^3*
f*x + (3*a^2*b - b^3)*c*d^2*f - (3*a*b^2*d^3 + (3*a^2*b - b^3)*d^3*f*x + (3
*a^2*b - b^3)*c*d^2*f)*cos(2*f*x + 2*e))*polylog(3, cos(2*f*x + 2*e) - I*si
n(2*f*x + 2*e)) - 12*(2*a*b^2*d^3*f^3*x^3 + 2*a*b^2*c^3*f^3 + b^3*c^2*d*f^2
+ (6*a*b^2*c*d^2*f^3 + b^3*d^3*f^2)*x^2 + 2*(3*a*b^2*c^2*d*f^3 + b^3*c*d^2
*f^2)*x)*sin(2*f*x + 2*e))/(f^4*cos(2*f*x + 2*e) - f^4)

```

Sympy [F]

$$\int (c + dx)^3 (a + b \cot(e + fx))^3 dx = \int (a + b \cot(e + fx))^3 (c + dx)^3 dx$$

```
[In] integrate((d*x+c)**3*(a+b*cot(f*x+e))**3,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**3*(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 11252 vs. $2(521) = 1042$.

Time = 15.55 (sec) , antiderivative size = 11252, normalized size of antiderivative = 18.66

$$\int (c + dx)^3 (a + b \cot(e + fx))^3 dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*a^3*c^3 + (f*x + e)^4*a^3*d^3/f^3 - 4*(f*x + e)^3*a^3*d^3*
e/f^3 + 6*(f*x + e)^2*a^3*d^3*e^2/f^3 - 4*(f*x + e)*a^3*d^3*e^3/f^3 + 4*(f*
x + e)^3*a^3*c*d^2/f^2 - 12*(f*x + e)^2*a^3*c*d^2*e/f^2 + 12*(f*x + e)*a^3*
c*d^2*e^2/f^2 + 6*(f*x + e)^2*a^3*c^2*d/f - 12*(f*x + e)*a^3*c^2*d*e/f + 12
*a^2*b*c^3*log(sin(f*x + e)) - 12*a^2*b*d^3*e^3*log(sin(f*x + e))/f^3 + 36*
a^2*b*c*d^2*e^2*log(sin(f*x + e))/f^2 - 36*a^2*b*c^2*d*e*log(sin(f*x + e))/
f - 4*(24*a*b^2*d^3*e^3 - 24*a*b^2*c^3*f^3 + (3*a^2*b - 3*I*a*b^2 - b^3)*(f
*x + e)^4*d^3 - 12*b^3*d^3*e^2 - 4*((3*a^2*b - 3*I*a*b^2 - b^3)*d^3*e - (3*
a^2*b - 3*I*a*b^2 - b^3)*c*d^2*f)*(f*x + e)^3 + 6*((3*a^2*b - 3*I*a*b^2 - b
^3)*d^3*e^2 - 2*(3*a^2*b - 3*I*a*b^2 - b^3)*c*d^2*e*f + (3*a^2*b - 3*I*a*b^
2 - b^3)*c^2*d*f^2)*(f*x + e)^2 + 12*(6*a*b^2*c^2*d*e - b^3*c^2*d)*f^2 - 4*
((-3*I*a*b^2 - b^3)*d^3*e^3 + 3*(3*I*a*b^2 + b^3)*c*d^2*e^2*f + 3*(-3*I*a*b
^2 - b^3)*c^2*d*e*f^2 + (3*I*a*b^2 + b^3)*c^3*f^3)*(f*x + e) - 24*(3*a*b^2*
c*d^2*e^2 - b^3*c*d^2*e)*f - 4*(b^3*d^3*e^3 - b^3*c^3*f^3 + 9*a*b^2*d^3*e^2
+ (3*a^2*b - b^3)*(f*x + e)^3*d^3 - 3*b^3*d^3*e + 3*(3*a*b^2*d^3 - (3*a^2*
b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x + e)^2 + 3*(b^3*c^2*d*e + 3*
a*b^2*c^2*d)*f^2 - 3*(6*a*b^2*d^3*e - b^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (
3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f*(
f*x + e) - 3*(b^3*c*d^2*e^2 + 6*a*b^2*c*d^2*e - b^3*c*d^2)*f + (b^3*d^3*e^3
- b^3*c^3*f^3 + 9*a*b^2*d^3*e^2 + (3*a^2*b - b^3)*(f*x + e)^3*d^3 - 3*b^3*
d^3*e + 3*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(
f*x + e)^2 + 3*(b^3*c^2*d*e + 3*a*b^2*c^2*d)*f^2 - 3*(6*a*b^2*d^3*e - b^3*d
^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*c*d^2
- (3*a^2*b - b^3)*c*d^2*e)*f*(f*x + e) - 3*(b^3*c*d^2*e^2 + 6*a*b^2*c*d^2
e - b^3*c*d^2)*f)*cos(4*f*x + 4*e) - 2*(b^3*d^3*e^3 - b^3*c^3*f^3 + 9*a*b^
2*d^3*e^2 + (3*a^2*b - b^3)*(f*x + e)^3*d^3 - 3*b^3*d^3*e + 3*(3*a*b^2*d^3
- (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x + e)^2 + 3*(b^3*c^2
*d*e + 3*a*b^2*c^2*d)*f^2 - 3*(6*a*b^2*d^3*e - b^3*d^3 - (3*a^2*b - b^3)*d^
3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^
2*e)*f*(f*x + e) - 3*(b^3*c*d^2*e^2 + 6*a*b^2*c*d^2*e - b^3*c*d^2)*f)*cos(
2*f*x + 2*e) + (I*b^3*d^3*e^3 - I*b^3*c^3*f^3 + 9*I*a*b^2*d^3*e^2 + (3*I*a^
2*b - I*b^3)*(f*x + e)^3*d^3 - 3*I*b^3*d^3*e + 3*(3*I*a*b^2*d^3 + (-3*I*a^2
*b + I*b^3)*d^3*e + (3*I*a^2*b - I*b^3)*c*d^2*f)*(f*x + e)^2 + 3*(I*b^3*c^2
*d*e + 3*I*a*b^2*c^2*d)*f^2 + 3*(-6*I*a*b^2*d^3*e + I*b^3*d^3 + (3*I*a^2*b
- I*b^3)*d^3*e^2 + (3*I*a^2*b - I*b^3)*c^2*d*f^2 + 2*(3*I*a*b^2*c*d^2 + (-3
*I*a^2*b + I*b^3)*c*d^2*e)*f*(f*x + e) + 3*(-I*b^3*c*d^2*e^2 - 6*I*a*b^2*c
*d^2*e + I*b^3*c*d^2)*f)*sin(4*f*x + 4*e) + 2*(-I*b^3*d^3*e^3 + I*b^3*c^3*f
^3 - 9*I*a*b^2*d^3*e^2 + (-3*I*a^2*b + I*b^3)*(f*x + e)^3*d^3 + 3*I*b^3*d^3
*e + 3*(-3*I*a*b^2*d^3 + (3*I*a^2*b - I*b^3)*d^3*e + (-3*I*a^2*b + I*b^3)*c
*d^2*f)*(f*x + e)^2 + 3*(-I*b^3*c^2*d*e - 3*I*a*b^2*c^2*d)*f^2 + 3*(6*I*a*b
^2*d^3*e - I*b^3*d^3 + (-3*I*a^2*b + I*b^3)*d^3*e^2 + (-3*I*a^2*b + I*b^3)*
c^2*d*f^2 + 2*(-3*I*a*b^2*c*d^2 + (3*I*a^2*b - I*b^3)*c*d^2*e)*f*(f*x + e)
+ 3*(I*b^3*c*d^2*e^2 + 6*I*a*b^2*c*d^2*e - I*b^3*c*d^2)*f)*sin(2*f*x + 2*e
))*arctan2(sin(f*x + e), cos(f*x + e) + 1) - 4*(b^3*d^3*e^3 - b^3*c^3*f^3 +
9*a*b^2*d^3*e^2 - 3*b^3*d^3*e + 3*(b^3*c^2*d*e + 3*a*b^2*c^2*d)*f^2 - 3*(b
```

$$\begin{aligned}
& ^3*c*d^2*e^2 + 6*a*b^2*c*d^2*e - b^3*c*d^2)*f + (b^3*d^3*e^3 - b^3*c^3*f^3 \\
& + 9*a*b^2*d^3*e^2 - 3*b^3*d^3*e + 3*(b^3*c^2*d*e + 3*a*b^2*c^2*d)*f^2 - 3*(\\
& b^3*c*d^2*e^2 + 6*a*b^2*c*d^2*e - b^3*c*d^2)*f)*\cos(4*f*x + 4*e) - 2*(b^3*d \\
& ^3*e^3 - b^3*c^3*f^3 + 9*a*b^2*d^3*e^2 - 3*b^3*d^3*e + 3*(b^3*c^2*d*e + 3*a \\
& *b^2*c^2*d)*f^2 - 3*(b^3*c*d^2*e^2 + 6*a*b^2*c*d^2*e - b^3*c*d^2)*f)*\cos(2* \\
& f*x + 2*e) + (I*b^3*d^3*e^3 - I*b^3*c^3*f^3 + 9*I*a*b^2*d^3*e^2 - 3*I*b^3*d \\
& ^3*e + 3*(I*b^3*c^2*d*e + 3*I*a*b^2*c^2*d)*f^2 + 3*(-I*b^3*c*d^2*e^2 - 6*I \\
& a*b^2*c*d^2*e + I*b^3*c*d^2)*f)*\sin(4*f*x + 4*e) + 2*(-I*b^3*d^3*e^3 + I*b^ \\
& 3*c^3*f^3 - 9*I*a*b^2*d^3*e^2 + 3*I*b^3*d^3*e + 3*(-I*b^3*c^2*d*e - 3*I*a*b \\
& ^2*c^2*d)*f^2 + 3*(I*b^3*c*d^2*e^2 + 6*I*a*b^2*c*d^2*e - I*b^3*c*d^2)*f)*\si \\
& n(2*f*x + 2*e))*\arctan2(\sin(f*x + e), \cos(f*x + e) - 1) + 4*((3*a^2*b - b^3 \\
&)*(f*x + e)^3*d^3 + 3*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3 \\
&)*c*d^2*f)*(f*x + e)^2 - 3*(6*a*b^2*d^3*e - b^3*d^3 - (3*a^2*b - b^3)*d^3*e \\
& ^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^2*e \\
&)*f)*(f*x + e) + ((3*a^2*b - b^3)*(f*x + e)^3*d^3 + 3*(3*a*b^2*d^3 - (3*a^2 \\
& *b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x + e)^2 - 3*(6*a*b^2*d^3*e - \\
& b^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2 \\
& *c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)*(f*x + e))*\cos(4*f*x + 4*e) - 2*((3*a^ \\
& 2*b - b^3)*(f*x + e)^3*d^3 + 3*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^ \\
& 2*b - b^3)*c*d^2*f)*(f*x + e)^2 - 3*(6*a*b^2*d^3*e - b^3*d^3 - (3*a^2*b - b \\
& ^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3 \\
&)*c*d^2*e)*f)*(f*x + e))*\cos(2*f*x + 2*e) - ((-3*I*a^2*b + I*b^3)*(f*x + e) \\
& ^3*d^3 + 3*(-3*I*a*b^2*d^3 + (3*I*a^2*b - I*b^3)*d^3*e + (-3*I*a^2*b + I*b^ \\
& 3)*c*d^2*f)*(f*x + e)^2 + 3*(6*I*a*b^2*d^3*e - I*b^3*d^3 + (-3*I*a^2*b + I \\
& b^3)*d^3*e^2 + (-3*I*a^2*b + I*b^3)*c^2*d*f^2 + 2*(-3*I*a*b^2*c*d^2 + (3*I \\
& a^2*b - I*b^3)*c*d^2*e)*f)*(f*x + e))*\sin(4*f*x + 4*e) - 2*((3*I*a^2*b - I \\
& b^3)*(f*x + e)^3*d^3 + 3*(3*I*a*b^2*d^3 + (-3*I*a^2*b + I*b^3)*d^3*e + (3*I \\
& *a^2*b - I*b^3)*c*d^2*f)*(f*x + e)^2 + 3*(-6*I*a*b^2*d^3*e + I*b^3*d^3 + (3 \\
& *I*a^2*b - I*b^3)*d^3*e^2 + (3*I*a^2*b - I*b^3)*c^2*d*f^2 + 2*(3*I*a*b^2*c \\
& d^2 + (-3*I*a^2*b + I*b^3)*c*d^2*e)*f)*(f*x + e))*\sin(2*f*x + 2*e))*\arctan2 \\
& (\sin(f*x + e), -\cos(f*x + e) + 1) + ((3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^ \\
& 4*d^3 + 4*(6*a*b^2*d^3 - (3*a^2*b - 3*I*a*b^2 - b^3)*d^3*e + (3*a^2*b - 3*I \\
& *a*b^2 - b^3)*c*d^2*f)*(f*x + e)^3 - 6*(12*a*b^2*d^3*e - 2*b^3*d^3 - (3*a^2 \\
& *b - 3*I*a*b^2 - b^3)*d^3*e^2 - (3*a^2*b - 3*I*a*b^2 - b^3)*c^2*d*f^2 - 2*(\\
& 6*a*b^2*c*d^2 - (3*a^2*b - 3*I*a*b^2 - b^3)*c*d^2*e)*f)*(f*x + e)^2 + 4*(18 \\
& *a*b^2*d^3*e^2 - 6*b^3*d^3*e - (-3*I*a*b^2 - b^3)*d^3*e^3 - (3*I*a*b^2 + b^ \\
& 3)*c^3*f^3 + 3*(6*a*b^2*c^2*d - (-3*I*a*b^2 - b^3)*c^2*d*e)*f^2 - 3*(12*a*b \\
& ^2*c*d^2*e - 2*b^3*c*d^2 + (3*I*a*b^2 + b^3)*c*d^2*e^2)*f)*(f*x + e))*\cos(4 \\
& *f*x + 4*e) - 2*((3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^4*d^3 - 6*b^3*d^3*e^ \\
& 2 + 4*(3*a*b^2 + I*b^3)*d^3*e^3 - 4*(3*a*b^2 + I*b^3)*c^3*f^3 - 4*((3*a^2*b \\
& - 3*I*a*b^2 - b^3)*d^3*e - (3*a^2*b - 3*I*a*b^2 - b^3)*c*d^2*f - (3*a*b^2 \\
& - I*b^3)*d^3)*(f*x + e)^3 + 6*(b^3*d^3 + (3*a^2*b - 3*I*a*b^2 - b^3)*d^3*e^ \\
& 2 + (3*a^2*b - 3*I*a*b^2 - b^3)*c^2*d*f^2 - 2*(3*a*b^2 - I*b^3)*d^3*e - 2*(\\
& (3*a^2*b - 3*I*a*b^2 - b^3)*c*d^2*e - (3*a*b^2 - I*b^3)*c*d^2)*f)*(f*x + e) \\
& ^2 - 6*(b^3*c^2*d - 2*(3*a*b^2 + I*b^3)*c^2*d*e)*f^2 - 4*(3*b^3*d^3*e - (3*
\end{aligned}$$

$$\begin{aligned}
& I*a*b^2 + b^3)*d^3*e^3 - (-3*I*a*b^2 - b^3)*c^3*f^3 - 3*(3*a*b^2 - I*b^3)*d \\
& ^3*e^2 - 3*((3*I*a*b^2 + b^3)*c^2*d*e + (3*a*b^2 - I*b^3)*c^2*d)*f^2 - 3*(b \\
& ^3*c*d^2 + (-3*I*a*b^2 - b^3)*c*d^2*e^2 - 2*(3*a*b^2 - I*b^3)*c*d^2*e)*f*(\\
& f*x + e) + 12*(b^3*c*d^2*e - (3*a*b^2 + I*b^3)*c*d^2*e^2)*f)*\cos(2*f*x + 2* \\
& e) - 12*(6*a*b^2*d^3*e - (3*a^2*b - b^3)*(f*x + e)^2*d^3 - b^3*d^3 - (3*a^2 \\
& *b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*d^3 - (3*a^2*b - \\
& b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x + e) - 2*(3*a*b^2*c*d^2 - (3*a^ \\
& 2*b - b^3)*c*d^2*e)*f + (6*a*b^2*d^3*e - (3*a^2*b - b^3)*(f*x + e)^2*d^3 - \\
& b^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2* \\
& d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x + e) - 2*(3*a*b \\
& ^2*c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)*\cos(4*f*x + 4*e) - 2*(6*a*b^2*d^3*e \\
& - (3*a^2*b - b^3)*(f*x + e)^2*d^3 - b^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3* \\
& a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b \\
& - b^3)*c*d^2*f)*(f*x + e) - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)* \\
& \cos(2*f*x + 2*e) + (6*I*a*b^2*d^3*e + (-3*I*a^2*b + I*b^3)*(f*x + e)^2*d^3 \\
& - I*b^3*d^3 + (-3*I*a^2*b + I*b^3)*d^3*e^2 + (-3*I*a^2*b + I*b^3)*c^2*d*f^2 \\
& + 2*(-3*I*a*b^2*d^3 + (3*I*a^2*b - I*b^3)*d^3*e + (-3*I*a^2*b + I*b^3)*c*d \\
& ^2*f)*(f*x + e) + 2*(-3*I*a*b^2*c*d^2 + (3*I*a^2*b - I*b^3)*c*d^2*e)*f)*\sin \\
& (4*f*x + 4*e) + 2*(-6*I*a*b^2*d^3*e + (3*I*a^2*b - I*b^3)*(f*x + e)^2*d^3 + \\
& I*b^3*d^3 + (3*I*a^2*b - I*b^3)*d^3*e^2 + (3*I*a^2*b - I*b^3)*c^2*d*f^2 + \\
& 2*(3*I*a*b^2*d^3 + (-3*I*a^2*b + I*b^3)*d^3*e + (3*I*a^2*b - I*b^3)*c*d^2*f \\
&)*(f*x + e) + 2*(3*I*a*b^2*c*d^2 + (-3*I*a^2*b + I*b^3)*c*d^2*e)*f)*\sin(2*f \\
& *x + 2*e))*\operatorname{dilog}(-e^{(I*f*x + I*e)}) - 12*(6*a*b^2*d^3*e - (3*a^2*b - b^3)*(f \\
& *x + e)^2*d^3 - b^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f \\
& ^2 - 2*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x \\
& + e) - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f + (6*a*b^2*d^3*e - (3 \\
& *a^2*b - b^3)*(f*x + e)^2*d^3 - b^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2* \\
& b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^ \\
& 3)*c*d^2*f)*(f*x + e) - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)*\cos(\\
& 4*f*x + 4*e) - 2*(6*a*b^2*d^3*e - (3*a^2*b - b^3)*(f*x + e)^2*d^3 - b^3*d^3 \\
& - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2*d^3 - (\\
& 3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*(f*x + e) - 2*(3*a*b^2*c*d^ \\
& 2 - (3*a^2*b - b^3)*c*d^2*e)*f)*\cos(2*f*x + 2*e) + (6*I*a*b^2*d^3*e + (-3*I \\
& *a^2*b + I*b^3)*(f*x + e)^2*d^3 - I*b^3*d^3 + (-3*I*a^2*b + I*b^3)*d^3*e^2 \\
& + (-3*I*a^2*b + I*b^3)*c^2*d*f^2 + 2*(-3*I*a*b^2*d^3 + (3*I*a^2*b - I*b^3)* \\
& d^3*e + (-3*I*a^2*b + I*b^3)*c*d^2*f)*(f*x + e) + 2*(-3*I*a*b^2*c*d^2 + (3* \\
& I*a^2*b - I*b^3)*c*d^2*e)*f)*\sin(4*f*x + 4*e) + 2*(-6*I*a*b^2*d^3*e + (3*I \\
& a^2*b - I*b^3)*(f*x + e)^2*d^3 + I*b^3*d^3 + (3*I*a^2*b - I*b^3)*d^3*e^2 + \\
& (3*I*a^2*b - I*b^3)*c^2*d*f^2 + 2*(3*I*a*b^2*d^3 + (-3*I*a^2*b + I*b^3)*d^3 \\
& *e + (3*I*a^2*b - I*b^3)*c*d^2*f)*(f*x + e) + 2*(3*I*a*b^2*c*d^2 + (-3*I*a^ \\
& 2*b + I*b^3)*c*d^2*e)*f)*\sin(2*f*x + 2*e))*\operatorname{dilog}(e^{(I*f*x + I*e)}) - 2*(-I*b \\
& ^3*d^3*e^3 + I*b^3*c^3*f^3 - 9*I*a*b^2*d^3*e^2 + (-3*I*a^2*b + I*b^3)*(f*x \\
& + e)^3*d^3 + 3*I*b^3*d^3*e + 3*(-3*I*a*b^2*d^3 + (3*I*a^2*b - I*b^3)*d^3*e \\
& + (-3*I*a^2*b + I*b^3)*c*d^2*f)*(f*x + e)^2 + 3*(-I*b^3*c^2*d*e - 3*I*a*b^2 \\
& *c^2*d)*f^2 + 3*(6*I*a*b^2*d^3*e - I*b^3*d^3 + (-3*I*a^2*b + I*b^3)*d^3*e^2
\end{aligned}$$

$$\begin{aligned}
&)*(f*x + e) + 3*(-I*b^3*c*d^2*e^2 - 6*I*a*b^2*c*d^2*e + I*b^3*c*d^2)*f*\cos \\
& (2*f*x + 2*e) + (b^3*d^3*e^3 - b^3*c^3*f^3 + 9*a*b^2*d^3*e^2 + (3*a^2*b - b^3) \\
& *(f*x + e)^3*d^3 - 3*b^3*d^3*e + 3*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e \\
& + (3*a^2*b - b^3)*c*d^2*f)*(f*x + e)^2 + 3*(b^3*c^2*d*e + 3*a*b^2*c^2*d)*f^2 \\
& - 3*(6*a*b^2*d^3*e - b^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)* \\
& c^2*d*f^2 - 2*(3*a*b^2*c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)*(f*x + e) - 3*(b \\
& ^3*c*d^2*e^2 + 6*a*b^2*c*d^2*e - b^3*c*d^2)*f*\sin(4*f*x + 4*e) - 2*(b^3*d^3 \\
& *e^3 - b^3*c^3*f^3 + 9*a*b^2*d^3*e^2 + (3*a^2*b - b^3)*(f*x + e)^3*d^3 - 3 \\
& *b^3*d^3*e + 3*(3*a*b^2*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2 \\
& *f)*(f*x + e)^2 + 3*(b^3*c^2*d*e + 3*a*b^2*c^2*d)*f^2 - 3*(6*a*b^2*d^3*e - \\
& b^3*d^3 - (3*a^2*b - b^3)*d^3*e^2 - (3*a^2*b - b^3)*c^2*d*f^2 - 2*(3*a*b^2* \\
& c*d^2 - (3*a^2*b - b^3)*c*d^2*e)*f)*(f*x + e) - 3*(b^3*c*d^2*e^2 + 6*a*b^2* \\
& c*d^2*e - b^3*c*d^2)*f*\sin(2*f*x + 2*e))*\log(\cos(f*x + e)^2 + \sin(f*x + e) \\
& ^2 - 2*\cos(f*x + e) + 1) - 24*((3*a^2*b - b^3)*d^3*\cos(4*f*x + 4*e) - 2*(3* \\
& a^2*b - b^3)*d^3*\cos(2*f*x + 2*e) + (3*I*a^2*b - I*b^3)*d^3*\sin(4*f*x + 4*e) \\
&) + 2*(-3*I*a^2*b + I*b^3)*d^3*\sin(2*f*x + 2*e) + (3*a^2*b - b^3)*d^3)*\text{poly} \\
& \log(4, -e^{(I*f*x + I*e)}) - 24*((3*a^2*b - b^3)*d^3*\cos(4*f*x + 4*e) - 2*(3* \\
& a^2*b - b^3)*d^3*\cos(2*f*x + 2*e) + (3*I*a^2*b - I*b^3)*d^3*\sin(4*f*x + 4*e) \\
&) + 2*(-3*I*a^2*b + I*b^3)*d^3*\sin(2*f*x + 2*e) + (3*a^2*b - b^3)*d^3)*\text{poly} \\
& \log(4, e^{(I*f*x + I*e)}) - 24*(-3*I*a*b^2*d^3 + (-3*I*a^2*b + I*b^3)*(f*x + \\
& e)*d^3 + (3*I*a^2*b - I*b^3)*d^3*e + (-3*I*a^2*b + I*b^3)*c*d^2*f + (-3*I*a \\
& *b^2*d^3 + (-3*I*a^2*b + I*b^3)*(f*x + e)*d^3 + (3*I*a^2*b - I*b^3)*d^3*e + \\
& (-3*I*a^2*b + I*b^3)*c*d^2*f)*\cos(4*f*x + 4*e) + 2*(3*I*a*b^2*d^3 + (3*I*a \\
& ^2*b - I*b^3)*(f*x + e)*d^3 + (-3*I*a^2*b + I*b^3)*d^3*e + (3*I*a^2*b - I*b \\
& ^3)*c*d^2*f)*\cos(2*f*x + 2*e) + (3*a*b^2*d^3 + (3*a^2*b - b^3)*(f*x + e)*d^3 \\
& - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*\sin(4*f*x + 4*e) - 2*(\\
& 3*a*b^2*d^3 + (3*a^2*b - b^3)*(f*x + e)*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^ \\
& 2*b - b^3)*c*d^2*f)*\sin(2*f*x + 2*e))*\text{polylog}(3, -e^{(I*f*x + I*e)}) - 24*(-3 \\
& *I*a*b^2*d^3 + (-3*I*a^2*b + I*b^3)*(f*x + e)*d^3 + (3*I*a^2*b - I*b^3)*d^3 \\
& *e + (-3*I*a^2*b + I*b^3)*c*d^2*f + (-3*I*a*b^2*d^3 + (-3*I*a^2*b + I*b^3)* \\
& (f*x + e)*d^3 + (3*I*a^2*b - I*b^3)*d^3*e + (-3*I*a^2*b + I*b^3)*c*d^2*f)*\cos \\
& (4*f*x + 4*e) + 2*(3*I*a*b^2*d^3 + (3*I*a^2*b - I*b^3)*(f*x + e)*d^3 + (- \\
& 3*I*a^2*b + I*b^3)*d^3*e + (3*I*a^2*b - I*b^3)*c*d^2*f)*\cos(2*f*x + 2*e) + \\
& (3*a*b^2*d^3 + (3*a^2*b - b^3)*(f*x + e)*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a \\
& ^2*b - b^3)*c*d^2*f)*\sin(4*f*x + 4*e) - 2*(3*a*b^2*d^3 + (3*a^2*b - b^3)*(f \\
& *x + e)*d^3 - (3*a^2*b - b^3)*d^3*e + (3*a^2*b - b^3)*c*d^2*f)*\sin(2*f*x + \\
& 2*e))*\text{polylog}(3, e^{(I*f*x + I*e)}) + ((3*I*a^2*b + 3*a*b^2 - I*b^3)*(f*x + e) \\
&)^4*d^3 - 4*(-6*I*a*b^2*d^3 + (3*I*a^2*b + 3*a*b^2 - I*b^3)*d^3*e + (-3*I*a \\
& ^2*b - 3*a*b^2 + I*b^3)*c*d^2*f)*(f*x + e)^3 - 6*(12*I*a*b^2*d^3*e - 2*I*b^3 \\
& *d^3 + (-3*I*a^2*b - 3*a*b^2 + I*b^3)*d^3*e^2 + (-3*I*a^2*b - 3*a*b^2 + I \\
& b^3)*c^2*d*f^2 + 2*(-6*I*a*b^2*c*d^2 + (3*I*a^2*b + 3*a*b^2 - I*b^3)*c*d^2* \\
& e)*f)*(f*x + e)^2 - 4*(-18*I*a*b^2*d^3*e^2 + 6*I*b^3*d^3*e + (3*a*b^2 - I*b \\
& ^3)*d^3*e^3 - (3*a*b^2 - I*b^3)*c^3*f^3 + 3*(-6*I*a*b^2*c^2*d + (3*a*b^2 - \\
& I*b^3)*c^2*d*e)*f^2 + 3*(12*I*a*b^2*c*d^2*e - 2*I*b^3*c*d^2 - (3*a*b^2 - I \\
& b^3)*c*d^2*e^2)*f)*(f*x + e))*\sin(4*f*x + 4*e) - 2*((3*I*a^2*b + 3*a*b^2 -
\end{aligned}$$

$$\begin{aligned}
& I^3 b^3 (f x + e)^4 d^3 - 6 I^3 b^3 d^3 e^2 + 4 (3 I^3 a b^2 - b^3) d^3 e^3 + 4 (-3 I^3 a b^2 + b^3) c^3 f^3 + 4 ((-3 I^3 a^2 b - 3 a b^2 + I^3 b^3) d^3 e + (3 I^3 a^2 b + 3 a b^2 - I^3 b^3) c d^2 f + (3 I^3 a b^2 + b^3) d^3) (f x + e)^3 + 6 (I^3 b^3 d^3 + (3 I^3 a^2 b + 3 a b^2 - I^3 b^3) d^3 e^2 + (3 I^3 a^2 b + 3 a b^2 - I^3 b^3) c^2 d f^2 + 2 (-3 I^3 a b^2 - b^3) d^3 e + 2 ((-3 I^3 a^2 b - 3 a b^2 + I^3 b^3) c d^2 e + (3 I^3 a b^2 + b^3) c d^2) f) (f x + e)^2 + 6 (-I^3 b^3 c^2 d + 2 (3 I^3 a b^2 - b^3) c^2 d e) f^2 + 4 (-3 I^3 b^3 d^3 e - (3 a b^2 - I^3 b^3) d^3 e^3 + (3 a b^2 - I^3 b^3) c^3 f^3 + 3 (3 I^3 a b^2 + b^3) d^3 e^2 - 3 ((3 a b^2 - I^3 b^3) c^2 d e - (3 I^3 a b^2 + b^3) c^2 d) f^2 + 3 (I^3 b^3 c d^2 + (3 a b^2 - I^3 b^3) c d^2 e^2 + 2 (-3 I^3 a b^2 - b^3) c d^2 e) f) (f x + e) + 12 (I^3 b^3 c d^2 e + (-3 I^3 a b^2 + b^3) c d^2 e^2) f \sin(2 f x + 2 e) / (-4 I^3 f^3 \cos(4 f x + 4 e) + 8 I^3 f^3 \cos(2 f x + 2 e) + 4 f^3 \sin(4 f x + 4 e) - 8 f^3 \sin(2 f x + 2 e) - 4 I^3 f^3) / f
\end{aligned}$$

Giac [F]

$$\int (c + dx)^3 (a + b \cot(e + fx))^3 dx = \int (dx + c)^3 (b \cot(fx + e) + a)^3 dx$$

[In] integrate((d*x+c)^3*(a+b*cot(f*x+e))^3,x, algorithm="giac")

[Out] integrate((d*x + c)^3*(b*cot(f*x + e) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int (c + dx)^3 (a + b \cot(e + fx))^3 dx = \int (a + b \cot(e + fx))^3 (c + dx)^3 dx$$

[In] int((a + b*cot(e + f*x))^3*(c + d*x)^3,x)

[Out] int((a + b*cot(e + f*x))^3*(c + d*x)^3, x)

3.48 $\int (c + dx)^2 (a + b \cot(e + fx))^3 dx$

Optimal result	331
Rubi [A] (verified)	332
Mathematica [B] (warning: unable to verify)	337
Maple [B] (verified)	339
Fricas [B] (verification not implemented)	340
Sympy [F]	341
Maxima [B] (verification not implemented)	341
Giac [F]	344
Mupad [F(-1)]	344

Optimal result

Integrand size = 20, antiderivative size = 433

$$\begin{aligned}
 \int (c + dx)^2 (a + b \cot(e + fx))^3 dx = & -\frac{b^3 c dx}{f} - \frac{b^3 d^2 x^2}{2f} - \frac{3iab^2(c + dx)^2}{f} + \frac{a^3(c + dx)^3}{3d} \\
 & - \frac{ia^2b(c + dx)^3}{d} - \frac{ab^2(c + dx)^3}{d} + \frac{ib^3(c + dx)^3}{3d} \\
 & - \frac{b^3d(c + dx) \cot(e + fx)}{f^2} - \frac{3ab^2(c + dx)^2 \cot(e + fx)}{f} \\
 & - \frac{b^3(c + dx)^2 \cot^2(e + fx)}{2f} \\
 & + \frac{6ab^2d(c + dx) \log(1 - e^{2i(e+fx)})}{f^2} \\
 & + \frac{3a^2b(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f} \\
 & - \frac{b^3(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f} \\
 & + \frac{b^3d^2 \log(\sin(e + fx))}{f^3} \\
 & - \frac{3iab^2d^2 \text{PolyLog}(2, e^{2i(e+fx)})}{f^3} \\
 & - \frac{3ia^2bd(c + dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} \\
 & + \frac{ib^3d(c + dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} \\
 & + \frac{3a^2bd^2 \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} \\
 & - \frac{b^3d^2 \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3}
 \end{aligned}$$

```

[Out] -b^3*c*d*x/f-1/2*b^3*d^2*x^2/f+I*b^3*d*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/
f^2+1/3*a^3*(d*x+c)^3/d-3*I*a*b^2*d^2*polylog(2,exp(2*I*(f*x+e)))/f^3-a*b^2
*(d*x+c)^3/d-3*I*a*b^2*(d*x+c)^2/f-b^3*d*(d*x+c)*cot(f*x+e)/f^2-3*a*b^2*(d*
x+c)^2*cot(f*x+e)/f-1/2*b^3*(d*x+c)^2*cot(f*x+e)^2/f+6*a*b^2*d*(d*x+c)*ln(1
-exp(2*I*(f*x+e)))/f^2+3*a^2*b*(d*x+c)^2*ln(1-exp(2*I*(f*x+e)))/f-b^3*(d*x+
c)^2*ln(1-exp(2*I*(f*x+e)))/f+b^3*d^2*ln(sin(f*x+e))/f^3-I*a^2*b*(d*x+c)^3/
d+1/3*I*b^3*(d*x+c)^3/d-3*I*a^2*b*d*(d*x+c)*polylog(2,exp(2*I*(f*x+e)))/f^2
+3/2*a^2*b*d^2*polylog(3,exp(2*I*(f*x+e)))/f^3-1/2*b^3*d^2*polylog(3,exp(2*
I*(f*x+e)))/f^3

```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3803, 3798, 2221, 2611, 2320, 6724, 3801, 2317, 2438, 32, 3556}

$$\int (c + dx)^2 (a + b \cot(e + fx))^3 dx = \frac{a^3(c + dx)^3}{3d} - \frac{3ia^2bd(c + dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{3a^2b(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f} - \frac{ia^2b(c + dx)^3}{d} + \frac{3a^2bd^2 \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} + \frac{6ab^2d(c + dx) \log(1 - e^{2i(e+fx)})}{f^2} - \frac{3ab^2(c + dx)^2 \cot(e + fx)}{f} - \frac{3iab^2(c + dx)^2}{f} - \frac{ab^2(c + dx)^3}{d} - \frac{3iab^2d^2 \text{PolyLog}(2, e^{2i(e+fx)})}{f^3} + \frac{ib^3d(c + dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} - \frac{b^3d(c + dx) \cot(e + fx)}{f^2} - \frac{b^3(c + dx)^2 \log(1 - e^{2i(e+fx)})}{f} - \frac{b^3(c + dx)^2 \cot^2(e + fx)}{2f} - \frac{b^3cdx}{f} + \frac{ib^3(c + dx)^3}{3d} - \frac{b^3d^2 \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} + \frac{b^3d^2 \log(\sin(e + fx))}{f^3} - \frac{b^3d^2x^2}{2f}$$

[In] Int[(c + d*x)^2*(a + b*Cot[e + f*x])^3,x]

[Out] -((b^3*c*d*x)/f) - (b^3*d^2*x^2)/(2*f) - ((3*I)*a*b^2*(c + d*x)^2)/f + (a^3*(c + d*x)^3)/(3*d) - (I*a^2*b*(c + d*x)^3)/d - (a*b^2*(c + d*x)^3)/d + ((I/3)*b^3*(c + d*x)^3)/d - (b^3*d*(c + d*x)*Cot[e + f*x])/f^2 - (3*a*b^2*(c + d*x)^2*Cot[e + f*x])/f - (b^3*(c + d*x)^2*Cot[e + f*x]^2)/(2*f) + (6*a*b^2*d*(c + d*x)*Log[1 - E^((2*I)*(e + f*x))])/f^2 + (3*a^2*b*(c + d*x)^2*Log[1 - E^((2*I)*(e + f*x))])/f - (b^3*(c + d*x)^2*Log[1 - E^((2*I)*(e + f*x))])/f + (b^3*d^2*Log[Sin[e + f*x]])/f^3 - ((3*I)*a*b^2*d^2*PolyLog[2, E^((2*I)*(e + f*x))])/f^3 - ((3*I)*a^2*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + (I*b^3*d*(c + d*x)*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + (3*a^2*b

$*d^2*PolyLog[3, E^((2*I)*(e + f*x))]/(2*f^3) - (b^3*d^2*PolyLog[3, E^((2*I)*(e + f*x))]/(2*f^3)$

Rule 32

$Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] \rightarrow Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] \&\& NeQ[m, -1]$

Rule 2221

$Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] \rightarrow Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] \&\& IGtQ[m, 0]$

Rule 2317

$Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] \rightarrow Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] \&\& GtQ[a, 0]$

Rule 2320

$Int[u_, x_Symbol] \rightarrow With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] \&\& !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] \&\& IntegerQ[m*n] \&\& !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] \&\& InverseFunctionQ[F[x]]]$

Rule 2438

$Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] \rightarrow Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] \&\& EqQ[c*d, 1]$

Rule 2611

$Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] \rightarrow Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] \&\& GtQ[m, 0]$

Rule 3556

$Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]$

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:= Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:= Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Dist[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:= Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:= Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (a^3(c + dx)^2 + 3a^2b(c + dx)^2 \cot(e + fx) + 3ab^2(c + dx)^2 \cot^2(e + fx) \\
&\quad + b^3(c + dx)^2 \cot^3(e + fx)) dx \\
&= \frac{a^3(c + dx)^3}{3d} + (3a^2b) \int (c + dx)^2 \cot(e + fx) dx \\
&\quad + (3ab^2) \int (c + dx)^2 \cot^2(e + fx) dx + b^3 \int (c + dx)^2 \cot^3(e + fx) dx \\
&= \frac{a^3(c + dx)^3}{3d} - \frac{ia^2b(c + dx)^3}{d} - \frac{3ab^2(c + dx)^2 \cot(e + fx)}{f} \\
&\quad - \frac{b^3(c + dx)^2 \cot^2(e + fx)}{2f} - (6ia^2b) \int \frac{e^{2i(e+fx)}(c + dx)^2}{1 - e^{2i(e+fx)}} dx \\
&\quad - (3ab^2) \int (c + dx)^2 dx - b^3 \int (c + dx)^2 \cot(e + fx) dx \\
&\quad + \frac{(6ab^2d) \int (c + dx) \cot(e + fx) dx}{f} + \frac{(b^3d) \int (c + dx) \cot^2(e + fx) dx}{f}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{3iab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{ia^2b(c+dx)^3}{d} - \frac{ab^2(c+dx)^3}{d} + \frac{ib^3(c+dx)^3}{3d} \\
&\quad - \frac{b^3d(c+dx)\cot(e+fx)}{f^2} - \frac{3ab^2(c+dx)^2\cot(e+fx)}{f} - \frac{b^3(c+dx)^2\cot^2(e+fx)}{2f} \\
&\quad + \frac{3a^2b(c+dx)^2\log(1-e^{2i(e+fx)})}{f} + (2ib^3) \int \frac{e^{2i(e+fx)}(c+dx)^2}{1-e^{2i(e+fx)}} dx \\
&\quad + \frac{(b^3d^2) \int \cot(e+fx) dx}{f^2} - \frac{(6a^2bd) \int (c+dx) \log(1-e^{2i(e+fx)}) dx}{f} \\
&\quad - \frac{(12iab^2d) \int \frac{e^{2i(e+fx)}(c+dx)}{1-e^{2i(e+fx)}} dx}{f} - \frac{(b^3d) \int (c+dx) dx}{f} \\
&= -\frac{b^3cdx}{f} - \frac{b^3d^2x^2}{2f} - \frac{3iab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{ia^2b(c+dx)^3}{d} \\
&\quad - \frac{ab^2(c+dx)^3}{d} + \frac{ib^3(c+dx)^3}{3d} - \frac{b^3d(c+dx)\cot(e+fx)}{f^2} \\
&\quad - \frac{3ab^2(c+dx)^2\cot(e+fx)}{f} - \frac{b^3(c+dx)^2\cot^2(e+fx)}{2f} \\
&\quad + \frac{6ab^2d(c+dx)\log(1-e^{2i(e+fx)})}{f^2} + \frac{3a^2b(c+dx)^2\log(1-e^{2i(e+fx)})}{f} \\
&\quad - \frac{b^3(c+dx)^2\log(1-e^{2i(e+fx)})}{f} + \frac{b^3d^2\log(\sin(e+fx))}{f^3} \\
&\quad - \frac{3ia^2bd(c+dx)\text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{(3ia^2bd^2) \int \text{PolyLog}(2, e^{2i(e+fx)}) dx}{f^2} \\
&\quad - \frac{(6ab^2d^2) \int \log(1-e^{2i(e+fx)}) dx}{f^2} + \frac{(2b^3d) \int (c+dx) \log(1-e^{2i(e+fx)}) dx}{f}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{b^3 c dx}{f} - \frac{b^3 d^2 x^2}{2f} - \frac{3iab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{ia^2b(c+dx)^3}{d} \\
&\quad - \frac{ab^2(c+dx)^3}{d} + \frac{ib^3(c+dx)^3}{3d} - \frac{b^3d(c+dx)\cot(e+fx)}{f^2} \\
&\quad - \frac{3ab^2(c+dx)^2\cot(e+fx)}{f} - \frac{b^3(c+dx)^2\cot^2(e+fx)}{2f} \\
&\quad + \frac{6ab^2d(c+dx)\log(1-e^{2i(e+fx)})}{f^2} + \frac{3a^2b(c+dx)^2\log(1-e^{2i(e+fx)})}{f} \\
&\quad - \frac{b^3(c+dx)^2\log(1-e^{2i(e+fx)})}{f} + \frac{b^3d^2\log(\sin(e+fx))}{f^3} \\
&\quad - \frac{3ia^2bd(c+dx)\text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{ib^3d(c+dx)\text{PolyLog}(2, e^{2i(e+fx)})}{f^2} \\
&\quad + \frac{(3a^2bd^2)\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x} dx, x, e^{2i(e+fx)}\right)}{2f^3} \\
&\quad + \frac{(3iab^2d^2)\text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2i(e+fx)}\right)}{f^3} - \frac{(ib^3d^2)\int \text{PolyLog}(2, e^{2i(e+fx)}) dx}{f^2} \\
&= -\frac{b^3 c dx}{f} - \frac{b^3 d^2 x^2}{2f} - \frac{3iab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{ia^2b(c+dx)^3}{d} - \frac{ab^2(c+dx)^3}{d} \\
&\quad + \frac{ib^3(c+dx)^3}{3d} - \frac{b^3d(c+dx)\cot(e+fx)}{f^2} - \frac{3ab^2(c+dx)^2\cot(e+fx)}{f} \\
&\quad - \frac{b^3(c+dx)^2\cot^2(e+fx)}{2f} + \frac{6ab^2d(c+dx)\log(1-e^{2i(e+fx)})}{f^2} \\
&\quad + \frac{3a^2b(c+dx)^2\log(1-e^{2i(e+fx)})}{f} - \frac{b^3(c+dx)^2\log(1-e^{2i(e+fx)})}{f} \\
&\quad + \frac{b^3d^2\log(\sin(e+fx))}{f^3} - \frac{3iab^2d^2\text{PolyLog}(2, e^{2i(e+fx)})}{f^3} \\
&\quad - \frac{3ia^2bd(c+dx)\text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{ib^3d(c+dx)\text{PolyLog}(2, e^{2i(e+fx)})}{f^2} \\
&\quad + \frac{3a^2bd^2\text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} - \frac{(b^3d^2)\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x} dx, x, e^{2i(e+fx)}\right)}{2f^3}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{b^3 c dx}{f} - \frac{b^3 d^2 x^2}{2f} - \frac{3iab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{ia^2b(c+dx)^3}{d} - \frac{ab^2(c+dx)^3}{d} \\
&+ \frac{ib^3(c+dx)^3}{3d} - \frac{b^3 d(c+dx) \cot(e+fx)}{f^2} - \frac{3ab^2(c+dx)^2 \cot(e+fx)}{f} \\
&- \frac{b^3(c+dx)^2 \cot^2(e+fx)}{2f} + \frac{6ab^2 d(c+dx) \log(1 - e^{2i(e+fx)})}{f^2} \\
&+ \frac{3a^2 b(c+dx)^2 \log(1 - e^{2i(e+fx)})}{f} - \frac{b^3(c+dx)^2 \log(1 - e^{2i(e+fx)})}{f} \\
&+ \frac{b^3 d^2 \log(\sin(e+fx))}{f^3} - \frac{3iab^2 d^2 \text{PolyLog}(2, e^{2i(e+fx)})}{f^3} \\
&- \frac{3ia^2 b d(c+dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} + \frac{ib^3 d(c+dx) \text{PolyLog}(2, e^{2i(e+fx)})}{f^2} \\
&+ \frac{3a^2 b d^2 \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3} - \frac{b^3 d^2 \text{PolyLog}(3, e^{2i(e+fx)})}{2f^3}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2029 vs. 2(433) = 866.

Time = 7.66 (sec) , antiderivative size = 2029, normalized size of antiderivative = 4.69

$$\int (c+dx)^2 (a+b \cot(e+fx))^3 dx = \text{Result too large to show}$$

[In] Integrate[(c + d*x)^2*(a + b*Cot[e + f*x])^3,x]

[Out]
$$\begin{aligned}
&-1/2*(a^2*b*d^2*E^{(I*e)}*Csc[e]*((2*f^3*x^3)/E^{((2*I)*e)} + (3*I)*(1 - E^{((-2*I)*e)})) * f^2*x^2*Log[1 - E^{((-I)*(e + f*x))}] + (3*I)*(1 - E^{((-2*I)*e)}) * f^2*x^2*Log[1 + E^{((-I)*(e + f*x))}] - 6*(1 - E^{((-2*I)*e)}) * f*x*PolyLog[2, -E^{((-I)*(e + f*x))}] - 6*(1 - E^{((-2*I)*e)}) * f*x*PolyLog[2, E^{((-I)*(e + f*x))}] + (6*I)*(1 - E^{((-2*I)*e)}) * PolyLog[3, -E^{((-I)*(e + f*x))}] + (6*I)*(1 - E^{((-2*I)*e)}) * PolyLog[3, E^{((-I)*(e + f*x))}]) / f^3 + (b^3*d^2*E^{(I*e)}*Csc[e]*((2*f^3*x^3)/E^{((2*I)*e)} + (3*I)*(1 - E^{((-2*I)*e)}) * f^2*x^2*Log[1 - E^{((-I)*(e + f*x))}] + (3*I)*(1 - E^{((-2*I)*e)}) * f^2*x^2*Log[1 + E^{((-I)*(e + f*x))}] - 6*(1 - E^{((-2*I)*e)}) * f*x*PolyLog[2, -E^{((-I)*(e + f*x))}] - 6*(1 - E^{((-2*I)*e)}) * f*x*PolyLog[2, E^{((-I)*(e + f*x))}] + (6*I)*(1 - E^{((-2*I)*e)}) * PolyLog[3, -E^{((-I)*(e + f*x))}] + (6*I)*(1 - E^{((-2*I)*e)}) * PolyLog[3, E^{((-I)*(e + f*x))}]) / (6*f^3) + (b^3*d^2*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e])) / (f^3*(Cos[e]^2 + Sin[e]^2)) + (6*a*b^2*c*d*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e])) / (f^2*(Cos[e]^2 + Sin[e]^2)) + (3*a^2*b*c^2*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e])) / (f*(Cos[e]^2 + Sin[e]^2)) - (b^3*c^2*Csc[e]*(-(f*x*Cos[e]) + Log[Cos[f*x]*Sin[e] + Cos[e]*Sin[f*x]]*Sin[e])) / (f*(Cos[e]^2 + Sin[e]^2))
\end{aligned}$$

$$\begin{aligned}
&^2 + \sin[e]^2)) + (\operatorname{Csc}[e] \operatorname{Csc}[e + f*x]^2 * (6*b^3*c*d*\operatorname{Cos}[e] + 18*a*b^2*c^2*f \\
&* \operatorname{Cos}[e] + 6*b^3*d^2*x*\operatorname{Cos}[e] + 36*a*b^2*c*d*f*x*\operatorname{Cos}[e] + 18*a^2*b*c^2*f^2*x \\
&* \operatorname{Cos}[e] - 6*b^3*c^2*f^2*x*\operatorname{Cos}[e] + 18*a*b^2*d^2*f*x^2*\operatorname{Cos}[e] + 18*a^2*b*c*d \\
&* f^2*x^2*\operatorname{Cos}[e] - 6*b^3*c*d*f^2*x^2*\operatorname{Cos}[e] + 6*a^2*b*d^2*f^2*x^3*\operatorname{Cos}[e] - 2 \\
&* b^3*d^2*f^2*x^3*\operatorname{Cos}[e] - 6*b^3*c*d*\operatorname{Cos}[e + 2*f*x] - 18*a*b^2*c^2*f*\operatorname{Cos}[e + \\
&2*f*x] - 6*b^3*d^2*x*\operatorname{Cos}[e + 2*f*x] - 36*a*b^2*c*d*f*x*\operatorname{Cos}[e + 2*f*x] - 9* \\
&a^2*b*c^2*f^2*x*\operatorname{Cos}[e + 2*f*x] + 3*b^3*c^2*f^2*x*\operatorname{Cos}[e + 2*f*x] - 18*a*b^2* \\
&d^2*f*x^2*\operatorname{Cos}[e + 2*f*x] - 9*a^2*b*c*d*f^2*x^2*\operatorname{Cos}[e + 2*f*x] + 3*b^3*c*d*f \\
&^2*x^2*\operatorname{Cos}[e + 2*f*x] - 3*a^2*b*d^2*f^2*x^3*\operatorname{Cos}[e + 2*f*x] + b^3*d^2*f^2*x^ \\
&3*\operatorname{Cos}[e + 2*f*x] - 9*a^2*b*c^2*f^2*x*\operatorname{Cos}[3*e + 2*f*x] + 3*b^3*c^2*f^2*x*\operatorname{Cos} \\
&[3*e + 2*f*x] - 9*a^2*b*c*d*f^2*x^2*\operatorname{Cos}[3*e + 2*f*x] + 3*b^3*c*d*f^2*x^2*\operatorname{Co} \\
&s[3*e + 2*f*x] - 3*a^2*b*d^2*f^2*x^3*\operatorname{Cos}[3*e + 2*f*x] + b^3*d^2*f^2*x^3*\operatorname{Cos} \\
&[3*e + 2*f*x] - 6*b^3*c^2*f*\operatorname{Sin}[e] - 12*b^3*c*d*f*x*\operatorname{Sin}[e] + 6*a^3*c^2*f^2*x \\
&*x*\operatorname{Sin}[e] - 18*a*b^2*c^2*f^2*x*\operatorname{Sin}[e] - 6*b^3*d^2*f*x^2*\operatorname{Sin}[e] + 6*a^3*c*d*f \\
&^2*x^2*\operatorname{Sin}[e] - 18*a*b^2*c*d*f^2*x^2*\operatorname{Sin}[e] + 2*a^3*d^2*f^2*x^3*\operatorname{Sin}[e] - 6* \\
&a*b^2*d^2*f^2*x^3*\operatorname{Sin}[e] + 3*a^3*c^2*f^2*x*\operatorname{Sin}[e + 2*f*x] - 9*a*b^2*c^2*f^2 \\
&>*x*\operatorname{Sin}[e + 2*f*x] + 3*a^3*c*d*f^2*x^2*\operatorname{Sin}[e + 2*f*x] - 9*a*b^2*c*d*f^2*x^2* \\
&\operatorname{Sin}[e + 2*f*x] + a^3*d^2*f^2*x^3*\operatorname{Sin}[e + 2*f*x] - 3*a*b^2*d^2*f^2*x^3*\operatorname{Sin}[e \\
&+ 2*f*x] - 3*a^3*c^2*f^2*x*\operatorname{Sin}[3*e + 2*f*x] + 9*a*b^2*c^2*f^2*x*\operatorname{Sin}[3*e + \\
&2*f*x] - 3*a^3*c*d*f^2*x^2*\operatorname{Sin}[3*e + 2*f*x] + 9*a*b^2*c*d*f^2*x^2*\operatorname{Sin}[3*e + \\
&2*f*x] - a^3*d^2*f^2*x^3*\operatorname{Sin}[3*e + 2*f*x] + 3*a*b^2*d^2*f^2*x^3*\operatorname{Sin}[3*e + \\
&2*f*x]))/(12*f^2) - (3*a*b^2*d^2*\operatorname{Csc}[e]*\operatorname{Sec}[e]*(E^(I*\operatorname{ArcTan}[\operatorname{Tan}[e]]))*f^2*x^ \\
&2 + ((I*f*x*(-\operatorname{Pi} + 2*\operatorname{ArcTan}[\operatorname{Tan}[e]]) - \operatorname{Pi}*\operatorname{Log}[1 + E^((-2*I)*f*x)] - 2*(f*x \\
&+ \operatorname{ArcTan}[\operatorname{Tan}[e]])*\operatorname{Log}[1 - E^((2*I)*(f*x + \operatorname{ArcTan}[\operatorname{Tan}[e]])])) + \operatorname{Pi}*\operatorname{Log}[\operatorname{Cos}[f*x] \\
&] + 2*\operatorname{ArcTan}[\operatorname{Tan}[e]]*\operatorname{Log}[\operatorname{Sin}[f*x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]]) + I*\operatorname{PolyLog}[2, E^((2* \\
&I)*(f*x + \operatorname{ArcTan}[\operatorname{Tan}[e]]))]*)*\operatorname{Tan}[e]/\operatorname{Sqrt}[1 + \operatorname{Tan}[e]^2]))/(f^3*\operatorname{Sqrt}[\operatorname{Sec}[e]^ \\
&2*(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2)] - (3*a^2*b*c*d*\operatorname{Csc}[e]*\operatorname{Sec}[e]*(E^(I*\operatorname{ArcTan}[\operatorname{Tan}[e] \\
&]))*f^2*x^2 + ((I*f*x*(-\operatorname{Pi} + 2*\operatorname{ArcTan}[\operatorname{Tan}[e]]) - \operatorname{Pi}*\operatorname{Log}[1 + E^((-2*I)*f*x)] - \\
&2*(f*x + \operatorname{ArcTan}[\operatorname{Tan}[e]])*\operatorname{Log}[1 - E^((2*I)*(f*x + \operatorname{ArcTan}[\operatorname{Tan}[e]])])) + \operatorname{Pi}*\operatorname{Lo} \\
&\operatorname{g}[\operatorname{Cos}[f*x]] + 2*\operatorname{ArcTan}[\operatorname{Tan}[e]]*\operatorname{Log}[\operatorname{Sin}[f*x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]]) + I*\operatorname{PolyL} \\
&\operatorname{og}[2, E^((2*I)*(f*x + \operatorname{ArcTan}[\operatorname{Tan}[e]]))]*)*\operatorname{Tan}[e]/\operatorname{Sqrt}[1 + \operatorname{Tan}[e]^2]))/(f^2*\operatorname{Sqrt} \\
&[\operatorname{Sec}[e]^2*(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2)] + (b^3*c*d*\operatorname{Csc}[e]*\operatorname{Sec}[e]*(E^(I*\operatorname{ArcTan}[\operatorname{Tan} \\
&[e]]))*f^2*x^2 + ((I*f*x*(-\operatorname{Pi} + 2*\operatorname{ArcTan}[\operatorname{Tan}[e]]) - \operatorname{Pi}*\operatorname{Log}[1 + E^((-2*I)*f*x) \\
&]) - 2*(f*x + \operatorname{ArcTan}[\operatorname{Tan}[e]])*\operatorname{Log}[1 - E^((2*I)*(f*x + \operatorname{ArcTan}[\operatorname{Tan}[e]])])) + P \\
&i*\operatorname{Log}[\operatorname{Cos}[f*x]] + 2*\operatorname{ArcTan}[\operatorname{Tan}[e]]*\operatorname{Log}[\operatorname{Sin}[f*x + \operatorname{ArcTan}[\operatorname{Tan}[e]]]]) + I*\operatorname{PolyL} \\
&\operatorname{og}[2, E^((2*I)*(f*x + \operatorname{ArcTan}[\operatorname{Tan}[e]]))]*)*\operatorname{Tan}[e]/\operatorname{Sqrt}[1 + \operatorname{Tan}[e]^2]))/(f^2* \\
&\operatorname{Sqrt}[\operatorname{Sec}[e]^2*(\operatorname{Cos}[e]^2 + \operatorname{Sin}[e]^2)])
\end{aligned}$$

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1787 vs. $2(399) = 798$.

Time = 0.93 (sec) , antiderivative size = 1788, normalized size of antiderivative = 4.13

method	result	size
risch	Expression too large to display	1788

[In] `int((d*x+c)^2*(a+b*cot(f*x+e))^3,x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -2/f^3*b^3*c*d*\ln(1-\exp(I*(f*x+e)))*x-6/f^3*b^3*e^2*a^2*d^2*\ln(\exp(I*(f*x+e)))+ \\ & 3/f^3*b^3*e^2*a^2*d^2*\ln(\exp(I*(f*x+e))-1)-4/f^2*b^3*e*d*c*\ln(\exp(I*(f*x+e))) \\ & +2*b^2*(-3*I*a*d^2*f*x^2*\exp(2*I*(f*x+e))-6*I*a*c*d*f*x*\exp(2*I*(f*x+e))+b \\ & d^2*f*x^2*\exp(2*I*(f*x+e))-3*I*a*c^2*f*\exp(2*I*(f*x+e))+3*I*a*d^2*f*x^2-I*b \\ & *d^2*x*\exp(2*I*(f*x+e))+2*b*c*d*f*x*\exp(2*I*(f*x+e))+6*I*a*c*d*f*x-I*b*c*d* \\ & \exp(2*I*(f*x+e))+b*c^2*f*\exp(2*I*(f*x+e))+3*I*a*c^2*f+I*b*d^2*x+I*b*c*d)/f^ \\ & 2/(\exp(2*I*(f*x+e))-1)^2+2/f^3*b^3*e^2*d^2*\ln(\exp(I*(f*x+e)))-1/f^3*b^3*e^2 \\ & *d^2*\ln(\exp(I*(f*x+e))-1)+1/f^3*b^3*d^2*\ln(1-\exp(I*(f*x+e)))*e^2+I*b^3*c*d* \\ & x^2-4/3*I/f^3*b^3*d^2*e^3-I*d^2*a^2*b*x^3-1/f*b^3*d^2*\ln(1-\exp(I*(f*x+e)))* \\ & x^2-1/f*b^3*d^2*\ln(\exp(I*(f*x+e))+1)*x^2+6/f^3*b^3*a^2*d^2*\text{polylog}(3,\exp(I*(f \\ & *x+e)))+6/f^3*b^3*a^2*d^2*\text{polylog}(3,-\exp(I*(f*x+e)))+3/f*b^3*a^2*c^2*\ln(\exp(I*(\\ & f*x+e))+1)-6/f*b^3*a^2*c^2*\ln(\exp(I*(f*x+e)))+3/f*b^3*a^2*c^2*\ln(\exp(I*(f*x+e)) \\ & -1)+2/f^2*b^3*e*d*c*\ln(\exp(I*(f*x+e))-1)-6*I/f^3*b^2*a*d^2*e^2-6*I/f^3*b^2*a \\ & *d^2*\text{polylog}(2,\exp(I*(f*x+e)))-2*I/f^2*b^3*d^2*e^2*x+2*I/f^2*b^3*d^2*\text{polyl} \\ & \text{og}(2,\exp(I*(f*x+e)))*x+2*I/f^2*b^3*d^2*\text{polylog}(2,-\exp(I*(f*x+e)))*x+2*I/f^2 \\ & *b^3*c*d*\text{polylog}(2,\exp(I*(f*x+e)))+2*I/f^2*b^3*c*d*\text{polylog}(2,-\exp(I*(f*x+e) \\ &))-6*I/f^3*b^2*a*d^2*\text{polylog}(2,-\exp(I*(f*x+e)))+4*I/f^3*b^3*a^2*d^2*e^3+2*I/f \\ & ^2*b^3*c*d*e^2-6*I/f*b^2*a*d^2*x^2+d*a^3*c*x^2+a^3*c^2*x-d^2*a*b^2*x^3-I*b^ \\ & 3*c^2*x-3*a*b^2*c^2*x-1/d*a*b^2*c^3-1/3*I/d*b^3*c^3+1/3*I*b^3*d^2*x^3+3*I*a \\ & ^2*b*c^2*x-12*I/f*b^3*a^2*c*d*e*x-3*d*a*b^2*c*x^2+I/d*a^2*b*c^3+1/f^3*b^3*d^2 \\ & *\ln(\exp(I*(f*x+e))+1)-2/f^3*b^3*d^2*\ln(\exp(I*(f*x+e)))+1/f^3*b^3*d^2*\ln(\exp \\ & (I*(f*x+e))-1)-2/f^3*b^3*d^2*\text{polylog}(3,\exp(I*(f*x+e)))-2/f^3*b^3*d^2*\text{polylo} \\ & \text{g}(3,-\exp(I*(f*x+e)))-1/f*b^3*c^2*\ln(\exp(I*(f*x+e))+1)+2/f*b^3*c^2*\ln(\exp(I* \\ & (f*x+e)))-1/f*b^3*c^2*\ln(\exp(I*(f*x+e))-1)-3*I*d*a^2*b*c*x^2+6/f*b^3*a^2*c*d* \\ & \ln(\exp(I*(f*x+e))+1)*x+12/f^2*b^3*e*d*a^2*c*\ln(\exp(I*(f*x+e)))-6/f^2*b^3*e*d*a^ \\ & 2*c*\ln(\exp(I*(f*x+e))-1)+6/f*b^3*a^2*c*d*\ln(1-\exp(I*(f*x+e)))*x+6/f^2*b^3*a^2*c \\ & *d*\ln(1-\exp(I*(f*x+e)))*e+4*I/f*b^3*c*d*e*x-12*I/f^2*b^2*a*d^2*e*x-6*I/f^2*b^2 \\ & *b^3*a^2*c*d*\text{polylog}(2,\exp(I*(f*x+e)))-6*I/f^2*b^3*a^2*c*d*\text{polylog}(2,-\exp(I*(f*x \\ & +e)))+6*I/f^2*b^3*a^2*d^2*e^2*x-6*I/f^2*b^3*a^2*d^2*\text{polylog}(2,\exp(I*(f*x+e)))*x \\ & -6*I/f^2*b^3*a^2*d^2*\text{polylog}(2,-\exp(I*(f*x+e)))*x-6*I/f^2*b^3*a^2*c*d*e^2-2/f*b \\ & ^3*c*d*\ln(\exp(I*(f*x+e))+1)*x+6/f^2*b^2*a*d^2*\ln(1-\exp(I*(f*x+e)))*x+12/f^3 \\ & *b^2*e*a*d^2*\ln(\exp(I*(f*x+e)))-3/f^3*b^3*a^2*d^2*\ln(1-\exp(I*(f*x+e)))*e^2+6/ \\ & f^2*b^2*a*c*d*\ln(\exp(I*(f*x+e))+1)-12/f^2*b^2*a*c*d*\ln(\exp(I*(f*x+e)))+6/f^ \\ & 2*b^2*a*c*d*\ln(\exp(I*(f*x+e))-1)+6/f^2*b^2*a*d^2*\ln(\exp(I*(f*x+e))+1)*x-2/f \end{aligned}$$

$$\begin{aligned} &^2*b^3*c*d*\ln(1-\exp(I*(f*x+e))) * e - 6/f^3*b^2*e*a*d^2*\ln(\exp(I*(f*x+e))-1) + 1/ \\ &3*d^2*a^3*x^3 + 1/3/d*a^3*c^3 + 6/f^3*b^2*a*d^2*\ln(1-\exp(I*(f*x+e))) * e + 3/f*b*a^ \\ &2*d^2*\ln(1-\exp(I*(f*x+e))) * x^2 + 3/f*b*a^2*d^2*\ln(\exp(I*(f*x+e))+1) * x^2 \end{aligned}$$

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1564 vs. $2(390) = 780$.

Time = 0.31 (sec) , antiderivative size = 1564, normalized size of antiderivative = 3.61

$$\int (c + dx)^2 (a + b \cot(e + fx))^3 dx = \text{Too large to display}$$

[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} &-1/12*(4*(a^3 - 3*a*b^2)*d^2*f^3*x^3 - 12*b^3*c^2*f^2 - 12*(b^3*d^2*f^2 - (\\ &a^3 - 3*a*b^2)*c*d*f^3)*x^2 - 12*(2*b^3*c*d*f^2 - (a^3 - 3*a*b^2)*c^2*f^3)* \\ &x - 4*((a^3 - 3*a*b^2)*d^2*f^3*x^3 + 3*(a^3 - 3*a*b^2)*c*d*f^3*x^2 + 3*(a^3 \\ &- 3*a*b^2)*c^2*f^3*x)*\cos(2*f*x + 2*e) + 6*(-3*I*a*b^2*d^2 - I*(3*a^2*b - \\ &b^3)*d^2*f*x - I*(3*a^2*b - b^3)*c*d*f + (3*I*a*b^2*d^2 + I*(3*a^2*b - b^3) \\ &*d^2*f*x + I*(3*a^2*b - b^3)*c*d*f)*\cos(2*f*x + 2*e))*\operatorname{dilog}(\cos(2*f*x + 2*e) \\ &) + I*\sin(2*f*x + 2*e)) + 6*(3*I*a*b^2*d^2 + I*(3*a^2*b - b^3)*d^2*f*x + I \\ &(3*a^2*b - b^3)*c*d*f + (-3*I*a*b^2*d^2 - I*(3*a^2*b - b^3)*d^2*f*x - I*(3 \\ &a^2*b - b^3)*c*d*f)*\cos(2*f*x + 2*e))*\operatorname{dilog}(\cos(2*f*x + 2*e) - I*\sin(2*f*x \\ &+ 2*e)) - 6*(6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b - b^3)*d^2*e^2 - (3*a^2*b - \\ &b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b - b^3)*c*d*e)*f - (6*a*b^2*d^2*e \\ &- b^3*d^2 - (3*a^2*b - b^3)*d^2*e^2 - (3*a^2*b - b^3)*c^2*f^2 - 2*(3*a*b^2* \\ &c*d - (3*a^2*b - b^3)*c*d*e)*f)*\cos(2*f*x + 2*e))*\log(-1/2*\cos(2*f*x + 2*e) \\ &+ 1/2*I*\sin(2*f*x + 2*e) + 1/2) - 6*(6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b - \\ &b^3)*d^2*e^2 - (3*a^2*b - b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b - b^3)*c \\ &*d*e)*f - (6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b - b^3)*d^2*e^2 - (3*a^2*b - b \\ &^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b - b^3)*c*d*e)*f)*\cos(2*f*x + 2*e))* \\ &\log(-1/2*\cos(2*f*x + 2*e) - 1/2*I*\sin(2*f*x + 2*e) + 1/2) + 6*((3*a^2*b - b \\ &^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*d^2*e^2 + 2*(3*a^2*b - b^ \\ &3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b - b^3)*c*d*f^2)*x - ((3*a^2*b - b^ \\ &3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*d^2*e^2 + 2*(3*a^2*b - b^3) \\ &)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b - b^3)*c*d*f^2)*x)*\cos(2*f*x + 2*e) \\ &))*\log(-\cos(2*f*x + 2*e) + I*\sin(2*f*x + 2*e) + 1) + 6*((3*a^2*b - b^3)*d^2* \\ &f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*d^2*e^2 + 2*(3*a^2*b - b^3)*c*d*e \\ &*f + 2*(3*a*b^2*d^2*f + (3*a^2*b - b^3)*c*d*f^2)*x - ((3*a^2*b - b^3)*d^2*f \\ &^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*d^2*e^2 + 2*(3*a^2*b - b^3)*c*d*e*f \\ &+ 2*(3*a*b^2*d^2*f + (3*a^2*b - b^3)*c*d*f^2)*x)*\cos(2*f*x + 2*e))*\log(-\cos \\ &(2*f*x + 2*e) - I*\sin(2*f*x + 2*e) + 1) - 3*((3*a^2*b - b^3)*d^2*\cos(2*f*x \\ &+ 2*e) - (3*a^2*b - b^3)*d^2)*\operatorname{polylog}(3, \cos(2*f*x + 2*e) + I*\sin(2*f*x + \\ &2*e)) - 3*((3*a^2*b - b^3)*d^2*\cos(2*f*x + 2*e) - (3*a^2*b - b^3)*d^2)*\operatorname{pol} \end{aligned}$$

$$\frac{\log(3, \cos(2fx + 2e) - I\sin(2fx + 2e)) - 12(3ab^2d^2f^2x^2 + 3a^2b^2c^2f^2 + b^3cd^2f + (6ab^2cd^2f^2 + b^3d^2f)x)\sin(2fx + 2e)}{(f^3\cos(2fx + 2e) - f^3)}$$

Sympy [F]

$$\int (c + dx)^2 (a + b \cot(e + fx))^3 dx = \int (a + b \cot(e + fx))^3 (c + dx)^2 dx$$

```
[In] integrate((d*x+c)**2*(a+b*cot(f*x+e))**3,x)
```

```
[Out] Integral((a + b*cot(e + f*x))**3*(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5429 vs. $2(390) = 780$.

Time = 2.64 (sec) , antiderivative size = 5429, normalized size of antiderivative = 12.54

$$\int (c + dx)^2 (a + b \cot(e + fx))^3 dx = \text{Too large to display}$$

```
[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a^3*c^2 + (f*x + e)^3*a^3*d^2/f^2 - 3*(f*x + e)^2*a^3*d^2*
e/f^2 + 3*(f*x + e)*a^3*d^2*e^2/f^2 + 3*(f*x + e)^2*a^3*c*d/f - 6*(f*x + e)
*a^3*c*d*e/f + 9*a^2*b*c^2*log(sin(f*x + e)) + 9*a^2*b*d^2*e^2*log(sin(f*x
+ e))/f^2 - 18*a^2*b*c*d*e*log(sin(f*x + e))/f + 3*(36*a*b^2*d^2*e^2 + 36*a
*b^2*c^2*f^2 - 2*(3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^3*d^2 - 12*b^3*d^2*e
+ 6*((3*a^2*b - 3*I*a*b^2 - b^3)*d^2*e - (3*a^2*b - 3*I*a*b^2 - b^3)*c*d*f
)*(f*x + e)^2 + 6*((3*I*a*b^2 + b^3)*d^2*e^2 + 2*(-3*I*a*b^2 - b^3)*c*d*e*f
+ (3*I*a*b^2 + b^3)*c^2*f^2)*(f*x + e) - 12*(6*a*b^2*c*d*e - b^3*c*d)*f -
6*(b^3*d^2*e^2 + b^3*c^2*f^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*(f*x + e)^2*
d^2 - b^3*d^2 - 2*(3*a*b^2*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c*
d*f)*(f*x + e) - 2*(b^3*c*d*e + 3*a*b^2*c*d)*f + (b^3*d^2*e^2 + b^3*c^2*f^2
+ 6*a*b^2*d^2*e - (3*a^2*b - b^3)*(f*x + e)^2*d^2 - b^3*d^2 - 2*(3*a*b^2*d
^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c*d*f)*(f*x + e) - 2*(b^3*c*d*
e + 3*a*b^2*c*d)*f)*cos(4*f*x + 4*e) - 2*(b^3*d^2*e^2 + b^3*c^2*f^2 + 6*a*b
^2*d^2*e - (3*a^2*b - b^3)*(f*x + e)^2*d^2 - b^3*d^2 - 2*(3*a*b^2*d^2 - (3*
a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c*d*f)*(f*x + e) - 2*(b^3*c*d*e + 3*a*
b^2*c*d)*f)*cos(2*f*x + 2*e) - (-I*b^3*d^2*e^2 - I*b^3*c^2*f^2 - 6*I*a*b^2*
d^2*e + (3*I*a^2*b - I*b^3)*(f*x + e)^2*d^2 + I*b^3*d^2 + 2*(3*I*a*b^2*d^2
+ (-3*I*a^2*b + I*b^3)*d^2*e + (3*I*a^2*b - I*b^3)*c*d*f)*(f*x + e) + 2*(I*
```

$$\begin{aligned}
& b^3 c d e + 3 I a b^2 c d) f) \sin(4 f x + 4 e) - 2 (I b^3 d^2 e^2 + I b^3 c \\
& ^2 f^2 + 6 I a b^2 d^2 e + (-3 I a^2 b + I b^3) (f x + e)^2 d^2 - I b^3 d^2 \\
& + 2 (-3 I a b^2 d^2 + (3 I a^2 b - I b^3) d^2 e + (-3 I a^2 b + I b^3) c d \\
& * f) (f x + e) + 2 (-I b^3 c d e - 3 I a b^2 c d) f) \sin(2 f x + 2 e)) * \arctan \\
& 2(\sin(f x + e), \cos(f x + e) + 1) - 6 (b^3 d^2 e^2 + b^3 c^2 f^2 + 6 a b^2 \\
& * d^2 e - b^3 d^2 - 2 (b^3 c d e + 3 a b^2 c d) f + (b^3 d^2 e^2 + b^3 c^2 f \\
& ^2 + 6 a b^2 d^2 e - b^3 d^2 - 2 (b^3 c d e + 3 a b^2 c d) f) * \cos(4 f x + \\
& 4 e) - 2 (b^3 d^2 e^2 + b^3 c^2 f^2 + 6 a b^2 d^2 e - b^3 d^2 - 2 (b^3 c d e \\
& + 3 a b^2 c d) f) * \cos(2 f x + 2 e) - (-I b^3 d^2 e^2 - I b^3 c^2 f^2 - 6 I \\
& a b^2 d^2 e + I b^3 d^2 + 2 (I b^3 c d e + 3 I a b^2 c d) f) \sin(4 f x + 4 \\
& e) - 2 (I b^3 d^2 e^2 + I b^3 c^2 f^2 + 6 I a b^2 d^2 e - I b^3 d^2 + 2 (- \\
& I b^3 c d e - 3 I a b^2 c d) f) \sin(2 f x + 2 e)) * \arctan 2(\sin(f x + e), \cos \\
& (f x + e) - 1) - 6 ((3 a^2 b - b^3) (f x + e)^2 d^2 + 2 (3 a b^2 d^2 - (3 a \\
& ^2 b - b^3) d^2 e + (3 a^2 b - b^3) c d f) (f x + e) + ((3 a^2 b - b^3) (f \\
& x + e)^2 d^2 + 2 (3 a b^2 d^2 - (3 a^2 b - b^3) d^2 e + (3 a^2 b - b^3) c d \\
& * f) (f x + e)) * \cos(4 f x + 4 e) - 2 ((3 a^2 b - b^3) (f x + e)^2 d^2 + 2 (3 \\
& a b^2 d^2 - (3 a^2 b - b^3) d^2 e + (3 a^2 b - b^3) c d f) (f x + e)) * \cos(\\
& 2 f x + 2 e) - ((-3 I a^2 b + I b^3) (f x + e)^2 d^2 + 2 (-3 I a b^2 d^2 + \\
& (3 I a^2 b - I b^3) d^2 e + (-3 I a^2 b + I b^3) c d f) (f x + e)) * \sin(4 f x \\
& + 4 e) - 2 ((3 I a^2 b - I b^3) (f x + e)^2 d^2 + 2 (3 I a b^2 d^2 + (-3 \\
& I a^2 b + I b^3) d^2 e + (3 I a^2 b - I b^3) c d f) (f x + e)) * \sin(2 f x + \\
& 2 e)) * \arctan 2(\sin(f x + e), -\cos(f x + e) + 1) - 2 ((3 a^2 b - 3 I a b^2 - \\
& b^3) (f x + e)^3 d^2 + 3 (6 a b^2 d^2 - (3 a^2 b - 3 I a b^2 - b^3) d^2 e + \\
& (3 a^2 b - 3 I a b^2 - b^3) c d f) (f x + e)^2 - 3 (12 a b^2 d^2 e - 2 b^3 \\
& * d^2 + (3 I a b^2 + b^3) d^2 e^2 + (3 I a b^2 + b^3) c^2 f^2 - 2 (6 a b^2 c \\
& * d - (-3 I a b^2 - b^3) c d e) f) (f x + e)) * \cos(4 f x + 4 e) + 4 ((3 a^2 b \\
& - 3 I a b^2 - b^3) (f x + e)^3 d^2 + 3 b^3 d^2 e - 3 (3 a b^2 + I b^3) d^2 \\
& * e^2 - 3 (3 a b^2 + I b^3) c^2 f^2 - 3 ((3 a^2 b - 3 I a b^2 - b^3) d^2 e - \\
& (3 a^2 b - 3 I a b^2 - b^3) c d f - (3 a b^2 - I b^3) d^2) (f x + e)^2 + 3 \\
& (b^3 d^2 + (-3 I a b^2 - b^3) d^2 e^2 + (-3 I a b^2 - b^3) c^2 f^2 - 2 (3 a \\
& b^2 - I b^3) d^2 e + 2 ((3 I a b^2 + b^3) c d e + (3 a b^2 - I b^3) c d) * \\
& f) (f x + e) - 3 (b^3 c d - 2 (3 a b^2 + I b^3) c d e) f) * \cos(2 f x + 2 e) \\
& - 12 (3 a b^2 d^2 + (3 a^2 b - b^3) (f x + e) d^2 - (3 a^2 b - b^3) d^2 e + \\
& (3 a^2 b - b^3) c d f + (3 a b^2 d^2 + (3 a^2 b - b^3) (f x + e) d^2 - (3 a \\
& a^2 b - b^3) d^2 e + (3 a^2 b - b^3) c d f) * \cos(4 f x + 4 e) - 2 (3 a b^2 d \\
& ^2 + (3 a^2 b - b^3) (f x + e) d^2 - (3 a^2 b - b^3) d^2 e + (3 a^2 b - b^3 \\
&) * c d f) * \cos(2 f x + 2 e) - (-3 I a b^2 d^2 + (-3 I a^2 b + I b^3) (f x + e \\
&) * d^2 + (3 I a^2 b - I b^3) d^2 e + (-3 I a^2 b + I b^3) c d f) * \sin(4 f x + \\
& 4 e) - 2 (3 I a b^2 d^2 + (3 I a^2 b - I b^3) (f x + e) d^2 + (-3 I a^2 b \\
& + I b^3) d^2 e + (3 I a^2 b - I b^3) c d f) * \sin(2 f x + 2 e)) * \operatorname{dilog}(-e^{(I f \\
& x + I e)}) - 12 (3 a b^2 d^2 + (3 a^2 b - b^3) (f x + e) d^2 - (3 a^2 b - b \\
& ^3) d^2 e + (3 a^2 b - b^3) c d f + (3 a b^2 d^2 + (3 a^2 b - b^3) (f x + e \\
&) * d^2 - (3 a^2 b - b^3) d^2 e + (3 a^2 b - b^3) c d f) * \cos(4 f x + 4 e) - 2 \\
& * (3 a b^2 d^2 + (3 a^2 b - b^3) (f x + e) d^2 - (3 a^2 b - b^3) d^2 e + (3 a \\
& a^2 b - b^3) c d f) * \cos(2 f x + 2 e) - (-3 I a b^2 d^2 + (-3 I a^2 b + I b^3
\end{aligned}$$

$$\begin{aligned}
& 3)*(f*x + e)*d^2 + (3*I*a^2*b - I*b^3)*d^2*e + (-3*I*a^2*b + I*b^3)*c*d*f)* \\
& \sin(4*f*x + 4*e) - 2*(3*I*a*b^2*d^2 + (3*I*a^2*b - I*b^3)*(f*x + e)*d^2 + (\\
& -3*I*a^2*b + I*b^3)*d^2*e + (3*I*a^2*b - I*b^3)*c*d*f)*\sin(2*f*x + 2*e))*di \\
& \log(e^{(I*f*x + I*e)}) + 3*(I*b^3*d^2*e^2 + I*b^3*c^2*f^2 + 6*I*a*b^2*d^2*e + \\
& (-3*I*a^2*b + I*b^3)*(f*x + e)^2*d^2 - I*b^3*d^2 + 2*(-3*I*a*b^2*d^2 + (3* \\
& I*a^2*b - I*b^3)*d^2*e + (-3*I*a^2*b + I*b^3)*c*d*f)*(f*x + e) + 2*(-I*b^3* \\
& c*d*e - 3*I*a*b^2*c*d)*f + (I*b^3*d^2*e^2 + I*b^3*c^2*f^2 + 6*I*a*b^2*d^2*e \\
& + (-3*I*a^2*b + I*b^3)*(f*x + e)^2*d^2 - I*b^3*d^2 + 2*(-3*I*a*b^2*d^2 + (\\
& 3*I*a^2*b - I*b^3)*d^2*e + (-3*I*a^2*b + I*b^3)*c*d*f)*(f*x + e) + 2*(-I*b^ \\
& 3*c*d*e - 3*I*a*b^2*c*d)*f)*\cos(4*f*x + 4*e) + 2*(-I*b^3*d^2*e^2 - I*b^3*c^ \\
& 2*f^2 - 6*I*a*b^2*d^2*e + (3*I*a^2*b - I*b^3)*(f*x + e)^2*d^2 + I*b^3*d^2 + \\
& 2*(3*I*a*b^2*d^2 + (-3*I*a^2*b + I*b^3)*d^2*e + (3*I*a^2*b - I*b^3)*c*d*f) \\
& *(f*x + e) + 2*(I*b^3*c*d*e + 3*I*a*b^2*c*d)*f)*\cos(2*f*x + 2*e) - (b^3*d^2 \\
& *e^2 + b^3*c^2*f^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*(f*x + e)^2*d^2 - b^3* \\
& d^2 - 2*(3*a*b^2*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c*d*f)*(f*x \\
& + e) - 2*(b^3*c*d*e + 3*a*b^2*c*d)*f)*\sin(4*f*x + 4*e) + 2*(b^3*d^2*e^2 + b \\
& ^3*c^2*f^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*(f*x + e)^2*d^2 - b^3*d^2 - 2* \\
& (3*a*b^2*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c*d*f)*(f*x + e) - 2 \\
& *(b^3*c*d*e + 3*a*b^2*c*d)*f)*\sin(2*f*x + 2*e))*\log(\cos(f*x + e)^2 + \sin(f* \\
& x + e)^2 + 2*\cos(f*x + e) + 1) + 3*(I*b^3*d^2*e^2 + I*b^3*c^2*f^2 + 6*I*a*b \\
& ^2*d^2*e + (-3*I*a^2*b + I*b^3)*(f*x + e)^2*d^2 - I*b^3*d^2 + 2*(-3*I*a*b^2 \\
& *d^2 + (3*I*a^2*b - I*b^3)*d^2*e + (-3*I*a^2*b + I*b^3)*c*d*f)*(f*x + e) + \\
& 2*(-I*b^3*c*d*e - 3*I*a*b^2*c*d)*f + (I*b^3*d^2*e^2 + I*b^3*c^2*f^2 + 6*I*a \\
& *b^2*d^2*e + (-3*I*a^2*b + I*b^3)*(f*x + e)^2*d^2 - I*b^3*d^2 + 2*(-3*I*a*b \\
& ^2*d^2 + (3*I*a^2*b - I*b^3)*d^2*e + (-3*I*a^2*b + I*b^3)*c*d*f)*(f*x + e) \\
& + 2*(-I*b^3*c*d*e - 3*I*a*b^2*c*d)*f)*\cos(4*f*x + 4*e) + 2*(-I*b^3*d^2*e^2 \\
& - I*b^3*c^2*f^2 - 6*I*a*b^2*d^2*e + (3*I*a^2*b - I*b^3)*(f*x + e)^2*d^2 + I \\
& *b^3*d^2 + 2*(3*I*a*b^2*d^2 + (-3*I*a^2*b + I*b^3)*d^2*e + (3*I*a^2*b - I*b \\
& ^3)*c*d*f)*(f*x + e) + 2*(I*b^3*c*d*e + 3*I*a*b^2*c*d)*f)*\cos(2*f*x + 2*e) \\
& - (b^3*d^2*e^2 + b^3*c^2*f^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*(f*x + e)^2* \\
& d^2 - b^3*d^2 - 2*(3*a*b^2*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c* \\
& d*f)*(f*x + e) - 2*(b^3*c*d*e + 3*a*b^2*c*d)*f)*\sin(4*f*x + 4*e) + 2*(b^3*d \\
& ^2*e^2 + b^3*c^2*f^2 + 6*a*b^2*d^2*e - (3*a^2*b - b^3)*(f*x + e)^2*d^2 - b^ \\
& 3*d^2 - 2*(3*a*b^2*d^2 - (3*a^2*b - b^3)*d^2*e + (3*a^2*b - b^3)*c*d*f)*(f* \\
& x + e) - 2*(b^3*c*d*e + 3*a*b^2*c*d)*f)*\sin(2*f*x + 2*e))*\log(\cos(f*x + e)^ \\
& 2 + \sin(f*x + e)^2 - 2*\cos(f*x + e) + 1) + 12*((-3*I*a^2*b + I*b^3)*d^2*\cos \\
& (4*f*x + 4*e) + 2*(3*I*a^2*b - I*b^3)*d^2*\cos(2*f*x + 2*e) + (3*a^2*b - b^3 \\
&)*d^2*\sin(4*f*x + 4*e) - 2*(3*a^2*b - b^3)*d^2*\sin(2*f*x + 2*e) + (-3*I*a^2 \\
& *b + I*b^3)*d^2)*\text{polylog}(3, -e^{(I*f*x + I*e)}) + 12*((-3*I*a^2*b + I*b^3)*d^ \\
& 2*\cos(4*f*x + 4*e) + 2*(3*I*a^2*b - I*b^3)*d^2*\cos(2*f*x + 2*e) + (3*a^2*b \\
& - b^3)*d^2*\sin(4*f*x + 4*e) - 2*(3*a^2*b - b^3)*d^2*\sin(2*f*x + 2*e) + (-3* \\
& I*a^2*b + I*b^3)*d^2)*\text{polylog}(3, e^{(I*f*x + I*e)}) + 2*((-3*I*a^2*b - 3*a*b^ \\
& 2 + I*b^3)*(f*x + e)^3*d^2 + 3*(-6*I*a*b^2*d^2 + (3*I*a^2*b + 3*a*b^2 - I*b \\
& ^3)*d^2*e + (-3*I*a^2*b - 3*a*b^2 + I*b^3)*c*d*f)*(f*x + e)^2 + 3*(12*I*a*b \\
& ^2*d^2*e - 2*I*b^3*d^2 - (3*a*b^2 - I*b^3)*d^2*e^2 - (3*a*b^2 - I*b^3)*c^2*
\end{aligned}$$

$f^2 + 2*(-6*I*a*b^2*c*d + (3*a*b^2 - I*b^3)*c*d*e)*f*(f*x + e))*\sin(4*f*x + 4*e) + 4*((3*I*a^2*b + 3*a*b^2 - I*b^3)*(f*x + e)^3*d^2 + 3*I*b^3*d^2*e + 3*(-3*I*a*b^2 + b^3)*d^2*e^2 + 3*(-3*I*a*b^2 + b^3)*c^2*f^2 + 3*((-3*I*a^2*b - 3*a*b^2 + I*b^3)*d^2*e + (3*I*a^2*b + 3*a*b^2 - I*b^3)*c*d*f + (3*I*a*b^2 + b^3)*d^2)*(f*x + e)^2 + 3*(I*b^3*d^2 + (3*a*b^2 - I*b^3)*d^2*e^2 + (3*a*b^2 - I*b^3)*c^2*f^2 + 2*(-3*I*a*b^2 - b^3)*d^2*e - 2*((3*a*b^2 - I*b^3)*c*d*e - (3*I*a*b^2 + b^3)*c*d)*f*(f*x + e) + 3*(-I*b^3*c*d + 2*(3*I*a*b^2 - b^3)*c*d*e)*f)*\sin(2*f*x + 2*e))/(-6*I*f^2*\cos(4*f*x + 4*e) + 12*I*f^2*\cos(2*f*x + 2*e) + 6*f^2*\sin(4*f*x + 4*e) - 12*f^2*\sin(2*f*x + 2*e) - 6*I*f^2))/f$

Giac [F]

$$\int (c + dx)^2 (a + b \cot(e + fx))^3 dx = \int (dx + c)^2 (b \cot(fx + e) + a)^3 dx$$

[In] integrate((d*x+c)^2*(a+b*cot(f*x+e))^3,x, algorithm="giac")

[Out] integrate((d*x + c)^2*(b*cot(f*x + e) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int (c + dx)^2 (a + b \cot(e + fx))^3 dx = \int (a + b \cot(e + fx))^3 (c + dx)^2 dx$$

[In] int((a + b*cot(e + f*x))^3*(c + d*x)^2,x)

[Out] int((a + b*cot(e + f*x))^3*(c + d*x)^2, x)

3.49 $\int (c + dx)(a + b \cot(e + fx))^3 dx$

Optimal result	345
Rubi [A] (verified)	346
Mathematica [A] (verified)	349
Maple [B] (verified)	349
Fricas [B] (verification not implemented)	350
Sympy [F]	351
Maxima [B] (verification not implemented)	351
Giac [F]	352
Mupad [F(-1)]	353

Optimal result

Integrand size = 18, antiderivative size = 278

$$\begin{aligned}
 \int (c + dx)(a + b \cot(e + fx))^3 dx = & -3ab^2cx - \frac{b^3dx}{2f} - \frac{3}{2}ab^2dx^2 + \frac{a^3(c + dx)^2}{2d} \\
 & - \frac{3ia^2b(c + dx)^2}{2d} + \frac{ib^3(c + dx)^2}{2d} - \frac{b^3d \cot(e + fx)}{2f^2} \\
 & - \frac{3ab^2(c + dx) \cot(e + fx)}{f} - \frac{b^3(c + dx) \cot^2(e + fx)}{2f} \\
 & + \frac{3a^2b(c + dx) \log(1 - e^{2i(e+fx)})}{f} \\
 & - \frac{b^3(c + dx) \log(1 - e^{2i(e+fx)})}{f} \\
 & + \frac{3ab^2d \log(\sin(e + fx))}{f^2} - \frac{3ia^2bd \operatorname{PolyLog}(2, e^{2i(e+fx)})}{2f^2} \\
 & + \frac{ib^3d \operatorname{PolyLog}(2, e^{2i(e+fx)})}{2f^2}
 \end{aligned}$$

```
[Out] -3*a*b^2*c*x-1/2*b^3*d*x/f-3/2*a*b^2*d*x^2+1/2*a^3*(d*x+c)^2/d-3/2*I*a^2*b*(d*x+c)^2/d+1/2*I*b^3*(d*x+c)^2/d-1/2*b^3*d*cot(f*x+e)/f^2-3*a*b^2*(d*x+c)*cot(f*x+e)/f-1/2*b^3*(d*x+c)*cot(f*x+e)^2/f+3*a^2*b*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f-b^3*(d*x+c)*ln(1-exp(2*I*(f*x+e)))/f+3*a*b^2*d*ln(sin(f*x+e))/f^2-3/2*I*a^2*b*d*polylog(2,exp(2*I*(f*x+e)))/f^2+1/2*I*b^3*d*polylog(2,exp(2*I*(f*x+e)))/f^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3803, 3798, 2221, 2317, 2438, 3801, 3556, 3554, 8}

$$\int (c + dx)(a + b \cot(e + fx))^3 dx = \frac{a^3(c + dx)^2}{2d} + \frac{3a^2b(c + dx) \log(1 - e^{2i(e+fx)})}{f} - \frac{3ia^2b(c + dx)^2}{2d} - \frac{3ia^2bd \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} - \frac{3ab^2(c + dx) \cot(e + fx)}{f} - 3ab^2cx + \frac{3ab^2d \log(\sin(e + fx))}{f^2} - \frac{3}{2}ab^2dx^2 - \frac{b^3(c + dx) \log(1 - e^{2i(e+fx)})}{f} - \frac{b^3(c + dx) \cot^2(e + fx)}{2f} + \frac{ib^3(c + dx)^2}{2d} + \frac{ib^3d \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} - \frac{b^3d \cot(e + fx)}{2f^2} - \frac{b^3dx}{2f}$$

[In] Int[(c + d*x)*(a + b*Cot[e + f*x])^3,x]

[Out] -3*a*b^2*c*x - (b^3*d*x)/(2*f) - (3*a*b^2*d*x^2)/2 + (a^3*(c + d*x)^2)/(2*d) - (((3*I)/2)*a^2*b*(c + d*x)^2)/d + ((I/2)*b^3*(c + d*x)^2)/d - (b^3*d*Cot[e + f*x])/(2*f^2) - (3*a*b^2*(c + d*x)*Cot[e + f*x])/f - (b^3*(c + d*x)*Cot[e + f*x]^2)/(2*f) + (3*a^2*b*(c + d*x)*Log[1 - E^((2*I)*(e + f*x))])/f - (b^3*(c + d*x)*Log[1 - E^((2*I)*(e + f*x))])/f + (3*a*b^2*d*Log[Sin[e + f*x]])/f^2 - (((3*I)/2)*a^2*b*d*PolyLog[2, E^((2*I)*(e + f*x))])/f^2 + ((I/2)*b^3*d*PolyLog[2, E^((2*I)*(e + f*x))])/f^2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2221

Int[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Dist[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3803

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int (a^3(c + dx) + 3a^2b(c + dx) \cot(e + fx) + 3ab^2(c + dx) \cot^2(e + fx) + b^3(c + dx) \cot^3(e + fx)) dx$$

$$\begin{aligned}
&= \frac{a^3(c+dx)^2}{2d} + (3a^2b) \int (c+dx) \cot(e+fx) dx \\
&\quad + (3ab^2) \int (c+dx) \cot^2(e+fx) dx + b^3 \int (c+dx) \cot^3(e+fx) dx \\
&= \frac{a^3(c+dx)^2}{2d} - \frac{3ia^2b(c+dx)^2}{2d} - \frac{3ab^2(c+dx) \cot(e+fx)}{f} \\
&\quad - \frac{b^3(c+dx) \cot^2(e+fx)}{2f} - (6ia^2b) \int \frac{e^{2i(e+fx)}(c+dx)}{1-e^{2i(e+fx)}} dx - (3ab^2) \int (c+dx) dx \\
&\quad - b^3 \int (c+dx) \cot(e+fx) dx + \frac{(3ab^2d) \int \cot(e+fx) dx}{f} + \frac{(b^3d) \int \cot^2(e+fx) dx}{2f} \\
&= -3ab^2cx - \frac{3}{2}ab^2dx^2 + \frac{a^3(c+dx)^2}{2d} - \frac{3ia^2b(c+dx)^2}{2d} + \frac{ib^3(c+dx)^2}{2d} \\
&\quad - \frac{b^3d \cot(e+fx)}{2f^2} - \frac{3ab^2(c+dx) \cot(e+fx)}{f} - \frac{b^3(c+dx) \cot^2(e+fx)}{2f} \\
&\quad + \frac{3a^2b(c+dx) \log(1-e^{2i(e+fx)})}{f} + \frac{3ab^2d \log(\sin(e+fx))}{f^2} \\
&\quad + (2ib^3) \int \frac{e^{2i(e+fx)}(c+dx)}{1-e^{2i(e+fx)}} dx - \frac{(3a^2bd) \int \log(1-e^{2i(e+fx)}) dx}{f} - \frac{(b^3d) \int 1 dx}{2f} \\
&= -3ab^2cx - \frac{b^3dx}{2f} - \frac{3}{2}ab^2dx^2 + \frac{a^3(c+dx)^2}{2d} - \frac{3ia^2b(c+dx)^2}{2d} \\
&\quad + \frac{ib^3(c+dx)^2}{2d} - \frac{b^3d \cot(e+fx)}{2f^2} - \frac{3ab^2(c+dx) \cot(e+fx)}{f} \\
&\quad - \frac{b^3(c+dx) \cot^2(e+fx)}{2f} + \frac{3a^2b(c+dx) \log(1-e^{2i(e+fx)})}{f} \\
&\quad - \frac{b^3(c+dx) \log(1-e^{2i(e+fx)})}{f} + \frac{3ab^2d \log(\sin(e+fx))}{f^2} \\
&\quad + \frac{(3ia^2bd) \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2i(e+fx)}\right)}{2f^2} + \frac{(b^3d) \int \log(1-e^{2i(e+fx)}) dx}{f} \\
&= -3ab^2cx - \frac{b^3dx}{2f} - \frac{3}{2}ab^2dx^2 + \frac{a^3(c+dx)^2}{2d} - \frac{3ia^2b(c+dx)^2}{2d} \\
&\quad + \frac{ib^3(c+dx)^2}{2d} - \frac{b^3d \cot(e+fx)}{2f^2} - \frac{3ab^2(c+dx) \cot(e+fx)}{f} \\
&\quad - \frac{b^3(c+dx) \cot^2(e+fx)}{2f} + \frac{3a^2b(c+dx) \log(1-e^{2i(e+fx)})}{f} \\
&\quad - \frac{b^3(c+dx) \log(1-e^{2i(e+fx)})}{f} + \frac{3ab^2d \log(\sin(e+fx))}{f^2} \\
&\quad - \frac{3ia^2bd \text{PolyLog}(2, e^{2i(e+fx)})}{2f^2} - \frac{(ib^3d) \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2i(e+fx)}\right)}{2f^2}
\end{aligned}$$

$$\begin{aligned}
&= -3ab^2cx - \frac{b^3dx}{2f} - \frac{3}{2}ab^2dx^2 + \frac{a^3(c+dx)^2}{2d} - \frac{3ia^2b(c+dx)^2}{2d} \\
&\quad + \frac{ib^3(c+dx)^2}{2d} - \frac{b^3d \cot(e+fx)}{2f^2} - \frac{3ab^2(c+dx) \cot(e+fx)}{f} \\
&\quad - \frac{b^3(c+dx) \cot^2(e+fx)}{2f} + \frac{3a^2b(c+dx) \log(1 - e^{2i(e+fx)})}{f} \\
&\quad - \frac{b^3(c+dx) \log(1 - e^{2i(e+fx)})}{f} + \frac{3ab^2d \log(\sin(e+fx))}{f^2} \\
&\quad - \frac{3ia^2bd \operatorname{PolyLog}(2, e^{2i(e+fx)})}{2f^2} + \frac{ib^3d \operatorname{PolyLog}(2, e^{2i(e+fx)})}{2f^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 12.40 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.56

$$\int (c+dx)(a+b \cot(e+fx))^3 dx$$

$$= \frac{(a+b \cot(e+fx))^3 \sin(e+fx) ((-a^3de^2 - 3ia^2bde^2 + 3ab^2de^2 + ib^3de^2 + 2a^3cef - 6ab^2cef - 6ia^2bdef$$

[In] Integrate[(c + d*x)*(a + b*Cot[e + f*x])^3,x]

[Out] ((a + b*Cot[e + f*x])^3*Sin[e + f*x]*((-a^3*d*e^2) - (3*I)*a^2*b*d*e^2 + 3*a*b^2*d*e^2 + I*b^3*d*e^2 + 2*a^3*c*e*f - 6*a*b^2*c*e*f - (6*I)*a^2*b*d*e*f*x + (2*I)*b^3*d*e*f*x + 2*a^3*c*f^2*x - 6*a*b^2*c*f^2*x + a^3*d*f^2*x^2 - (3*I)*a^2*b*d*f^2*x^2 - 3*a*b^2*d*f^2*x^2 + I*b^3*d*f^2*x^2 - 2*b*(-3*a^2 + b^2)*d*(e + f*x)*Log[1 - E^((2*I)*(e + f*x))] + 2*b*(3*a*b*d + b^2*(d*e - c*f) + a^2*(-3*d*e + 3*c*f))*Log[Cos[e + f*x]] + 6*a*b^2*d*Log[Tan[e + f*x]] - 6*a^2*b*d*e*Log[Tan[e + f*x]] + 2*b^3*d*e*Log[Tan[e + f*x]] + 6*a^2*b*c*f*Log[Tan[e + f*x]] - 2*b^3*c*f*Log[Tan[e + f*x]])*Sin[e + f*x]^2 + I*b*(-3*a^2 + b^2)*d*PolyLog[2, E^((2*I)*(e + f*x))]*Sin[e + f*x]^2 - (b^2*(2*b*f*(c + d*x) + (b*d + 6*a*f*(c + d*x))*Sin[2*(e + f*x)]))/2)/(2*f^2*(b*Cos[e + f*x] + a*Sin[e + f*x])^3)

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 744 vs. 2(248) = 496.

Time = 0.72 (sec) , antiderivative size = 745, normalized size of antiderivative = 2.68

method	result
risch	$-ib^3cx + \frac{ib^3dx^2}{2} + \frac{b^2(-6iadfx e^{2i(fx+e)} - 6iacf e^{2i(fx+e)} + 2bdfx e^{2i(fx+e)} + 6iadfx - ibd e^{2i(fx+e)} + 2bcf e^{2i(fx+e)} + 6iacf + ibd e^{2i(fx+e)} - 1)^2}{f^2(e^{2i(fx+e)} - 1)^2}$

[In] int((d*x+c)*(a+b*cot(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] $-I*b^3*c*x+6/f^2*b*e*d*a^2*\ln(\exp(I*(f*x+e)))-3/f^2*b*e*d*a^2*\ln(\exp(I*(f*x+e))-1)+3/f*b*a^2*d*\ln(1-\exp(I*(f*x+e)))*x+3/f*b*a^2*d*\ln(\exp(I*(f*x+e))+1)*x+3/f^2*b*a^2*d*\ln(1-\exp(I*(f*x+e)))*e-3*I/f^2*b*a^2*d*\text{polylog}(2,\exp(I*(f*x+e)))-3*I/f^2*b*a^2*d*\text{polylog}(2,-\exp(I*(f*x+e)))-3*I/f^2*b*a^2*d*e^2+2*I/f*b^3*d*e*x-6*I/f*b*a^2*d*e*x+1/2*a^3*d*x^2+a^3*c*x+1/2*I*b^3*d*x^2+b^2*(-6*I*a*d*f*x*\exp(2*I*(f*x+e))-6*I*a*c*f*\exp(2*I*(f*x+e))+2*b*d*f*x*\exp(2*I*(f*x+e))+6*I*a*d*f*x-I*b*d*\exp(2*I*(f*x+e))+2*b*c*f*\exp(2*I*(f*x+e))+6*I*a*c*f+I*b*d)/f^2/(\exp(2*I*(f*x+e))-1)^2-3*a*b^2*c*x-3/2*a*b^2*d*x^2-3/2*I*a^2*b*d*x^2-1/f*b^3*d*\ln(1-\exp(I*(f*x+e)))*x-1/f^2*b^3*d*\ln(1-\exp(I*(f*x+e)))*e+I/f^2*b^3*d*\text{polylog}(2,\exp(I*(f*x+e)))+I/f^2*b^3*d*\text{polylog}(2,-\exp(I*(f*x+e)))+3/f^2*b^2*a*d*\ln(\exp(I*(f*x+e))+1)-6/f^2*b^2*a*d*\ln(\exp(I*(f*x+e)))+3/f^2*b^2*a*d*\ln(\exp(I*(f*x+e))-1)+3/f*b*a^2*c*\ln(\exp(I*(f*x+e))+1)-6/f*b*a^2*c*\ln(\exp(I*(f*x+e)))+3/f*b*a^2*c*\ln(\exp(I*(f*x+e))-1)-2/f^2*b^3*e*d*\ln(\exp(I*(f*x+e)))+1/f^2*b^3*e*d*\ln(\exp(I*(f*x+e))-1)+I/f^2*b^3*d*e^2-1/f*b^3*d*\ln(\exp(I*(f*x+e))+1)*x+3*I*a^2*b*c*x-1/f*b^3*c*\ln(\exp(I*(f*x+e))+1)+2/f*b^3*c*\ln(\exp(I*(f*x+e)))-1/f*b^3*c*\ln(\exp(I*(f*x+e))-1)$

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(242) = 484$.

Time = 0.29 (sec) , antiderivative size = 721, normalized size of antiderivative = 2.59

$$\int (c + dx)(a + b \cot(e + fx))^3 dx = \frac{2(a^3 - 3ab^2)df^2x^2 - 4b^3cf - 4(b^3df - (a^3 - 3ab^2)cf^2)x - 2((a^3 - 3ab^2)df^2x^2 + 2(a^3 - 3ab^2)cf^2x) c}{}$$

[In] integrate((d*x+c)*(a+b*cot(f*x+e))^3,x, algorithm="fricas")

[Out] $-1/4*(2*(a^3 - 3*a*b^2)*d*f^2*x^2 - 4*b^3*c*f - 4*(b^3*d*f - (a^3 - 3*a*b^2)*c*f^2)*x - 2*((a^3 - 3*a*b^2)*d*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*f^2*x)*\cos(2*f*x + 2*e) - (-I*(3*a^2*b - b^3)*d*\cos(2*f*x + 2*e) + I*(3*a^2*b - b^3)*d)*\text{dilog}(\cos(2*f*x + 2*e) + I*\sin(2*f*x + 2*e)) - (I*(3*a^2*b - b^3)*d*\cos(2*f*x + 2*e) - I*(3*a^2*b - b^3)*d)*\text{dilog}(\cos(2*f*x + 2*e) - I*\sin(2*f*x + 2*e)) + 2*(3*a*b^2*d - (3*a^2*b - b^3)*d*e + (3*a^2*b - b^3)*c*f - (3*a*b^2*d - (3*a^2*b - b^3)*d*e + (3*a^2*b - b^3)*c*f)*\cos(2*f*x + 2*e))*\log(-1/2*\cos(2*f*x + 2*e) + 1/2*I*\sin(2*f*x + 2*e) + 1/2) + 2*(3*a*b^2*d - (3*a^2*b - b^3)*d*e + (3*a^2*b - b^3)*c*f - (3*a*b^2*d - (3*a^2*b - b^3)*d*e + (3*a^2*b - b^3)*c*f)*\cos(2*f*x + 2*e))*\log(-1/2*\cos(2*f*x + 2*e) - 1/2*I*\sin(2*f*x + 2*e) + 1/2) + 2*((3*a^2*b - b^3)*d*f*x + (3*a^2*b - b^3)*d*e - ((3*a^2*b - b^3)*d*f*x + (3*a^2*b - b^3)*d*e)*\cos(2*f*x + 2*e))*\log(-\cos(2*f*x + 2*e) + I*\sin(2*f*x + 2*e) + 1) + 2*((3*a^2*b - b^3)*d*f*x + (3*a^2*b - b^3)*d$

*e - ((3*a^2*b - b^3)*d*f*x + (3*a^2*b - b^3)*d*e)*cos(2*f*x + 2*e))*log(-cos(2*f*x + 2*e) - I*sin(2*f*x + 2*e) + 1) - 2*(6*a*b^2*d*f*x + 6*a*b^2*c*f + b^3*d)*sin(2*f*x + 2*e))/(f^2*cos(2*f*x + 2*e) - f^2)

Sympy [F]

$$\int (c + dx)(a + b \cot(e + fx))^3 dx = \int (a + b \cot(e + fx))^3 (c + dx) dx$$

[In] integrate((d*x+c)*(a+b*cot(f*x+e))**3,x)

[Out] Integral((a + b*cot(e + f*x))**3*(c + d*x), x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2017 vs. 2(242) = 484.

Time = 0.78 (sec) , antiderivative size = 2017, normalized size of antiderivative = 7.26

$$\int (c + dx)(a + b \cot(e + fx))^3 dx = \text{Too large to display}$$

[In] integrate((d*x+c)*(a+b*cot(f*x+e))^3,x, algorithm="maxima")

[Out] 1/2*(2*(f*x + e)*a^3*c + (f*x + e)^2*a^3*d/f - 2*(f*x + e)*a^3*d*e/f + 6*a^2*b*c*log(sin(f*x + e)) - 6*a^2*b*d*e*log(sin(f*x + e))/f - 2*(12*a*b^2*d*e - 12*a*b^2*c*f + (3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^2*d - 2*b^3*d - 2*(-3*I*a*b^2 - b^3)*d*e + (3*I*a*b^2 + b^3)*c*f)*(f*x + e) - 2*(b^3*d*e - b^3*c*f + 3*a*b^2*d + (3*a^2*b - b^3)*(f*x + e)*d + (b^3*d*e - b^3*c*f + 3*a*b^2*d + (3*a^2*b - b^3)*(f*x + e)*d)*cos(4*f*x + 4*e) - 2*(b^3*d*e - b^3*c*f + 3*a*b^2*d + (3*a^2*b - b^3)*(f*x + e)*d)*cos(2*f*x + 2*e) + (I*b^3*d*e - I*b^3*c*f + 3*I*a*b^2*d + (3*I*a^2*b - I*b^3)*(f*x + e)*d)*sin(4*f*x + 4*e) + 2*(-I*b^3*d*e + I*b^3*c*f - 3*I*a*b^2*d + (-3*I*a^2*b + I*b^3)*(f*x + e)*d)*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) - 2*(b^3*d*e - b^3*c*f + 3*a*b^2*d + (b^3*d*e - b^3*c*f + 3*a*b^2*d)*cos(4*f*x + 4*e) - 2*(b^3*d*e - b^3*c*f + 3*a*b^2*d)*cos(2*f*x + 2*e) + (I*b^3*d*e - I*b^3*c*f + 3*I*a*b^2*d)*sin(4*f*x + 4*e) + 2*(-I*b^3*d*e + I*b^3*c*f - 3*I*a*b^2*d)*sin(2*f*x + 2*e))*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*((3*a^2*b - b^3)*(f*x + e)*d*cos(4*f*x + 4*e) - 2*(3*a^2*b - b^3)*(f*x + e)*d*cos(2*f*x + 2*e) - (-3*I*a^2*b + I*b^3)*(f*x + e)*d*sin(4*f*x + 4*e) - 2*(3*I*a^2*b - I*b^3)*(f*x + e)*d*sin(2*f*x + 2*e) + (3*a^2*b - b^3)*(f*x + e)*d*arctan2(sin(f*x + e), -cos(f*x + e) + 1) + ((3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^2*d + 2*(6*a*b^2*d - (-3*I*a*b^2 - b^3)*d*e - (3*I*a*b^2 + b^3)*c*f)*(f*

```

x + e))*cos(4*f*x + 4*e) - 2*((3*a^2*b - 3*I*a*b^2 - b^3)*(f*x + e)^2*d - b
^3*d + 2*(3*a*b^2 + I*b^3)*d*e - 2*(3*a*b^2 + I*b^3)*c*f + 2*((3*I*a*b^2 +
b^3)*d*e + (-3*I*a*b^2 - b^3)*c*f + (3*a*b^2 - I*b^3)*d)*(f*x + e))*cos(2*f
*x + 2*e) + 2*((3*a^2*b - b^3)*d*cos(4*f*x + 4*e) - 2*(3*a^2*b - b^3)*d*cos
(2*f*x + 2*e) - (-3*I*a^2*b + I*b^3)*d*sin(4*f*x + 4*e) - 2*(3*I*a^2*b - I
b^3)*d*sin(2*f*x + 2*e) + (3*a^2*b - b^3)*d)*dilog(-e^(I*f*x + I*e)) + 2*((
3*a^2*b - b^3)*d*cos(4*f*x + 4*e) - 2*(3*a^2*b - b^3)*d*cos(2*f*x + 2*e) -
(-3*I*a^2*b + I*b^3)*d*sin(4*f*x + 4*e) - 2*(3*I*a^2*b - I*b^3)*d*sin(2*f*x
+ 2*e) + (3*a^2*b - b^3)*d)*dilog(e^(I*f*x + I*e)) + (I*b^3*d*e - I*b^3*c*
f + 3*I*a*b^2*d + (3*I*a^2*b - I*b^3)*(f*x + e)*d + (I*b^3*d*e - I*b^3*c*f
+ 3*I*a*b^2*d + (3*I*a^2*b - I*b^3)*(f*x + e)*d)*cos(4*f*x + 4*e) - 2*(I*b^
3*d*e - I*b^3*c*f + 3*I*a*b^2*d + (3*I*a^2*b - I*b^3)*(f*x + e)*d)*cos(2*f*
x + 2*e) - (b^3*d*e - b^3*c*f + 3*a*b^2*d + (3*a^2*b - b^3)*(f*x + e)*d)*si
n(4*f*x + 4*e) + 2*(b^3*d*e - b^3*c*f + 3*a*b^2*d + (3*a^2*b - b^3)*(f*x +
e)*d)*sin(2*f*x + 2*e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e
) + 1) + (I*b^3*d*e - I*b^3*c*f + 3*I*a*b^2*d + (3*I*a^2*b - I*b^3)*(f*x +
e)*d + (I*b^3*d*e - I*b^3*c*f + 3*I*a*b^2*d + (3*I*a^2*b - I*b^3)*(f*x + e
)*d)*cos(4*f*x + 4*e) - 2*(I*b^3*d*e - I*b^3*c*f + 3*I*a*b^2*d + (3*I*a^2*b
- I*b^3)*(f*x + e)*d)*cos(2*f*x + 2*e) - (b^3*d*e - b^3*c*f + 3*a*b^2*d + (
3*a^2*b - b^3)*(f*x + e)*d)*sin(4*f*x + 4*e) + 2*(b^3*d*e - b^3*c*f + 3*a*b
^2*d + (3*a^2*b - b^3)*(f*x + e)*d)*sin(2*f*x + 2*e))*log(cos(f*x + e)^2 +
sin(f*x + e)^2 - 2*cos(f*x + e) + 1) + ((3*I*a^2*b + 3*a*b^2 - I*b^3)*(f*x
+ e)^2*d - 2*(-6*I*a*b^2*d + (3*a*b^2 - I*b^3)*d*e - (3*a*b^2 - I*b^3)*c*f)
*(f*x + e))*sin(4*f*x + 4*e) - 2*((3*I*a^2*b + 3*a*b^2 - I*b^3)*(f*x + e)^2
*d - I*b^3*d + 2*(3*I*a*b^2 - b^3)*d*e + 2*(-3*I*a*b^2 + b^3)*c*f - 2*((3*a
*b^2 - I*b^3)*d*e - (3*a*b^2 - I*b^3)*c*f - (3*I*a*b^2 + b^3)*d)*(f*x + e))
*sin(2*f*x + 2*e))/(-2*I*f*cos(4*f*x + 4*e) + 4*I*f*cos(2*f*x + 2*e) + 2*f*
sin(4*f*x + 4*e) - 4*f*sin(2*f*x + 2*e) - 2*I*f))/f

```

Giac [F]

$$\int (c + dx)(a + b \cot(e + fx))^3 dx = \int (dx + c)(b \cot(fx + e) + a)^3 dx$$

```
[In] integrate((d*x+c)*(a+b*cot(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*cot(f*x + e) + a)^3, x)
```


Mupad [F(-1)]

Timed out.

$$\int (c + dx)(a + b \cot(e + fx))^3 dx = \int (a + b \cot(e + fx))^3 (c + dx) dx$$

```
[In] int((a + b*cot(e + f*x))^3*(c + d*x),x)
```

```
[Out] int((a + b*cot(e + f*x))^3*(c + d*x), x)
```

3.50 $\int \frac{(a+b \cot(e+fx))^3}{c+dx} dx$

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Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a + b \cot(e + fx))^3}{c + dx} dx = \text{Int}\left(\frac{(a + b \cot(e + fx))^3}{c + dx}, x\right)$$

[Out] Unintegrable((a+b*cot(f*x+e))^3/(d*x+c),x)

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a + b \cot(e + fx))^3}{c + dx} dx = \int \frac{(a + b \cot(e + fx))^3}{c + dx} dx$$

[In] Int[(a + b*Cot[e + f*x])^3/(c + d*x),x]

[Out] Defer[Int][(a + b*Cot[e + f*x])^3/(c + d*x), x]

Rubi steps

$$\text{integral} = \int \frac{(a + b \cot(e + fx))^3}{c + dx} dx$$

Mathematica [N/A]

Not integrable

Time = 9.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \cot(e + fx))^3}{c + dx} dx = \int \frac{(a + b \cot(e + fx))^3}{c + dx} dx$$

[In] Integrate[(a + b*Cot[e + f*x])^3/(c + d*x), x]

[Out] Integrate[(a + b*Cot[e + f*x])^3/(c + d*x), x]

Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \cot(fx + e))^3}{dx + c} dx$$

[In] int((a+b*cot(f*x+e))^3/(d*x+c), x)

[Out] int((a+b*cot(f*x+e))^3/(d*x+c), x)

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{(a + b \cot(e + fx))^3}{c + dx} dx = \int \frac{(b \cot(fx + e) + a)^3}{dx + c} dx$$

[In] integrate((a+b*cot(f*x+e))^3/(d*x+c), x, algorithm="fricas")

[Out] integral((b^3*cot(f*x + e)^3 + 3*a*b^2*cot(f*x + e)^2 + 3*a^2*b*cot(f*x + e) + a^3)/(d*x + c), x)

Sympy [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(a + b \cot(e + fx))^3}{c + dx} dx = \int \frac{(a + b \cot(e + fx))^3}{c + dx} dx$$

[In] integrate((a+b*cot(f*x+e))**3/(d*x+c),x)

[Out] Integral((a + b*cot(e + f*x))**3/(c + d*x), x)

Maxima [N/A]

Not integrable

Time = 5.49 (sec) , antiderivative size = 2585, normalized size of antiderivative = 129.25

$$\int \frac{(a + b \cot(e + fx))^3}{c + dx} dx = \int \frac{(b \cot(fx + e) + a)^3}{dx + c} dx$$

[In] integrate((a+b*cot(f*x+e))^3/(d*x+c),x, algorithm="maxima")

```
[Out] (((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*cos(4*f*x + 4*e)^2*log(d*x + c) + ((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c)*sin(4*f*x + 4*e)^2 - 4*(b^3*d^2*f*x + b^3*c*d*f - ((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c))*cos(2*f*x + 2*e)^2 - 4*(b^3*d^2*f*x + b^3*c*d*f - ((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c))*sin(2*f*x + 2*e)^2 + (2*(b^3*d^2*f*x + b^3*c*d*f - 2*((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c))*cos(2*f*x + 2*e) + 2*((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c) + (6*a*b^2*d^2*f*x + 6*a*b^2*c*d*f - b^3*d^2)*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) + 2*(b^3*d^2*f*x + b^3*c*d*f - 2*((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*log(d*x + c))*cos(2*f*x + 2*e) + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(4*f*x + 4*e)^2 + 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e)^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sin(4*f*x + 4*e)^2 - 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sin(2*f*x + 2*e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2) - 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) - 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cos(2*f*x + 2*e))*integrate(-((3*a^2*b - b^3)*d^2*f^2*x^2 - 3*a*b^2*c*d*f +
```

$$\begin{aligned}
& b^3 d^2 + (3a^2 b - b^3) c^2 f^2 - (3a b^2 d^2 f - 2(3a^2 b - b^3) c d \\
& f^2) x) \sin(fx + e) / (d^3 f^2 x^3 + 3c d^2 f^2 x^2 + 3c^2 d f^2 x + c^3 f^2 \\
& f^2 + (d^3 f^2 x^3 + 3c d^2 f^2 x^2 + 3c^2 d f^2 x + c^3 f^2) \cos(fx + e) \\
&)^2 + (d^3 f^2 x^3 + 3c d^2 f^2 x^2 + 3c^2 d f^2 x + c^3 f^2) \sin(fx + e) \\
&)^2 + 2(d^3 f^2 x^3 + 3c d^2 f^2 x^2 + 3c^2 d f^2 x + c^3 f^2) \cos(fx + \\
& e), x) - (d^3 f^2 x^2 + 2c d^2 f^2 x + c^2 d f^2 + (d^3 f^2 x^2 + 2c d^2 \\
& f^2 x + c^2 d f^2) \cos(4fx + 4e))^2 + 4(d^3 f^2 x^2 + 2c d^2 f^2 x + \\
& c^2 d f^2) \cos(2fx + 2e))^2 + (d^3 f^2 x^2 + 2c d^2 f^2 x + c^2 d f^2) s \\
& \sin(4fx + 4e))^2 - 4(d^3 f^2 x^2 + 2c d^2 f^2 x + c^2 d f^2) \sin(4fx + \\
& 4e) \sin(2fx + 2e) + 4(d^3 f^2 x^2 + 2c d^2 f^2 x + c^2 d f^2) \sin(2 \\
& fx + 2e))^2 + 2(d^3 f^2 x^2 + 2c d^2 f^2 x + c^2 d f^2 - 2(d^3 f^2 x^2 \\
& + 2c d^2 f^2 x + c^2 d f^2) \cos(2fx + 2e)) \cos(4fx + 4e) - 4(d^3 f^2 \\
& 2x^2 + 2c d^2 f^2 x + c^2 d f^2) \cos(2fx + 2e)) \int -((3a^2 b - \\
& b^3) d^2 f^2 x^2 - 3a b^2 c d f + b^3 d^2 + (3a^2 b - b^3) c^2 f^2 - (3 \\
& a b^2 d^2 f - 2(3a^2 b - b^3) c d f^2) x) \sin(fx + e) / (d^3 f^2 x^3 + 3c \\
& d^2 f^2 x^2 + 3c^2 d f^2 x + c^3 f^2 + (d^3 f^2 x^3 + 3c d^2 f^2 x^2 + 3 \\
& c^2 d f^2 x + c^3 f^2) \cos(fx + e))^2 + (d^3 f^2 x^3 + 3c d^2 f^2 x^2 + 3 \\
& c^2 d f^2 x + c^3 f^2) \sin(fx + e))^2 - 2(d^3 f^2 x^3 + 3c d^2 f^2 x^2 + \\
& 3c^2 d f^2 x + c^3 f^2) \cos(fx + e)), x) + ((a^3 - 3a b^2) d^2 f^2 x^2 \\
& + 2(a^3 - 3a b^2) c d f^2 x + (a^3 - 3a b^2) c^2 f^2) \log(dx + c) + (6 \\
& a b^2 d^2 f x + 6a b^2 c d f - b^3 d^2 - (6a b^2 d^2 f x + 6a b^2 c d f \\
& - b^3 d^2) \cos(2fx + 2e) + 2(b^3 d^2 f x + b^3 c d f - 2((a^3 - 3a b^2 \\
& 2) d^2 f^2 x^2 + 2(a^3 - 3a b^2) c d f^2 x + (a^3 - 3a b^2) c^2 f^2) \log \\
& (dx + c)) \sin(2fx + 2e)) \sin(4fx + 4e) - (6a b^2 d^2 f x + 6a b^2 \\
& c d f - b^3 d^2) \sin(2fx + 2e)) / (d^3 f^2 x^2 + 2c d^2 f^2 x + c^2 d f^2 \\
& + (d^3 f^2 x^2 + 2c d^2 f^2 x + c^2 d f^2) \cos(4fx + 4e))^2 + 4(d^3 f^2 \\
& 2x^2 + 2c d^2 f^2 x + c^2 d f^2) \cos(2fx + 2e))^2 + (d^3 f^2 x^2 + 2c \\
& d^2 f^2 x + c^2 d f^2) \sin(4fx + 4e))^2 - 4(d^3 f^2 x^2 + 2c d^2 f^2 x \\
& + c^2 d f^2) \sin(4fx + 4e) \sin(2fx + 2e) + 4(d^3 f^2 x^2 + 2c d^2 f^2 \\
& ^2 x + c^2 d f^2) \sin(2fx + 2e))^2 + 2(d^3 f^2 x^2 + 2c d^2 f^2 x + c^2 \\
& d f^2 - 2(d^3 f^2 x^2 + 2c d^2 f^2 x + c^2 d f^2) \cos(2fx + 2e)) \cos(\\
& 4fx + 4e) - 4(d^3 f^2 x^2 + 2c d^2 f^2 x + c^2 d f^2) \cos(2fx + 2e) \\
&)
\end{aligned}$$

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \cot(e + fx))^3}{c + dx} dx = \int \frac{(b \cot(fx + e) + a)^3}{dx + c} dx$$

[In] integrate((a+b*cot(f*x+e))^3/(d*x+c),x, algorithm="giac")

[Out] integrate((b*cot(f*x + e) + a)^3/(d*x + c), x)

Mupad [N/A]

Not integrable

Time = 12.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \cot(e + f x))^3}{c + d x} dx = \int \frac{(a + b \cot(e + f x))^3}{c + d x} dx$$

```
[In] int((a + b*cot(e + f*x))^3/(c + d*x),x)
```

```
[Out] int((a + b*cot(e + f*x))^3/(c + d*x), x)
```

$$3.51 \quad \int \frac{(a+b \cot(e+fx))^3}{(c+dx)^2} dx$$

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Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a + b \cot(e + fx))^3}{(c + dx)^2} dx = \text{Int}\left(\frac{(a + b \cot(e + fx))^3}{(c + dx)^2}, x\right)$$

[Out] Unintegrable((a+b*cot(f*x+e))^3/(d*x+c)^2,x)

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a + b \cot(e + fx))^3}{(c + dx)^2} dx = \int \frac{(a + b \cot(e + fx))^3}{(c + dx)^2} dx$$

[In] Int[(a + b*Cot[e + f*x])^3/(c + d*x)^2,x]

[Out] Defer[Int] [(a + b*Cot[e + f*x])^3/(c + d*x)^2, x]

Rubi steps

$$\text{integral} = \int \frac{(a + b \cot(e + fx))^3}{(c + dx)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 12.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \cot(e + fx))^3}{(c + dx)^2} dx = \int \frac{(a + b \cot(e + fx))^3}{(c + dx)^2} dx$$

[In] Integrate[(a + b*Cot[e + f*x])^3/(c + d*x)^2,x]

[Out] Integrate[(a + b*Cot[e + f*x])^3/(c + d*x)^2, x]

Maple [N/A] (verified)

Not integrable

Time = 0.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \cot(fx + e))^3}{(dx + c)^2} dx$$

[In] int((a+b*cot(f*x+e))^3/(d*x+c)^2,x)

[Out] int((a+b*cot(f*x+e))^3/(d*x+c)^2,x)

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.15

$$\int \frac{(a + b \cot(e + fx))^3}{(c + dx)^2} dx = \int \frac{(b \cot(fx + e) + a)^3}{(dx + c)^2} dx$$

[In] integrate((a+b*cot(f*x+e))^3/(d*x+c)^2,x, algorithm="fricas")

[Out] integral((b^3*cot(f*x + e)^3 + 3*a*b^2*cot(f*x + e)^2 + 3*a^2*b*cot(f*x + e) + a^3)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(a + b \cot(e + fx))^3}{(c + dx)^2} dx = \int \frac{(a + b \cot(e + fx))^3}{(c + dx)^2} dx$$

[In] integrate((a+b*cot(f*x+e))**3/(d*x+c)**2,x)

[Out] Integral((a + b*cot(e + f*x))**3/(c + d*x)**2, x)

Maxima [N/A]

Not integrable

Time = 15.25 (sec) , antiderivative size = 3017, normalized size of antiderivative = 150.85

$$\int \frac{(a + b \cot(e + fx))^3}{(c + dx)^2} dx = \int \frac{(b \cot(fx + e) + a)^3}{(dx + c)^2} dx$$

[In] integrate((a+b*cot(f*x+e))^3/(d*x+c)^2,x, algorithm="maxima")

```
[Out] -((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2 + ((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*cos(4*f*x + 4*e)^2 + 4*((a^3 - 3*a*b^2)*d^2*f^2*x^2 + b^3*c*d*f + (a^3 - 3*a*b^2)*c^2*f^2 + (b^3*d^2*f + 2*(a^3 - 3*a*b^2)*c*d*f^2)*x)*cos(2*f*x + 2*e)^2 + ((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2)*sin(4*f*x + 4*e)^2 + 4*((a^3 - 3*a*b^2)*d^2*f^2*x^2 + b^3*c*d*f + (a^3 - 3*a*b^2)*c^2*f^2 + (b^3*d^2*f + 2*(a^3 - 3*a*b^2)*c*d*f^2)*x)*sin(2*f*x + 2*e)^2 + 2*((a^3 - 3*a*b^2)*d^2*f^2*x^2 + 2*(a^3 - 3*a*b^2)*c*d*f^2*x + (a^3 - 3*a*b^2)*c^2*f^2 - (2*(a^3 - 3*a*b^2)*d^2*f^2*x^2 + b^3*c*d*f + 2*(a^3 - 3*a*b^2)*c^2*f^2 + (b^3*d^2*f + 4*(a^3 - 3*a*b^2)*c*d*f^2)*x)*cos(2*f*x + 2*e) - (3*a*b^2*d^2*f*x + 3*a*b^2*c*d*f - b^3*d^2)*sin(2*f*x + 2*e))*cos(4*f*x + 4*e) - 2*(2*(a^3 - 3*a*b^2)*d^2*f^2*x^2 + b^3*c*d*f + 2*(a^3 - 3*a*b^2)*c^2*f^2 + (b^3*d^2*f + 4*(a^3 - 3*a*b^2)*c*d*f^2)*x)*cos(2*f*x + 2*e) - (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(4*f*x + 4*e)^2 + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(2*f*x + 2*e)^2 + (d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x + 4*e)^2 - 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*sin(2*f*x + 2*e)^2 + 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2 - 2*(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2)*cos(2*f*x
```

$$\begin{aligned}
& x + 2e)) \cos(4fx + 4e) - 4(d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x^2 + c^3d^2f^2) \cos(2fx + 2e)) \int -((3a^2b - b^3)d^2f^2x^2 - 6ab^2cdf + 3b^3d^2 + (3a^2b - b^3)c^2f^2 - 2(3ab^2d^2f - (3a^2b - b^3)cdf^2)x) \sin(fx + e) / (d^4f^2x^4 + 4cd^3f^2x^3 + 6c^2d^2f^2x^2 + 4c^3d^2f^2x + c^4f^2 + (d^4f^2x^4 + 4cd^3f^2x^3 + 6c^2d^2f^2x^2 + 4c^3d^2f^2x + c^4f^2) \cos(fx + e)^2 + (d^4f^2x^4 + 4cd^3f^2x^3 + 6c^2d^2f^2x^2 + 4c^3d^2f^2x + c^4f^2) \sin(fx + e)^2 + 2(d^4f^2x^4 + 4cd^3f^2x^3 + 6c^2d^2f^2x^2 + 4c^3d^2f^2x + c^4f^2) \cos(fx + e)), x) + (d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2) \cos(4fx + 4e)^2 + 4(d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2) \cos(2fx + 2e)^2 + (d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2) \sin(4fx + 4e)^2 - 4(d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2) \sin(4fx + 4e) \sin(2fx + 2e) + 4(d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2) \sin(2fx + 2e)^2 + 2(d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2 - 2(d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2) \cos(2fx + 2e)) \cos(4fx + 4e) - 4(d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2) \cos(2fx + 2e)) \int -((3a^2b - b^3)d^2f^2x^2 - 6ab^2cdf + 3b^3d^2 + (3a^2b - b^3)c^2f^2 - 2(3ab^2d^2f - (3a^2b - b^3)cdf^2)x) \sin(fx + e) / (d^4f^2x^4 + 4cd^3f^2x^3 + 6c^2d^2f^2x^2 + 4c^3d^2f^2x + c^4f^2 + (d^4f^2x^4 + 4cd^3f^2x^3 + 6c^2d^2f^2x^2 + 4c^3d^2f^2x + c^4f^2) \cos(fx + e)^2 + (d^4f^2x^4 + 4cd^3f^2x^3 + 6c^2d^2f^2x^2 + 4c^3d^2f^2x + c^4f^2) \sin(fx + e)^2 - 2(d^4f^2x^4 + 4cd^3f^2x^3 + 6c^2d^2f^2x^2 + 4c^3d^2f^2x + c^4f^2) \cos(fx + e)), x) - 2(3ab^2d^2fx + 3ab^2cdf - b^3d^2 - (3ab^2d^2fx + 3ab^2cdf - b^3d^2) \cos(2fx + 2e) + (2(a^3 - 3ab^2)d^2f^2x^2 + b^3cdf + 2(a^3 - 3ab^2)c^2f^2 + (b^3d^2f + 4(a^3 - 3ab^2)cdf^2)x) \sin(2fx + 2e)) \sin(4fx + 4e) + 2(3ab^2d^2fx + 3ab^2cdf - b^3d^2) \sin(2fx + 2e)) / (d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2 + (d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2) \cos(4fx + 4e)^2 + 4(d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2) \cos(2fx + 2e)^2 + (d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2) \sin(4fx + 4e)^2 - 4(d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2) \sin(4fx + 4e) \sin(2fx + 2e) + 4(d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2) \sin(2fx + 2e)^2 + 2(d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2) \cos(2fx + 2e)) \cos(4fx + 4e) - 4(d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2) \cos(2fx + 2e))
\end{aligned}$$

Giac [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \cot(e + fx))^3}{(c + dx)^2} dx = \int \frac{(b \cot(fx + e) + a)^3}{(dx + c)^2} dx$$

[In] integrate((a+b*cot(f*x+e))^3/(d*x+c)^2,x, algorithm="giac")

[Out] integrate((b*cot(f*x + e) + a)^3/(d*x + c)^2, x)

Mupad [N/A]

Not integrable

Time = 13.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \cot(e + fx))^3}{(c + dx)^2} dx = \int \frac{(a + b \cot(e + fx))^3}{(c + dx)^2} dx$$

[In] int((a + b*cot(e + f*x))^3/(c + d*x)^2,x)

[Out] int((a + b*cot(e + f*x))^3/(c + d*x)^2, x)

3.52 $\int \frac{(c+dx)^3}{a+b \cot(e+fx)} dx$

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Optimal result

Integrand size = 20, antiderivative size = 242

$$\int \frac{(c+dx)^3}{a+b \cot(e+fx)} dx = \frac{(c+dx)^4}{4(a-ib)d} - \frac{b(c+dx)^3 \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)f} + \frac{3ibd(c+dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2(a^2+b^2)f^2} - \frac{3bd^2(c+dx) \text{PolyLog}\left(3, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2(a^2+b^2)f^3} - \frac{3ibd^3 \text{PolyLog}\left(4, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{4(a^2+b^2)f^4}$$

```
[Out] 1/4*(d*x+c)^4/(a-I*b)/d-b*(d*x+c)^3*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)/f+3/2*I*b*d*(d*x+c)^2*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)/f^2-3/2*b*d^2*(d*x+c)*polylog(3,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)/f^3-3/4*I*b*d^3*polylog(4,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)/f^4
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3812, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{(c + dx)^3}{a + b \cot(e + fx)} dx = -\frac{3bd^2(c + dx) \text{PolyLog}\left(3, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^3(a^2 + b^2)} + \frac{3ibd(c + dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^2(a^2 + b^2)} - \frac{b(c + dx)^3 \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f(a^2 + b^2)} - \frac{3ibd^3 \text{PolyLog}\left(4, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{4f^4(a^2 + b^2)} + \frac{(c + dx)^4}{4d(a - ib)}$$

[In] Int[(c + d*x)^3/(a + b*Cot[e + f*x]),x]

[Out] (c + d*x)^4/(4*(a - I*b)*d) - (b*(c + d*x)^3*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)]/(a^2 + b^2)*f) + (((3*I)/2)*b*d*(c + d*x)^2*PolyLog[2, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)]/(a^2 + b^2)*f^2) - (3*b*d^2*(c + d*x)*PolyLog[3, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)]/(2*(a^2 + b^2)*f^3) - (((3*I)/4)*b*d^3*PolyLog[4, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)]/(a^2 + b^2)*f^4)

Rule 2221

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp[(((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2320

Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2611

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)*(x_))^(m_), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +

$b*x)))^n/(b*c*n*\text{Log}[F]), x] + \text{Dist}[g*(m/(b*c*n*\text{Log}[F])), \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, (-e)*(F^{(c*(a + b*x)))^n}], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

Rule 3812

$\text{Int}[(c + d*x)^{(m+1)}/(d*(m+1)*(a + I*b)), x] + \text{Dist}[2*I*b, \text{Int}[(c + d*x)^m * E^{(2*I*k*Pi)} * (E^{\text{Simp}[2*I*(e + f*x), x]}/(a + I*b)^2 + (a^2 + b^2) * E^{(2*I*k*Pi)} * E^{\text{Simp}[2*I*(e + f*x), x]}), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IntegerQ}[4*k] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{IGtQ}[m, 0]$

Rule 6724

$\text{Int}[\text{PolyLog}[n, (c + d*x)^{(a + b*x)^p}]/(d + e*x), x_Symbol] :> \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x] \&\& \text{EqQ}[b*d, a*e]$

Rule 6744

$\text{Int}[(e + f*x)^{(m+1)}*\text{PolyLog}[n, (d + e*x)^{(c + b*x)^p}], x_Symbol] :> \text{Simp}[(e + f*x)^m * (\text{PolyLog}[n + 1, d*(F^{(c*(a + b*x)})^p]/(b*c*p*\text{Log}[F])), x] - \text{Dist}[f*(m/(b*c*p*\text{Log}[F])), \text{Int}[(e + f*x)^{(m-1)}*\text{PolyLog}[n + 1, d*(F^{(c*(a + b*x)})^p], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, n, p\}, x] \&\& \text{GtQ}[m, 0]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{(c + dx)^4}{4(a - ib)d} + (2ib) \int \frac{e^{2i(e+fx)}(c + dx)^3}{(a - ib)^2 + (-a^2 - b^2) e^{2i(e+fx)}} dx \\
 &= \frac{(c + dx)^4}{4(a - ib)d} - \frac{b(c + dx)^3 \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2 + b^2) f} \\
 &\quad + \frac{(3bd) \int (c + dx)^2 \log\left(1 + \frac{(-a^2-b^2)e^{2i(e+fx)}}{(a-ib)^2}\right) dx}{(a^2 + b^2) f} \\
 &= \frac{(c + dx)^4}{4(a - ib)d} - \frac{b(c + dx)^3 \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2 + b^2) f} \\
 &\quad + \frac{3ibd(c + dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2(a^2 + b^2) f^2} \\
 &\quad - \frac{(3ibd^2) \int (c + dx) \text{PolyLog}\left(2, -\frac{(-a^2-b^2)e^{2i(e+fx)}}{(a-ib)^2}\right) dx}{(a^2 + b^2) f^2}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{(c+dx)^4}{4(a-ib)d} - \frac{b(c+dx)^3 \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)f} + \frac{3ibd(c+dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2(a^2+b^2)f^2} \\
&\quad - \frac{3bd^2(c+dx) \text{PolyLog}\left(3, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2(a^2+b^2)f^3} + \frac{(3bd^3) \int \text{PolyLog}\left(3, -\frac{(-a^2-b^2)e^{2i(e+fx)}}{(a-ib)^2}\right) dx}{2(a^2+b^2)f^3} \\
&= \frac{(c+dx)^4}{4(a-ib)d} - \frac{b(c+dx)^3 \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)f} \\
&\quad + \frac{3ibd(c+dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2(a^2+b^2)f^2} \\
&\quad - \frac{3bd^2(c+dx) \text{PolyLog}\left(3, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2(a^2+b^2)f^3} \\
&\quad - \frac{(3ibd^3) \text{Subst}\left(\int \frac{\text{PolyLog}\left(3, \frac{(a+ib)x}{a-ib}\right)}{x} dx, x, e^{2i(e+fx)}\right)}{4(a^2+b^2)f^4} \\
&= \frac{(c+dx)^4}{4(a-ib)d} - \frac{b(c+dx)^3 \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)f} + \frac{3ibd(c+dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2(a^2+b^2)f^2} \\
&\quad - \frac{3bd^2(c+dx) \text{PolyLog}\left(3, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2(a^2+b^2)f^3} - \frac{3ibd^3 \text{PolyLog}\left(4, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{4(a^2+b^2)f^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.43

$$\begin{aligned}
&\int \frac{(c+dx)^3}{a+b \cot(e+fx)} dx \\
&= b \left(\frac{2i(c+dx)^4}{(a+ib)d} - \frac{4(a(-1+e^{2ie})+ib(1+e^{2ie}))(c+dx)^3 \log\left(1 + \frac{(-a+ib)e^{-2i(e+fx)}}{a+ib}\right)}{(a^2+b^2)f} + \frac{3d(-ia(-1+e^{2ie})+b(1+e^{2ie}))}{4(a^2+b^2)f} \left(2f^2(c+dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right) \right. \right. \\
&\quad \left. \left. + \frac{x(4c^3 + 6c^2dx + 4cd^2x^2 + d^3x^3) \sin(e)}{4(b \cos(e) + a \sin(e))} \right) \right)
\end{aligned}$$

[In] Integrate[(c + d*x)^3/(a + b*Cot[e + f*x]),x]

[Out] (b*(((2*I)*(c + d*x)^4)/((a + I*b)*d) - (4*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(c + d*x)^3*Log[1 + (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x))])))/((a^2 + b^2)*f) + (3*d*((-I)*a*(-1 + E^((2*I)*e)) + b*(1 + E^((2*I)*e)))*(2*f^2*(c + d*x)^2*PolyLog[2, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x))]]) + d*((-2*I)*f*(c + d*x)*PolyLog[3, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x))]))

)] - d*PolyLog[4, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]/((a^2 + b^2)*f^4))/(4*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))) + (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Sin[e])/(4*(b*Cos[e] + a*SIN[e]))

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1401 vs. 2(215) = 430.

Time = 0.43 (sec) , antiderivative size = 1402, normalized size of antiderivative = 5.79

method	result	size
risch	Expression too large to display	1402

[In] int((d*x+c)^3/(a+b*cot(f*x+e)),x,method=_RETURNVERBOSE)

[Out] $\frac{1}{4}d^3/(a+I*b)*x^4 + \frac{1}{(a+I*b)*c^3*x + 1/4/d/(a+I*b)*c^4 + I/f^4/(b-I*a)*b*e^3*d^3/(I*b-a)*\ln(\exp(2*I*(f*x+e))*a+I*b*\exp(2*I*(f*x+e))-a+I*b) + 6/f/(b-I*a)*b/(a-I*b)*c^2*d*e*x + I/f^4/(b-I*a)*b/(a-I*b)*e^3*d^3*\ln(1-(a+I*b)*\exp(2*I*(f*x+e)))/(a-I*b) - 6/f^2/(b-I*a)*b/(a-I*b)*e^2*c*d^2*x + I/f/(b-I*a)*b/(a-I*b)*d^3*\ln(1-(a+I*b)*\exp(2*I*(f*x+e)))/(a-I*b) *x^3 + 3/f^2/(b-I*a)*b/(a-I*b)*d^2*c*\text{polylog}(2, (a+I*b)*\exp(2*I*(f*x+e))/(a-I*b)) *x - 2*I/f^4/(b-I*a)*b*e^3*d^3/(I*b-a)*\ln(\exp(I*(f*x+e))) + 3/2*I/f^3/(b-I*a)*b/(a-I*b)*d^2*c*\text{polylog}(3, (a+I*b)*\exp(2*I*(f*x+e))/(a-I*b)) + 3/2*I/f^3/(b-I*a)*b/(a-I*b)*d^3*\text{polylog}(3, (a+I*b)*\exp(2*I*(f*x+e))/(a-I*b)) *x + 3/f^2/(b-I*a)*b/(a-I*b)*c^2*d*e^2 + 3/2/f^2/(b-I*a)*b/(a-I*b)*c^2*d*\text{polylog}(2, (a+I*b)*\exp(2*I*(f*x+e))/(a-I*b)) + 2/f^3/(b-I*a)*b/(a-I*b)*d^3*e^3*x - 4/f^3/(b-I*a)*b/(a-I*b)*e^3*c*d^2 + 3/2/f^2/(b-I*a)*b/(a-I*b)*d^3*\text{polylog}(2, (a+I*b)*\exp(2*I*(f*x+e))/(a-I*b)) *x^2 + 2*I/f/(b-I*a)*b*c^3/(I*b-a)*\ln(\exp(I*(f*x+e))) - I/f/(b-I*a)*b*c^3/(I*b-a)*\ln(\exp(2*I*(f*x+e)))*a + I*b*\exp(2*I*(f*x+e)) - a + I*b + 6*I/f^3/(b-I*a)*b*e^2*c*d^2/(I*b-a)*\ln(\exp(I*(f*x+e))) - 3*I/f^3/(b-I*a)*b*e^2*c*d^2/(I*b-a)*\ln(\exp(2*I*(f*x+e)))*a + I*b*\exp(2*I*(f*x+e)) - a + I*b - 6*I/f^2/(b-I*a)*b*e*c^2*d/(I*b-a)*\ln(\exp(I*(f*x+e))) + 3*I/f^2/(b-I*a)*b*e*c^2*d/(I*b-a)*\ln(\exp(2*I*(f*x+e)))*a + I*b*\exp(2*I*(f*x+e)) - a + I*b + 3*I/f/(b-I*a)*b/(a-I*b)*c^2*d*\ln(1-(a+I*b)*\exp(2*I*(f*x+e)))/(a-I*b) *x + 3*I/f^2/(b-I*a)*b/(a-I*b)*c^2*d*\ln(1-(a+I*b)*\exp(2*I*(f*x+e)))/(a-I*b) *e - 3*I/f^3/(b-I*a)*b/(a-I*b)*e^2*c*d^2*\ln(1-(a+I*b)*\exp(2*I*(f*x+e)))/(a-I*b) + 3*I/f/(b-I*a)*b/(a-I*b)*d^2*c*\ln(1-(a+I*b)*\exp(2*I*(f*x+e)))/(a-I*b) *x^2 + 1/2/(b-I*a)*b/(a-I*b)*d^3*x^4 + 3/(b-I*a)*b/(a-I*b)*c^2*d*x^2 + 2/(b-I*a)*b/(a-I*b)*d^2*c*x^3 + 3/2/f^4/(b-I*a)*b/(a-I*b)*d^3*e^4 - 3/4/f^4/(b-I*a)*b/(a-I*b)*d^3*\text{polylog}(4, (a+I*b)*\exp(2*I*(f*x+e))/(a-I*b)) + d^2/(a+I*b)*c*x^3 + 3/2*d/(a+I*b)*c^2*x^2$

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(203) = 406$.

Time = 0.34 (sec) , antiderivative size = 1031, normalized size of antiderivative = 4.26

$$\int \frac{(c + dx)^3}{a + b \cot(e + fx)} dx = \text{Too large to display}$$

[In] integrate((d*x+c)^3/(a+b*cot(f*x+e)),x, algorithm="fricas")

[Out] $\frac{1}{8}(2ad^3f^4x^4 + 8ac^2d^2f^4x^3 + 12a^2c^2d^2f^4x^2 + 8a^2c^3f^4x - 3Ibd^3\text{polylog}(4, ((a^2 + 2Iab - b^2)\cos(2fx + 2e) + (Ia^2 - 2ab - Ib^2)\sin(2fx + 2e))/(a^2 + b^2)) + 3Ibd^3\text{polylog}(4, ((a^2 - 2Iab - b^2)\cos(2fx + 2e) + (-Ia^2 - 2ab + Ib^2)\sin(2fx + 2e))/(a^2 + b^2)) - 6(-Ibd^3f^2x^2 - 2Ib^2cd^2f^2x - Ib^2c^2d^2f^2)\text{dilog}(- (a^2 + b^2 - (a^2 + 2Iab - b^2)\cos(2fx + 2e) + (-Ia^2 + 2ab + Ib^2)\sin(2fx + 2e))/(a^2 + b^2) + 1) - 6(Ibd^3f^2x^2 + 2Ib^2cd^2f^2x + Ib^2c^2d^2f^2)\text{dilog}(- (a^2 + b^2 - (a^2 - 2Iab - b^2)\cos(2fx + 2e) + (Ia^2 + 2ab - Ib^2)\sin(2fx + 2e))/(a^2 + b^2) + 1) + 4(bd^3e^3 - 3b^2cd^2e^2f + 3b^2c^2d^2e^2f^2 - b^2c^3f^3)\log(1/2a^2 + Iab - 1/2b^2 - 1/2(a^2 + b^2)\cos(2fx + 2e) + 1/2(Ia^2 + Ib^2)\sin(2fx + 2e)) + 4(bd^3e^3 - 3b^2cd^2e^2f + 3b^2c^2d^2e^2f^2 - b^2c^3f^3)\log(-1/2a^2 + Iab + 1/2b^2 + 1/2(a^2 + b^2)\cos(2fx + 2e) + 1/2(Ia^2 + Ib^2)\sin(2fx + 2e)) - 4(bd^3f^3x^3 + 3b^2cd^2f^3x^2 + 3b^2c^2d^2f^3x + bd^3e^3 - 3b^2cd^2e^2f + 3b^2c^2d^2e^2f^2)\log((a^2 + b^2 - (a^2 + 2Iab - b^2)\cos(2fx + 2e) + (-Ia^2 + 2ab + Ib^2)\sin(2fx + 2e))/(a^2 + b^2)) - 4(bd^3f^3x^3 + 3b^2cd^2f^3x^2 + 3b^2c^2d^2f^3x + bd^3e^3 - 3b^2cd^2e^2f + 3b^2c^2d^2e^2f^2)\log((a^2 + b^2 - (a^2 - 2Iab - b^2)\cos(2fx + 2e) + (Ia^2 + 2ab - Ib^2)\sin(2fx + 2e))/(a^2 + b^2)) - 6(bd^3fx + b^2cd^2f)\text{polylog}(3, ((a^2 + 2Iab - b^2)\cos(2fx + 2e) + (Ia^2 - 2ab - Ib^2)\sin(2fx + 2e))/(a^2 + b^2)) - 6(bd^3fx + b^2cd^2f)\text{polylog}(3, ((a^2 - 2Iab - b^2)\cos(2fx + 2e) + (-Ia^2 - 2ab + Ib^2)\sin(2fx + 2e))/(a^2 + b^2)))/((a^2 + b^2)f^4)$

Sympy [F]

$$\int \frac{(c + dx)^3}{a + b \cot(e + fx)} dx = \int \frac{(c + dx)^3}{a + b \cot(e + fx)} dx$$

[In] integrate((d*x+c)**3/(a+b*cot(f*x+e)),x)

[Out] Integral((c + d*x)**3/(a + b*cot(e + f*x)), x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(203) = 406$.

Time = 0.57 (sec) , antiderivative size = 990, normalized size of antiderivative = 4.09

$$\int \frac{(c + dx)^3}{a + b \cot(e + fx)} dx = \text{Too large to display}$$

[In] integrate((d*x+c)^3/(a+b*cot(f*x+e)),x, algorithm="maxima")

[Out]
$$-1/12*(18*c^2*d*e*(2*(f*x + e)*a/((a^2 + b^2)*f) - 2*b*\log(a*\tan(f*x + e) + b)/((a^2 + b^2)*f) + b*\log(\tan(f*x + e)^2 + 1)/((a^2 + b^2)*f)) - 6*(2*(f*x + e)*a/(a^2 + b^2) - 2*b*\log(a*\tan(f*x + e) + b)/(a^2 + b^2) + b*\log(\tan(f*x + e)^2 + 1)/(a^2 + b^2))*c^3 - (3*(f*x + e)^4*(a + I*b)*d^3 - 12*I*b*d^3*\text{polylog}(4, (I*a - b)*e^{(2*I*f*x + 2*I*e)/(I*a + b)}) - 12*((a + I*b)*d^3*e - (a + I*b)*c*d^2*f)*(f*x + e)^3 + 18*((a + I*b)*d^3*e^2 - 2*(a + I*b)*c*d^2*e*f + (a + I*b)*c^2*d*f^2)*(f*x + e)^2 - 12*((a + I*b)*d^3*e^3 - 3*(a + I*b)*c*d^2*e^2*f)*(f*x + e) + 12*(I*b*d^3*e^3 - 3*I*b*c*d^2*e^2*f)*\arctan2(b*\cos(2*f*x + 2*e) + a*\sin(2*f*x + 2*e) + b, a*\cos(2*f*x + 2*e) - b*\sin(2*f*x + 2*e) - a) + 4*(-4*I*(f*x + e)^3*b*d^3 + 9*(I*b*d^3*e - I*b*c*d^2*f)*(f*x + e)^2 + 9*(-I*b*d^3*e^2 + 2*I*b*c*d^2*e*f - I*b*c^2*d*f^2)*(f*x + e))*\arctan2(-(2*a*b*\cos(2*f*x + 2*e) + (a^2 - b^2)*\sin(2*f*x + 2*e))/(a^2 + b^2), (2*a*b*\sin(2*f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)*\cos(2*f*x + 2*e))/(a^2 + b^2)) + 6*(4*I*(f*x + e)^2*b*d^3 + 3*I*b*d^3*e^2 - 6*I*b*c*d^2*e*f + 3*I*b*c^2*d*f^2 + 6*(-I*b*d^3*e + I*b*c*d^2*f)*(f*x + e))*\text{dilog}((I*a - b)*e^{(2*I*f*x + 2*I*e)/(I*a + b)}) + 6*(b*d^3*e^3 - 3*b*c*d^2*e^2*f)*\log((a^2 + b^2)*\cos(2*f*x + 2*e)^2 + 4*a*b*\sin(2*f*x + 2*e) + (a^2 + b^2)*\sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*\cos(2*f*x + 2*e)) - 2*(4*(f*x + e)^3*b*d^3 - 9*(b*d^3*e - b*c*d^2*f)*(f*x + e)^2 + 9*(b*d^3*e^2 - 2*b*c*d^2*e*f + b*c^2*d*f^2)*(f*x + e))*\log(((a^2 + b^2)*\cos(2*f*x + 2*e)^2 + 4*a*b*\sin(2*f*x + 2*e) + (a^2 + b^2)*\sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*\cos(2*f*x + 2*e))/(a^2 + b^2)) - 6*(4*(f*x + e)*b*d^3 - 3*b*d^3*e + 3*b*c*d^2*f)*\text{polylog}(3, (I*a - b)*e^{(2*I*f*x + 2*I*e)/(I*a + b)})/((a^2 + b^2)*f^3))/f$$

Giac [F]

$$\int \frac{(c + dx)^3}{a + b \cot(e + fx)} dx = \int \frac{(dx + c)^3}{b \cot(fx + e) + a} dx$$

[In] integrate((d*x+c)^3/(a+b*cot(f*x+e)),x, algorithm="giac")

[Out] integrate((d*x + c)^3/(b*cot(f*x + e) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx)^3}{a + b \cot(e + fx)} dx = \int \frac{(c + dx)^3}{a + b \cot(e + fx)} dx$$

```
[In] int((c + d*x)^3/(a + b*cot(e + f*x)),x)
```

```
[Out] int((c + d*x)^3/(a + b*cot(e + f*x)), x)
```

3.53 $\int \frac{(c+dx)^2}{a+b \cot(e+fx)} dx$

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Optimal result

Integrand size = 20, antiderivative size = 181

$$\int \frac{(c+dx)^2}{a+b \cot(e+fx)} dx = \frac{(c+dx)^3}{3(a-ib)d} - \frac{b(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)f} + \frac{ibd(c+dx) \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)f^2} - \frac{bd^2 \operatorname{PolyLog}\left(3, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2(a^2+b^2)f^3}$$

[Out] 1/3*(d*x+c)^3/(a-I*b)/d-b*(d*x+c)^2*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)/f+I*b*d*(d*x+c)*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)/f^2-1/2*b*d^2*polylog(3,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)/f^3

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3812, 2221, 2611, 2320, 6724}

$$\int \frac{(c+dx)^2}{a+b \cot(e+fx)} dx = \frac{ibd(c+dx) \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f^2(a^2+b^2)} - \frac{b(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f(a^2+b^2)} - \frac{bd^2 \operatorname{PolyLog}\left(3, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^3(a^2+b^2)} + \frac{(c+dx)^3}{3d(a-ib)}$$

[In] Int[(c + d*x)^2/(a + b*Cot[e + f*x]),x]

[Out] (c + d*x)^3/(3*(a - I*b)*d) - (b*(c + d*x)^2*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/((a^2 + b^2)*f) + (I*b*d*(c + d*x)*PolyLog[2, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/((a^2 + b^2)*f^2) - (b*d^2*PolyLog[3, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/(2*(a^2 + b^2)*f^3)

Rule 2221

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[(c + d*x)^m/(b*f*g*n*Log[F])*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2320

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2611

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*(f_) + (g_)*(x_)]^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 3812

Int[(((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*tan[(e_) + Pi*(k_) + (f_)*(x_)]), x_Symbol] := Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist[2*I*b, Int[(c + d*x)^m*E^(2*I*k*Pi)*(E^Simp[2*I*(e + f*x), x])/((a + I*b)^2 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

Rule 6724

Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c+dx)^3}{3(a-ib)d} + (2ib) \int \frac{e^{2i(e+fx)}(c+dx)^2}{(a-ib)^2 + (-a^2-b^2)e^{2i(e+fx)}} dx \\
&= \frac{(c+dx)^3}{3(a-ib)d} - \frac{b(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)f} \\
&\quad + \frac{(2bd) \int (c+dx) \log\left(1 + \frac{(-a^2-b^2)e^{2i(e+fx)}}{(a-ib)^2}\right) dx}{(a^2+b^2)f} \\
&= \frac{(c+dx)^3}{3(a-ib)d} - \frac{b(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)f} \\
&\quad + \frac{ibd(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)f^2} \\
&\quad - \frac{(ibd^2) \int \text{PolyLog}\left(2, -\frac{(-a^2-b^2)e^{2i(e+fx)}}{(a-ib)^2}\right) dx}{(a^2+b^2)f^2} \\
&= \frac{(c+dx)^3}{3(a-ib)d} - \frac{b(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)f} \\
&\quad + \frac{ibd(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)f^2} \\
&\quad - \frac{(bd^2) \text{Subst}\left(\int \frac{\text{PolyLog}\left(2, \frac{(a+ib)x}{a-ib}\right)}{x} dx, x, e^{2i(e+fx)}\right)}{2(a^2+b^2)f^3} \\
&= \frac{(c+dx)^3}{3(a-ib)d} - \frac{b(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)f} \\
&\quad + \frac{ibd(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)f^2} - \frac{bd^2 \text{PolyLog}\left(3, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2(a^2+b^2)f^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.60

$$\int \frac{(c + dx)^2}{a + b \cot(e + fx)} dx$$

$$= \frac{b \left(\frac{4i(c+dx)^3}{(a+ib)d} - \frac{6(a(-1+e^{2ie})+ib(1+e^{2ie}))(c+dx)^2 \log\left(1 + \frac{(-a+ib)e^{-2i(e+fx)}}{a+ib}\right)}{(a^2+b^2)f} \right) + \frac{3d(-ia(-1+e^{2ie})+b(1+e^{2ie})) \left(2f(c+dx) \text{PolyLog}\left(2, \frac{(-a+ib)e^{-2i(e+fx)}}{a+ib}\right) \right)}{(a^2+b^2)f}}{6(a(-1+e^{2ie})+ib(1+e^{2ie}))} + \frac{x(3c^2 + 3cdx + d^2x^2) \sin(e)}{3(b \cos(e) + a \sin(e))}$$

`[In] Integrate[(c + d*x)^2/(a + b*Cot[e + f*x]),x]`

```
[Out] (b*(((4*I)*(c + d*x)^3)/((a + I*b)*d) - (6*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(c + d*x)^2*Log[1 + (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x))])))/((a^2 + b^2)*f) + (3*d*((-I)*a*(-1 + E^((2*I)*e)) + b*(1 + E^((2*I)*e)))*(2*f*(c + d*x)*PolyLog[2, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))] - I*d*PolyLog[3, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]))/((a^2 + b^2)*f^3)))/(6*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Sin[e])/(3*(b*Cos[e] + a*Sin[e]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 896 vs. 2(163) = 326.

Time = 0.32 (sec) , antiderivative size = 897, normalized size of antiderivative = 4.96

method	result
risch	$\frac{d^2x^3}{3ib+3a} + \frac{dcx^2}{ib+a} + \frac{c^2x}{ib+a} + \frac{c^3}{3d(ib+a)} + \frac{2bd^2x^3}{3(-ia+b)(-ib+a)} + \frac{2ibc^2 \ln(e^{i(fx+e)})}{f(-ia+b)(ib-a)} - \frac{ibe^2d^2 \ln\left(1 - \frac{(ib+a)e^{2i(fx+e)}}{-ib+a}\right)}{f^3(-ia+b)(-ib+a)} + \frac{2ibc}{f^3(-ia+b)(-ib+a)}$

`[In] int((d*x+c)^2/(a+b*cot(f*x+e)),x,method=_RETURNVERBOSE)`

```
[Out] 1/3*d^2/(a+I*b)*x^3+d/(a+I*b)*c*x^2+1/(a+I*b)*c^2*x+1/3/d/(a+I*b)*c^3+2/3/(b-I*a)*b/(a-I*b)*d^2*x^3+2*I/f/(b-I*a)*b*c^2/(I*b-a)*ln(exp(I*(f*x+e)))-I/f^3/(b-I*a)*b/(a-I*b)*e^2*d^2*ln(1-(a+I*b)*exp(2*I*(f*x+e)))/(a-I*b))+2*I/f^2/(b-I*a)*b/(a-I*b)*c*d*ln(1-(a+I*b)*exp(2*I*(f*x+e)))/(a-I*b)*e-I/f^3/(b-I*a)*b*e^2*d^2/(I*b-a)*ln(exp(2*I*(f*x+e))*a+I*b*exp(2*I*(f*x+e))-a+I*b)-4*I/f^2/(b-I*a)*b*e*c*d/(I*b-a)*ln(exp(I*(f*x+e)))-2/f^2/(b-I*a)*b/(a-I*b)*e^2*d^2*x+1/f^2/(b-I*a)*b/(a-I*b)*d^2*polylog(2,(a+I*b)*exp(2*I*(f*x+e)))/(a-I*b)*x+2*I/f^3/(b-I*a)*b*e^2*d^2/(I*b-a)*ln(exp(I*(f*x+e)))+2*I/f^2/(b-I*a)*b*e*c*d/(I*b-a)*ln(exp(2*I*(f*x+e))*a+I*b*exp(2*I*(f*x+e))-a+I*b)+2/(b-I*a)*
```

$$\begin{aligned} & b/(a-I*b)*c*d*x^2+4/f/(b-I*a)*b/(a-I*b)*c*d*e*x+2/f^2/(b-I*a)*b/(a-I*b)*c*d \\ & *e^2+1/f^2/(b-I*a)*b/(a-I*b)*c*d*polylog(2, (a+I*b)*exp(2*I*(f*x+e)))/(a-I*b) \\ &)+1/2*I/f^3/(b-I*a)*b/(a-I*b)*d^2*polylog(3, (a+I*b)*exp(2*I*(f*x+e)))/(a-I*b) \\ &))-I/f/(b-I*a)*b*c^2/(I*b-a)*ln(exp(2*I*(f*x+e))*a+I*b*exp(2*I*(f*x+e))-a+I \\ & *b)+2*I/f/(b-I*a)*b/(a-I*b)*c*d*ln(1-(a+I*b)*exp(2*I*(f*x+e)))/(a-I*b))*x-4/ \\ & 3/f^3/(b-I*a)*b/(a-I*b)*e^3*d^2+I/f/(b-I*a)*b/(a-I*b)*d^2*ln(1-(a+I*b)*exp(\\ & 2*I*(f*x+e)))/(a-I*b))*x^2 \end{aligned}$$

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 732 vs. $2(154) = 308$.

Time = 0.31 (sec) , antiderivative size = 732, normalized size of antiderivative = 4.04

$$\int \frac{(c + dx)^2}{a + b \cot(e + fx)} dx$$

$$= \frac{4ad^2 f^3 x^3 + 12acdf^3 x^2 + 12ac^2 f^3 x - 3bd^2 \operatorname{polylog}\left(3, \frac{(a^2 + 2iab - b^2) \cos(2fx + 2e) + (ia^2 - 2ab - ib^2) \sin(2fx + 2e)}{a^2 + b^2}\right) - 3b}{}$$

[In] integrate((d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="fricas")

[Out] $\frac{1}{12}*(4*a*d^2*f^3*x^3 + 12*a*c*d*f^3*x^2 + 12*a*c^2*f^3*x - 3*b*d^2*polylog(3, ((a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 - 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 3*b*d^2*polylog(3, ((a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 - 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 6*(-I*b*d^2*f*x - I*b*c*d*f)*dilog(-(a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - 6*(I*b*d^2*f*x + I*b*c*d*f)*dilog(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - 6*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(1/2*a^2 + I*a*b - 1/2*b^2 - 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 6*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(-1/2*a^2 + I*a*b + 1/2*b^2 + 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log((a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 6*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log((a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)))/((a^2 + b^2)*f^3)$

SymPy [F]

$$\int \frac{(c + dx)^2}{a + b \cot(e + fx)} dx = \int \frac{(c + dx)^2}{a + b \cot(e + fx)} dx$$

```
[In] integrate((d*x+c)**2/(a+b*cot(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**2/(a + b*cot(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(154) = 308$.

Time = 0.54 (sec) , antiderivative size = 720, normalized size of antiderivative = 3.98

$$\int \frac{(c + dx)^2}{a + b \cot(e + fx)} dx =$$

$$6cde \left(\frac{2(fx+e)a}{(a^2+b^2)f} - \frac{2b \log(a \tan(fx+e)+b)}{(a^2+b^2)f} + \frac{b \log(\tan(fx+e)^2+1)}{(a^2+b^2)f} \right) - 3 \left(\frac{2(fx+e)a}{a^2+b^2} - \frac{2b \log(a \tan(fx+e)+b)}{a^2+b^2} + \frac{b \log(\tan(fx+e)^2+1)}{a^2+b^2} \right)$$

```
[In] integrate((d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/6*(6*c*d*e*(2*(f*x + e)*a/((a^2 + b^2)*f) - 2*b*log(a*tan(f*x + e) + b)/
((a^2 + b^2)*f) + b*log(tan(f*x + e)^2 + 1)/((a^2 + b^2)*f)) - 3*(2*(f*x +
e)*a/(a^2 + b^2) - 2*b*log(a*tan(f*x + e) + b)/(a^2 + b^2) + b*log(tan(f*x
+ e)^2 + 1)/(a^2 + b^2))*c^2 - (2*(f*x + e)^3*(a + I*b)*d^2 + 6*(f*x + e)*(
a + I*b)*d^2*e^2 - 6*I*b*d^2*e^2*arctan2(b*cos(2*f*x + 2*e) + a*sin(2*f*x +
2*e) + b, a*cos(2*f*x + 2*e) - b*sin(2*f*x + 2*e) - a) - 3*b*d^2*e^2*log((
a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*f*x + 2*e) + (a^2 + b^2)*sin(2*
f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*f*x + 2*e)) - 3*b*d^2*polylo
g(3, (I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) - 6*((a + I*b)*d^2*e - (a + I
*b)*c*d*f)*(f*x + e)^2 + 6*(-I*(f*x + e)^2*b*d^2 + 2*(I*b*d^2*e - I*b*c*d*f
)*(f*x + e))*arctan2(-(2*a*b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f*x + 2*e
))/(a^2 + b^2), (2*a*b*sin(2*f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)*cos(2*f*x
+ 2*e))/(a^2 + b^2)) + 6*(I*(f*x + e)*b*d^2 - I*b*d^2*e + I*b*c*d*f)*dilog
((I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) - 3*((f*x + e)^2*b*d^2 - 2*(b*d^2
*e - b*c*d*f)*(f*x + e))*log(((a^2 + b^2)*cos(2*f*x + 2*e)^2 + 4*a*b*sin(2*
f*x + 2*e) + (a^2 + b^2)*sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos
(2*f*x + 2*e))/(a^2 + b^2)))/((a^2 + b^2)*f^2))/f
```

Giac [F]

$$\int \frac{(c + dx)^2}{a + b \cot(e + fx)} dx = \int \frac{(dx + c)^2}{b \cot(fx + e) + a} dx$$

[In] integrate((d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="giac")

[Out] integrate((d*x + c)^2/(b*cot(f*x + e) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx)^2}{a + b \cot(e + fx)} dx = \int \frac{(c + dx)^2}{a + b \cot(e + fx)} dx$$

[In] int((c + d*x)^2/(a + b*cot(e + f*x)),x)

[Out] int((c + d*x)^2/(a + b*cot(e + f*x)), x)

3.54 $\int \frac{c+dx}{a+b \cot(e+fx)} dx$

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Optimal result

Integrand size = 18, antiderivative size = 126

$$\int \frac{c+dx}{a+b \cot(e+fx)} dx = \frac{(c+dx)^2}{2(a-ib)d} - \frac{b(c+dx) \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)f} + \frac{ibd \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2(a^2+b^2)f^2}$$

[Out] 1/2*(d*x+c)^2/(a-I*b)/d-b*(d*x+c)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)/f+1/2*I*b*d*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)/f^2

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3812, 2221, 2317, 2438}

$$\int \frac{c+dx}{a+b \cot(e+fx)} dx = -\frac{b(c+dx) \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f(a^2+b^2)} + \frac{ibd \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2f^2(a^2+b^2)} + \frac{(c+dx)^2}{2d(a-ib)}$$

[In] Int[(c + d*x)/(a + b*Cot[e + f*x]),x]

[Out] (c + d*x)^2/(2*(a - I*b)*d) - (b*(c + d*x)*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/((a^2 + b^2)*f) + ((I/2)*b*d*PolyLog[2, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/((a^2 + b^2)*f^2)

Rule 2221

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3812

```
Int[(((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*tan[(e_) + Pi*(k_) + (f_)*(
x_)]), x_Symbol] := Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist
[2*I*b, Int[(c + d*x)^m*E^(2*I*k*Pi)*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2
+ (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])), x], x] /; FreeQ[{a,
b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c + dx)^2}{2(a - ib)d} + (2ib) \int \frac{e^{2i(e+fx)}(c + dx)}{(a - ib)^2 + (-a^2 - b^2) e^{2i(e+fx)}} dx \\
&= \frac{(c + dx)^2}{2(a - ib)d} - \frac{b(c + dx) \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2 + b^2) f} + \frac{(bd) \int \log\left(1 + \frac{(-a^2-b^2)e^{2i(e+fx)}}{(a-ib)^2}\right) dx}{(a^2 + b^2) f} \\
&= \frac{(c + dx)^2}{2(a - ib)d} - \frac{b(c + dx) \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2 + b^2) f} \\
&\quad - \frac{(ibd) \text{Subst}\left(\int \frac{\log\left(1 + \frac{(-a^2-b^2)x}{(a-ib)^2}\right)}{x} dx, x, e^{2i(e+fx)}\right)}{2(a^2 + b^2) f^2} \\
&= \frac{(c + dx)^2}{2(a - ib)d} - \frac{b(c + dx) \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2 + b^2) f} + \frac{ibd \text{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{2(a^2 + b^2) f^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.44

$$\int \frac{c + dx}{a + b \cot(e + fx)} dx = \frac{1}{2} b \left(\frac{2i(c + dx)^2}{(a + ib)d(a(-1 + e^{2ie}) + ib(1 + e^{2ie}))} - \frac{2(c + dx) \log \left(1 + \frac{(-a+ib)e^{-2i(e+fx)}}{a+ib} \right)}{(a^2 + b^2) f} - \frac{id \operatorname{PolyLog} \left(2, \frac{(a-ib)e^{-2i(e+fx)}}{a+ib} \right)}{(a^2 + b^2) f^2} \right) + \frac{x(2c + dx) \sin(e)}{2(b \cos(e) + a \sin(e))}$$

[In] Integrate[(c + d*x)/(a + b*Cot[e + f*x]),x]

[Out] (b*(((2*I)*(c + d*x)^2)/((a + I*b)*d*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))) - (2*(c + d*x)*Log[1 + (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x))]))/((a^2 + b^2)*f) - (I*d*PolyLog[2, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]))/((a^2 + b^2)*f^2))/2 + (x*(2*c + d*x)*Sin[e])/(2*(b*Cos[e] + a*Ssin[e]))

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(112) = 224.

Time = 0.38 (sec) , antiderivative size = 445, normalized size of antiderivative = 3.53

method	result
risch	$\frac{dx^2}{2ib+2a} + \frac{xc}{ib+a} - \frac{2bc \ln(e^{i(fx+e)})}{f(ib+a)(ib-a)} + \frac{bc \ln(e^{2i(fx+e)} a + ib e^{2i(fx+e)} - a + ib)}{f(ib+a)(ib-a)} - \frac{bd \ln \left(1 - \frac{(ib+a)e^{2i(fx+e)}}{-ib+a} \right) x}{f(ib+a)(-ib+a)} + \frac{ibd x^2}{(ib+a)(-ib+a)}$

[In] int((d*x+c)/(a+b*cot(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 1/2/(a+I*b)*d*x^2+1/(a+I*b)*x*c-2/f*b/(a+I*b)*c/(I*b-a)*ln(exp(I*(f*x+e)))+1/f*b/(a+I*b)*c/(I*b-a)*ln(exp(2*I*(f*x+e))*a+I*b*exp(2*I*(f*x+e))-a+I*b)-1/f*b/(a+I*b)/(a-I*b)*d*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*x+I*b/(a+I*b)/(a-I*b)*d*x^2-1/f^2*b/(a+I*b)/(a-I*b)*d*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))*e+2*I/f*b/(a+I*b)/(a-I*b)*d*e*x+I/f^2*b/(a+I*b)/(a-I*b)*d*e^2+1/2*I/f^2*b/(a+I*b)/(a-I*b)*d*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))+2/f^2*b/(a+I*b)*d*e/(I*b-a)*ln(exp(I*(f*x+e)))-1/f^2*b/(a+I*b)*d*e/(I*b-a)*ln(exp(2*I*(f*x+e))*a+I*b*exp(2*I*(f*x+e))-a+I*b)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(105) = 210$.

Time = 0.29 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.77

$$\int \frac{c + dx}{a + b \cot(e + fx)} dx$$

$$= \frac{2adf^2x^2 + 4acf^2x + i bd \operatorname{Li}_2\left(-\frac{a^2+b^2-(a^2+2iab-b^2)\cos(2fx+2e)+(-ia^2+2ab+ib^2)\sin(2fx+2e)}{a^2+b^2} + 1\right) - i bd \operatorname{Li}_2\left(-\frac{a^2+b^2-(a^2+2iab-b^2)\cos(2fx+2e)+(-ia^2+2ab+ib^2)\sin(2fx+2e)}{a^2+b^2} + 1\right)}{2adf^2x^2 + 4acf^2x + i bd \operatorname{Li}_2\left(-\frac{a^2+b^2-(a^2+2iab-b^2)\cos(2fx+2e)+(-ia^2+2ab+ib^2)\sin(2fx+2e)}{a^2+b^2} + 1\right) - i bd \operatorname{Li}_2\left(-\frac{a^2+b^2-(a^2+2iab-b^2)\cos(2fx+2e)+(-ia^2+2ab+ib^2)\sin(2fx+2e)}{a^2+b^2} + 1\right)}$$

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d*f^2*x^2 + 4*a*c*f^2*x + I*b*d*dilog(-(a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - I*b*d*dilog(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) + 2*(b*d*e - b*c*f)*log(1/2*a^2 + I*a*b - 1/2*b^2 - 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) + 2*(b*d*e - b*c*f)*log(-1/2*a^2 + I*a*b + 1/2*b^2 + 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 2*(b*d*f*x + b*d*e)*log((a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 2*(b*d*f*x + b*d*e)*log((a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)))/((a^2 + b^2)*f^2)
```

Sympy [F]

$$\int \frac{c + dx}{a + b \cot(e + fx)} dx = \int \frac{c + dx}{a + b \cot(e + fx)} dx$$

```
[In] integrate((d*x+c)/(a+b*cot(f*x+e)),x)
```

```
[Out] Integral((c + d*x)/(a + b*cot(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(105) = 210$.

Time = 0.45 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.22

$$\int \frac{c + dx}{a + b \cot(e + fx)} dx$$

$$= \frac{(a + ib)df^2x^2 + 2(a + ib)cf^2x - 2ibdfx \arctan\left(-\frac{2ab \cos(2fx+2e) + (a^2 - b^2) \sin(2fx+2e)}{a^2 + b^2}, \frac{2ab \sin(2fx+2e) + a^2 + b^2}{a^2 + b^2}\right)}{f^2}$$

[In] integrate((d*x+c)/(a+b*cot(f*x+e)),x, algorithm="maxima")

[Out] $\frac{1}{2}((a + I*b)*d*f^2*x^2 + 2*(a + I*b)*c*f^2*x - 2*I*b*d*f*x*\arctan2(- (2*a*b*\cos(2*f*x + 2*e) + (a^2 - b^2)*\sin(2*f*x + 2*e))/(a^2 + b^2), (2*a*b*\sin(2*f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)*\cos(2*f*x + 2*e))/(a^2 + b^2)) - b*d*f*x*\log(((a^2 + b^2)*\cos(2*f*x + 2*e)^2 + 4*a*b*\sin(2*f*x + 2*e) + (a^2 + b^2)*\sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*\cos(2*f*x + 2*e))/(a^2 + b^2)) - 2*I*b*c*f*\arctan2(b*\cos(2*f*x + 2*e) + a*\sin(2*f*x + 2*e) + b, a*\cos(2*f*x + 2*e) - b*\sin(2*f*x + 2*e) - a) - b*c*f*\log((a^2 + b^2)*\cos(2*f*x + 2*e)^2 + 4*a*b*\sin(2*f*x + 2*e) + (a^2 + b^2)*\sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*\cos(2*f*x + 2*e)) + I*b*d*\operatorname{dilog}((I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)))/(a^2 + b^2)*f^2$

Giac [F]

$$\int \frac{c + dx}{a + b \cot(e + fx)} dx = \int \frac{dx + c}{b \cot(fx + e) + a} dx$$

[In] integrate((d*x+c)/(a+b*cot(f*x+e)),x, algorithm="giac")

[Out] integrate((d*x + c)/(b*cot(f*x + e) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx}{a + b \cot(e + fx)} dx = \int \frac{c + dx}{a + b \cot(e + fx)} dx$$

[In] int((c + d*x)/(a + b*cot(e + f*x)),x)

[Out] int((c + d*x)/(a + b*cot(e + f*x)), x)

3.55 $\int \frac{1}{(c+dx)(a+b \cot(e+fx))} dx$

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Maxima [N/A]	386
Giac [N/A]	386
Mupad [N/A]	386

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b \cot(e+fx))} dx = \text{Int}\left(\frac{1}{(c+dx)(a+b \cot(e+fx))}, x\right)$$

[Out] Unintegrable(1/(d*x+c)/(a+b*cot(f*x+e)),x)

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b \cot(e+fx))} dx = \int \frac{1}{(c+dx)(a+b \cot(e+fx))} dx$$

[In] Int[1/((c + d*x)*(a + b*Cot[e + f*x])),x]

[Out] Defer[Int][1/((c + d*x)*(a + b*Cot[e + f*x])), x]

Rubi steps

$$\text{integral} = \int \frac{1}{(c+dx)(a+b \cot(e+fx))} dx$$

Mathematica [N/A]

Not integrable

Time = 3.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c + dx)(a + b \cot(e + fx))} dx = \int \frac{1}{(c + dx)(a + b \cot(e + fx))} dx$$

[In] Integrate[1/((c + d*x)*(a + b*Cot[e + f*x])),x]

[Out] Integrate[1/((c + d*x)*(a + b*Cot[e + f*x])), x]

Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx + c)(a + b \cot(fx + e))} dx$$

[In] int(1/(d*x+c)/(a+b*cot(f*x+e)),x)

[Out] int(1/(d*x+c)/(a+b*cot(f*x+e)),x)

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c + dx)(a + b \cot(e + fx))} dx = \int \frac{1}{(dx + c)(b \cot(fx + e) + a)} dx$$

[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e)),x, algorithm="fricas")

[Out] integral(1/(a*d*x + a*c + (b*d*x + b*c)*cot(f*x + e)), x)

Sympy [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(c + dx)(a + b \cot(e + fx))} dx = \int \frac{1}{(a + b \cot(e + fx))(c + dx)} dx$$

[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e)),x)

[Out] Integral(1/((a + b*cot(e + f*x))*(c + d*x)), x)

Maxima [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 279, normalized size of antiderivative = 13.95

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))} dx = \int \frac{1}{(dx+c)(b\cot(fx+e)+a)} dx$$

[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e)),x, algorithm="maxima")

```
[Out] (2*(a^2*b + b^3)*d*integrate(-(2*a*b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f*x + 2*e))/((a^4 + 2*a^2*b^2 + b^4)*d*x + ((a^4 + 2*a^2*b^2 + b^4)*d*x + (a^4 + 2*a^2*b^2 + b^4)*c)*cos(2*f*x + 2*e)^2 + ((a^4 + 2*a^2*b^2 + b^4)*d*x + (a^4 + 2*a^2*b^2 + b^4)*c)*sin(2*f*x + 2*e)^2 + (a^4 + 2*a^2*b^2 + b^4)*c - 2*((a^4 - b^4)*d*x + (a^4 - b^4)*c)*cos(2*f*x + 2*e) + 4*((a^3*b + a*b^3)*d*x + (a^3*b + a*b^3)*c)*sin(2*f*x + 2*e)), x) + a*log(d*x + c))/((a^2 + b^2)*d)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))} dx = \int \frac{1}{(dx+c)(b\cot(fx+e)+a)} dx$$

[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e)),x, algorithm="giac")

[Out] integrate(1/((d*x + c)*(b*cot(f*x + e) + a)), x)

Mupad [N/A]

Not integrable

Time = 12.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\cot(e+fx))} dx = \int \frac{1}{(a+b\cot(e+fx))(c+dx)} dx$$

[In] int(1/((a + b*cot(e + f*x))*(c + d*x)),x)

[Out] int(1/((a + b*cot(e + f*x))*(c + d*x)), x)

$$3.56 \quad \int \frac{1}{(c+dx)^2(a+b \cot(e+fx))} dx$$

Optimal result	387
Rubi [N/A]	387
Mathematica [N/A]	388
Maple [N/A] (verified)	388
Fricas [N/A]	388
Sympy [N/A]	389
Maxima [N/A]	389
Giac [N/A]	389
Mupad [N/A]	390

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b \cot(e+fx))} dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+b \cot(e+fx))}, x\right)$$

[Out] Unintegrable(1/(d*x+c)^2/(a+b*cot(f*x+e)), x)

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b \cot(e+fx))} dx = \int \frac{1}{(c+dx)^2(a+b \cot(e+fx))} dx$$

[In] Int[1/((c + d*x)^2*(a + b*Cot[e + f*x])), x]

[Out] Defer[Int][1/((c + d*x)^2*(a + b*Cot[e + f*x])), x]

Rubi steps

$$\text{integral} = \int \frac{1}{(c+dx)^2(a+b \cot(e+fx))} dx$$

Mathematica [N/A]

Not integrable

Time = 5.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c + dx)^2(a + b \cot(e + fx))} dx = \int \frac{1}{(c + dx)^2(a + b \cot(e + fx))} dx$$

[In] Integrate[1/((c + d*x)^2*(a + b*Cot[e + f*x])),x]

[Out] Integrate[1/((c + d*x)^2*(a + b*Cot[e + f*x])), x]

Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx + c)^2(a + b \cot(fx + e))} dx$$

[In] int(1/(d*x+c)^2/(a+b*cot(f*x+e)),x)

[Out] int(1/(d*x+c)^2/(a+b*cot(f*x+e)),x)

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{1}{(c + dx)^2(a + b \cot(e + fx))} dx = \int \frac{1}{(dx + c)^2(b \cot(fx + e) + a)} dx$$

[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="fricas")

[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cot(f*x + e)), x)

Sympy [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))} dx = \int \frac{1}{(a+b\cot(e+fx))(c+dx)^2} dx$$

[In] integrate(1/(d*x+c)**2/(a+b*cot(f*x+e)),x)

[Out] Integral(1/((a + b*cot(e + f*x))*(c + d*x)**2), x)

Maxima [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 424, normalized size of antiderivative = 21.20

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))} dx = \int \frac{1}{(dx+c)^2(b\cot(fx+e)+a)} dx$$

[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="maxima")

[Out] (2*((a^2*b + b^3)*d^2*x + (a^2*b + b^3)*c*d)*integrate(-(2*a*b*cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f*x + 2*e))/((a^4 + 2*a^2*b^2 + b^4)*d^2*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*c*d*x + (a^4 + 2*a^2*b^2 + b^4)*c^2 + ((a^4 + 2*a^2*b^2 + b^4)*d^2*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*c*d*x + (a^4 + 2*a^2*b^2 + b^4)*c^2)*cos(2*f*x + 2*e)^2 + ((a^4 + 2*a^2*b^2 + b^4)*d^2*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*c*d*x + (a^4 + 2*a^2*b^2 + b^4)*c^2)*sin(2*f*x + 2*e)^2 - 2*((a^4 - b^4)*d^2*x^2 + 2*(a^4 - b^4)*c*d*x + (a^4 - b^4)*c^2)*cos(2*f*x + 2*e) + 4*((a^3*b + a*b^3)*d^2*x^2 + 2*(a^3*b + a*b^3)*c*d*x + (a^3*b + a*b^3)*c^2)*sin(2*f*x + 2*e)), x) - a)/((a^2 + b^2)*d^2*x + (a^2 + b^2)*c*d)

Giac [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))} dx = \int \frac{1}{(dx+c)^2(b\cot(fx+e)+a)} dx$$

[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e)),x, algorithm="giac")

[Out] integrate(1/((d*x + c)^2*(b*cot(f*x + e) + a)), x)

Mupad [N/A]

Not integrable

Time = 12.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c + dx)^2(a + b \cot(e + fx))} dx = \int \frac{1}{(a + b \cot(e + fx)) (c + dx)^2} dx$$

```
[In] int(1/((a + b*cot(e + f*x))*(c + d*x)^2),x)
```

```
[Out] int(1/((a + b*cot(e + f*x))*(c + d*x)^2), x)
```

3.57 $\int \frac{(c+dx)^3}{(a+b \cot(e+fx))^2} dx$

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Rubi [A] (verified)	393
Mathematica [B] (warning: unable to verify)	401
Maple [B] (verified)	403
Fricas [B] (verification not implemented)	403
Sympy [F]	405
Maxima [B] (verification not implemented)	406
Giac [F]	408
Mupad [F(-1)]	408

Optimal result

Integrand size = 20, antiderivative size = 839

$$\begin{aligned}
 \int \frac{(c+dx)^3}{(a+b \cot(e+fx))^2} dx = & -\frac{2ib^2(c+dx)^3}{(a^2+b^2)^2 f} - \frac{2b^2(c+dx)^3}{(a-ib)(a+ib)^2 (ia+b-(ia-b)e^{2ie+2ifx}) f} \\
 & + \frac{(c+dx)^4}{4(a+ib)^2 d} - \frac{b(c+dx)^4}{(a+ib)^2 (ia+b)d} - \frac{b^2(c+dx)^4}{(a^2+b^2)^2 d} \\
 & + \frac{3b^2 d(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} \\
 & - \frac{2b(c+dx)^3 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a-ib)(a+ib)^2 f} \\
 & - \frac{2ib^2(c+dx)^3 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f} \\
 & - \frac{3ib^2 d^2(c+dx) \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^3} \\
 & - \frac{3bd(c+dx)^2 \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)^2 (ia+b) f^2} \\
 & - \frac{3b^2 d(c+dx)^2 \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} \\
 & + \frac{3b^2 d^3 \operatorname{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{2(a^2+b^2)^2 f^4} \\
 & - \frac{3bd^2(c+dx) \operatorname{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a-ib)(a+ib)^2 f^3} \\
 & - \frac{3ib^2 d^2(c+dx) \operatorname{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^3} \\
 & + \frac{3bd^3 \operatorname{PolyLog}\left(4, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{2(a+ib)^2 (ia+b) f^4} \\
 & + \frac{3b^2 d^3 \operatorname{PolyLog}\left(4, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{2(a^2+b^2)^2 f^4}
 \end{aligned}$$

[Out] $-2*I*b^2*(d*x+c)^3*\ln(1-(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f-2*b^2*(d*x+c)^3/(a-I*b)/(a+I*b)^2/(I*a+b-(I*a-b)*\exp(2*I*e+2*I*f*x))/f+1/4*(d*x+c)^4/(a+I*b)^2/d-b*(d*x+c)^4/(a+I*b)^2/(I*a+b)/d-b^2*(d*x+c)^4/(a^2+b^2)^2/d+3*b^2*d*(d*x+c)^2*\ln(1-(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2$

$$\begin{aligned} & 2/f^2-2*b*(d*x+c)^3*\ln(1-(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a-I*b)/(a+I*b) \\ &)^2/f-3*I*b^2*d^2*(d*x+c)*\text{polylog}(3,(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f-3*I*b^2*d^2*(d*x+c)^3/(a^2+b^2)^2/f-3*b*d*(d*x+c)^2*\text{polylog}(2,(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a+I*b)^2/(I*a+b)/f^2-3*b^2*d*(d*x+c)^2*\text{polylog}(2,(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^2+3/2*b^2*d^3*\text{polylog}(3,(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^4-3*b*d^2*(d*x+c)*\text{polylog}(3,(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a-I*b)/(a+I*b)^2/f^3-3*I*b^2*d^2*(d*x+c)*\text{polylog}(2,(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^3+3/2*b*d^3*\text{polylog}(4,(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a+I*b)^2/(I*a+b)/f^4+3/2*b^2*d^3*\text{polylog}(4,(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^4 \end{aligned}$$

Rubi [A] (verified)

Time = 2.32 (sec) , antiderivative size = 839, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used

= {3815, 2216, 2215, 2221, 2611, 6744, 2320, 6724, 2222}

$$\int \frac{(c+dx)^3}{(a+b \cot(e+fx))^2} dx = -\frac{b(c+dx)^4}{(a+ib)^2(ia+b)d} + \frac{(c+dx)^4}{4(a+ib)^2d} - \frac{b^2(c+dx)^4}{(a^2+b^2)^2d}$$

$$-\frac{2b \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)^3}{(a-ib)(a+ib)^2f}$$

$$-\frac{2ib^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)^3}{(a^2+b^2)^2f}$$

$$-\frac{2b^2(c+dx)^3}{(a-ib)(a+ib)^2(ia-(ia-b)e^{2ie+2ifx}+b)f}$$

$$-\frac{2ib^2(c+dx)^3}{(a^2+b^2)^2f} + \frac{3b^2d \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)^2}{(a^2+b^2)^2f^2}$$

$$-\frac{3bd \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)^2}{(a+ib)^2(ia+b)f^2}$$

$$-\frac{3b^2d \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)^2}{(a^2+b^2)^2f^2}$$

$$-\frac{3ib^2d^2 \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)}{(a^2+b^2)^2f^3}$$

$$-\frac{3bd^2 \operatorname{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)}{(a-ib)(a+ib)^2f^3}$$

$$-\frac{3ib^2d^2 \operatorname{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)(c+dx)}{(a^2+b^2)^2f^3}$$

$$+\frac{3b^2d^3 \operatorname{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{2(a^2+b^2)^2f^4}$$

$$+\frac{3bd^3 \operatorname{PolyLog}\left(4, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{2(a+ib)^2(ia+b)f^4}$$

$$+\frac{3b^2d^3 \operatorname{PolyLog}\left(4, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{2(a^2+b^2)^2f^4}$$

[In] Int[(c + d*x)^3/(a + b*Cot[e + f*x])^2,x]

[Out] ((-2*I)*b^2*(c + d*x)^3)/((a^2 + b^2)^2*f) - (2*b^2*(c + d*x)^3)/((a - I*b)*(a + I*b)^2*(I*a + b - (I*a - b)*E^((2*I)*e + (2*I)*f*x))*f) + (c + d*x)^4/(4*(a + I*b)^2*d) - (b*(c + d*x)^4)/((a + I*b)^2*(I*a + b)*d) - (b^2*(c + d*x)^4)/((a^2 + b^2)^2*d) + (3*b^2*d*(c + d*x)^2*Log[1 - ((a + I*b)*E^((2*I)

```

)*e + (2*I)*f*x))/(a - I*b)]/((a^2 + b^2)^2*f^2) - (2*b*(c + d*x)^3*Log[1
- ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)]/((a - I*b)*(a + I*b)^2*f)
- ((2*I)*b^2*(c + d*x)^3*Log[1 - ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a -
I*b)]/((a^2 + b^2)^2*f) - ((3*I)*b^2*d^2*(c + d*x)*PolyLog[2, ((a + I*b)*E
^((2*I)*e + (2*I)*f*x))/(a - I*b)]/((a^2 + b^2)^2*f^3) - (3*b*d*(c + d*x)^
2*PolyLog[2, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)]/((a + I*b)^2*(
I*a + b)*f^2) - (3*b^2*d*(c + d*x)^2*PolyLog[2, ((a + I*b)*E^((2*I)*e + (2*
I)*f*x))/(a - I*b)]/((a^2 + b^2)^2*f^2) + (3*b^2*d^3*PolyLog[3, ((a + I*b)
*e^((2*I)*e + (2*I)*f*x))/(a - I*b)]/(2*(a^2 + b^2)^2*f^4) - (3*b*d^2*(c +
d*x)*PolyLog[3, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)]/((a - I*b)
*(a + I*b)^2*f^3) - ((3*I)*b^2*d^2*(c + d*x)*PolyLog[3, ((a + I*b)*E^((2*I)
*e + (2*I)*f*x))/(a - I*b)]/((a^2 + b^2)^2*f^3) + (3*b*d^3*PolyLog[4, ((a
+ I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)]/(2*(a + I*b)^2*(I*a + b)*f^4) +
(3*b^2*d^3*PolyLog[4, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)]/(2*(
a^2 + b^2)^2*f^4)

```

Rule 2215

```

Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

Rule 2216

```

Int[((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^p)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] := Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
x] && ILtQ[p, 0] && IGtQ[m, 0]

```

Rule 2221

```

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

Rule 2222

```

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*
(e_.) + (f_.)*(x_))))^(n_.))^p)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :=
Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m

```

, n, p}, x] && NeQ[p, -1]

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 3815

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - 2*I*(b/(a^2 +
b^2 + (a - I*b)^2*E^(2*I*(e + f*x))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
)*(x_)))]^(p_.), x_Symbol] := Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(c + dx)^3}{(a + ib)^2} - \frac{4b^2(c + dx)^3}{(ia - b)^2 \left(ia \left(1 - \frac{ib}{a} \right) - ia \left(1 + \frac{ib}{a} \right) e^{2ie+2ifx} \right)^2} + \frac{4b(c + dx)^3}{(a + ib)^2 \left(-ia \left(1 - \frac{ib}{a} \right) + ia \left(1 + \frac{ib}{a} \right) e^{2ie+2ifx} \right)} \right) dx$$

$$\begin{aligned}
&= \frac{(c+dx)^4}{4(a+ib)^2 d} + \frac{(4b) \int \frac{(c+dx)^3}{-ia(1-\frac{ib}{a})+ia(1+\frac{ib}{a})e^{2ie+2ifx}} dx}{(a+ib)^2} - \frac{(4b^2) \int \frac{(c+dx)^3}{(ia(1-\frac{ib}{a})-ia(1+\frac{ib}{a})e^{2ie+2ifx})^2} dx}{(ia-b)^2} \\
&= \frac{(c+dx)^4}{4(a+ib)^2 d} - \frac{b(c+dx)^4}{(a+ib)^2(ia+b)d} + \frac{(4b^2) \int \frac{(c+dx)^3}{ia(1-\frac{ib}{a})-ia(1+\frac{ib}{a})e^{2ie+2ifx}} dx}{(a+ib)^2(ia+b)} \\
&\quad + \frac{(4b) \int \frac{e^{2ie+2ifx}(c+dx)^3}{-ia(1-\frac{ib}{a})+ia(1+\frac{ib}{a})e^{2ie+2ifx}} dx}{a^2+b^2} + \frac{(4b^2) \int \frac{e^{2ie+2ifx}(c+dx)^3}{(ia(1-\frac{ib}{a})-ia(1+\frac{ib}{a})e^{2ie+2ifx})^2} dx}{a^2+b^2} \\
&= -\frac{2b^2(c+dx)^3}{(a+ib)(a^2+b^2)(ia+b-(ia-b)e^{2ie+2ifx})f} + \frac{(c+dx)^4}{4(a+ib)^2 d} - \frac{b(c+dx)^4}{(a+ib)^2(ia+b)d} \\
&\quad - \frac{b^2(c+dx)^4}{(a^2+b^2)^2 d} - \frac{2b(c+dx)^3 \log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)(a^2+b^2)f} + \frac{(4b^2) \int \frac{e^{2ie+2ifx}(c+dx)^3}{ia(1-\frac{ib}{a})-ia(1+\frac{ib}{a})e^{2ie+2ifx}} dx}{(ia-b)(a-ib)^2} \\
&\quad + \frac{(6bd) \int (c+dx)^2 \log\left(1-\frac{(1+\frac{ib}{a})e^{2ie+2ifx}}{1-\frac{ib}{a}}\right) dx}{(a+ib)(a^2+b^2)f} + \frac{(6b^2 d) \int \frac{(c+dx)^2}{ia(1-\frac{ib}{a})-ia(1+\frac{ib}{a})e^{2ie+2ifx}} dx}{(a+ib)(a^2+b^2)f} \\
&= -\frac{2ib^2(c+dx)^3}{(a^2+b^2)^2 f} - \frac{2b^2(c+dx)^3}{(a+ib)(a^2+b^2)(ia+b-(ia-b)e^{2ie+2ifx})f} + \frac{(c+dx)^4}{4(a+ib)^2 d} \\
&\quad - \frac{b(c+dx)^4}{(a+ib)^2(ia+b)d} - \frac{b^2(c+dx)^4}{(a^2+b^2)^2 d} - \frac{2ib^2(c+dx)^3 \log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f} \\
&\quad - \frac{2b(c+dx)^3 \log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)(a^2+b^2)f} - \frac{3bd(c+dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)^2(ia+b)f^2} \\
&\quad + \frac{(6bd^2) \int (c+dx) \text{PolyLog}\left(2, \frac{(1+\frac{ib}{a})e^{2ie+2ifx}}{1-\frac{ib}{a}}\right) dx}{(a+ib)^2(ia+b)f^2} \\
&\quad + \frac{(6b^2 d) \int \frac{e^{2ie+2ifx}(c+dx)^2}{ia(1-\frac{ib}{a})-ia(1+\frac{ib}{a})e^{2ie+2ifx}} dx}{(a-ib)^2(a+ib)f} \\
&\quad + \frac{(6ib^2 d) \int (c+dx)^2 \log\left(1-\frac{(1+\frac{ib}{a})e^{2ie+2ifx}}{1-\frac{ib}{a}}\right) dx}{(a^2+b^2)^2 f}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2ib^2(c+dx)^3}{(a^2+b^2)^2 f} - \frac{2b^2(c+dx)^3}{(a+ib)(a^2+b^2)(ia+b-(ia-b)e^{2ie+2ifx}) f} \\
&+ \frac{(c+dx)^4}{4(a+ib)^2 d} - \frac{b(c+dx)^4}{(a+ib)^2(ia+b)d} - \frac{b^2(c+dx)^4}{(a^2+b^2)^2 d} \\
&+ \frac{3b^2d(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} - \frac{2ib^2(c+dx)^3 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f} \\
&- \frac{2b(c+dx)^3 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)(a^2+b^2) f} - \frac{3bd(c+dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)^2(ia+b)f^2} \\
&- \frac{3b^2d(c+dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} \\
&- \frac{3bd^2(c+dx) \text{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a-ib)(a+ib)^2 f^3} \\
&+ \frac{(3bd^3) \int \text{PolyLog}\left(3, \frac{\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}{1-\frac{ib}{a}}\right) dx}{(a-ib)(a+ib)^2 f^3} \\
&- \frac{(6b^2d^2) \int (c+dx) \log\left(1 - \frac{\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}{1-\frac{ib}{a}}\right) dx}{(a^2+b^2)^2 f^2} \\
&+ \frac{(6b^2d^2) \int (c+dx) \text{PolyLog}\left(2, \frac{\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}{1-\frac{ib}{a}}\right) dx}{(a^2+b^2)^2 f^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2ib^2(c+dx)^3}{(a^2+b^2)^2 f} - \frac{2b^2(c+dx)^3}{(a+ib)(a^2+b^2)(ia+b-(ia-b)e^{2ie+2ifx}) f} \\
&+ \frac{(c+dx)^4}{4(a+ib)^2 d} - \frac{b(c+dx)^4}{(a+ib)^2(ia+b)d} - \frac{b^2(c+dx)^4}{(a^2+b^2)^2 d} \\
&+ \frac{3b^2 d(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} - \frac{2ib^2(c+dx)^3 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f} \\
&- \frac{2b(c+dx)^3 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)(a^2+b^2) f} - \frac{3ib^2 d^2(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^3} \\
&- \frac{3bd(c+dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)^2(ia+b) f^2} \\
&- \frac{3b^2 d(c+dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} \\
&- \frac{3bd^2(c+dx) \text{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a-ib)(a+ib)^2 f^3} \\
&- \frac{3ib^2 d^2(c+dx) \text{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^3} \\
&+ \frac{(3bd^3) \text{Subst}\left(\int \frac{\text{PolyLog}\left(3, \frac{(a+ib)x}{a-ib}\right)}{x} dx, x, e^{2ie+2ifx}\right)}{2(a+ib)^2(ia+b) f^4} \\
&+ \frac{(3ib^2 d^3) \int \text{PolyLog}\left(2, \frac{\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}{1-\frac{ib}{a}}\right) dx}{(a^2+b^2)^2 f^3} \\
&+ \frac{(3ib^2 d^3) \int \text{PolyLog}\left(3, \frac{\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}{1-\frac{ib}{a}}\right) dx}{(a^2+b^2)^2 f^3}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2ib^2(c+dx)^3}{(a^2+b^2)^2 f} - \frac{2b^2(c+dx)^3}{(a+ib)(a^2+b^2)(ia+b-(ia-b)e^{2ie+2ifx}) f} \\
&+ \frac{(c+dx)^4}{4(a+ib)^2 d} - \frac{b(c+dx)^4}{(a+ib)^2(ia+b)d} - \frac{b^2(c+dx)^4}{(a^2+b^2)^2 d} \\
&+ \frac{3b^2d(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} - \frac{2ib^2(c+dx)^3 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f} \\
&- \frac{2b(c+dx)^3 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)(a^2+b^2) f} - \frac{3ib^2d^2(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^3} \\
&- \frac{3bd(c+dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)^2(ia+b) f^2} \\
&- \frac{3b^2d(c+dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} \\
&- \frac{3bd^2(c+dx) \text{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a-ib)(a+ib)^2 f^3} \\
&- \frac{3ib^2d^2(c+dx) \text{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^3} + \frac{3bd^3 \text{PolyLog}\left(4, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{2(a+ib)^2(ia+b) f^4} \\
&+ \frac{(3b^2d^3) \text{Subst}\left(\int \frac{\text{PolyLog}\left(2, \frac{(a+ib)x}{a-ib}\right)}{x} dx, x, e^{2ie+2ifx}\right)}{2(a^2+b^2)^2 f^4} \\
&+ \frac{(3b^2d^3) \text{Subst}\left(\int \frac{\text{PolyLog}\left(3, \frac{(a+ib)x}{a-ib}\right)}{x} dx, x, e^{2ie+2ifx}\right)}{2(a^2+b^2)^2 f^4}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2ib^2(c+dx)^3}{(a^2+b^2)^2 f} - \frac{2b^2(c+dx)^3}{(a+ib)(a^2+b^2)(ia+b-(ia-b)e^{2ie+2ifx}) f} \\
&+ \frac{(c+dx)^4}{4(a+ib)^2 d} - \frac{b(c+dx)^4}{(a+ib)^2(ia+b)d} - \frac{b^2(c+dx)^4}{(a^2+b^2)^2 d} \\
&+ \frac{3b^2 d(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} - \frac{2ib^2(c+dx)^3 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f} \\
&- \frac{2b(c+dx)^3 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)(a^2+b^2) f} - \frac{3ib^2 d^2(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^3} \\
&- \frac{3bd(c+dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)^2(ia+b) f^2} \\
&- \frac{3b^2 d(c+dx)^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} \\
&+ \frac{3b^2 d^3 \text{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{2(a^2+b^2)^2 f^4} - \frac{3bd^2(c+dx) \text{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a-ib)(a+ib)^2 f^3} \\
&- \frac{3ib^2 d^2(c+dx) \text{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^3} \\
&+ \frac{3bd^3 \text{PolyLog}\left(4, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{2(a+ib)^2(ia+b) f^4} + \frac{3b^2 d^3 \text{PolyLog}\left(4, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{2(a^2+b^2)^2 f^4}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1733 vs. $2(839) = 1678$.

Time = 11.55 (sec) , antiderivative size = 1733, normalized size of antiderivative = 2.07

$$\int \frac{(c + dx)^3}{(a + b \cot(e + fx))^2} dx$$

$$= b \left(\frac{4c^2(a(-1+e^{2ie})+ib(1+e^{2ie}))(-3bd+2acf)x}{a^2+b^2} - \frac{4b(c+dx)^3}{a+ib} + \frac{2af(c+dx)^4}{(a+ib)d} + \frac{12cd(a(-1+e^{2ie})+ib(1+e^{2ie}))(-bd+acf)x \log\left(1+\frac{(-a+ib)}{a+ib}\right)}{(a-ib)(-ia+ib)f} \right)$$

$$+ \frac{3x^2(-ac^2d - ibc^2d + ac^2d \cos(2e) - ibc^2d \cos(2e) + iac^2d \sin(2e) + bc^2d \sin(2e))}{2(a-ib)(a+ib)(-a+ib+a \cos(2e) + ib \cos(2e) + ia \sin(2e) - b \sin(2e))}$$

$$+ \frac{x^3(-acd^2 - ibcd^2 + acd^2 \cos(2e) - ibcd^2 \cos(2e) + iacd^2 \sin(2e) + bcd^2 \sin(2e))}{(a-ib)(a+ib)(-a+ib+a \cos(2e) + ib \cos(2e) + ia \sin(2e) - b \sin(2e))}$$

$$+ \frac{x^4(-ad^3 - ibd^3 + ad^3 \cos(2e) - ibd^3 \cos(2e) + iad^3 \sin(2e) + bd^3 \sin(2e))}{4(a-ib)(a+ib)(-a+ib+a \cos(2e) + ib \cos(2e) + ia \sin(2e) - b \sin(2e))}$$

$$+ x \left(\frac{c^3}{a^2 - 2iab - b^2 + a^2 \cos(4e) + 2iab \cos(4e) - b^2 \cos(4e) + ia^2 \sin(4e) - 2ab \sin(4e) - ib^2 \sin(4e)} \right.$$

$$\left. + \frac{(-ia - b - ia \cos(2e) + b \cos(2e) + a \sin(2e) + ib \sin(2e)) (4abc)}{(a-ib)(a+ib)(-a+ib+a \cos(2e) + ib \cos(2e) + ia \sin(2e) - b \sin(2e)) (a^2 - 2iab - b^2 + a^2 \cos(4e) + 2iab \cos(4e) - b^2 \cos(4e) + ia^2 \sin(4e) - 2ab \sin(4e) - ib^2 \sin(4e))} \right.$$

$$\left. + \frac{c^3 \cos(4e) + ic^3 \sin(4e)}{a^2 - 2iab - b^2 + a^2 \cos(4e) + 2iab \cos(4e) - b^2 \cos(4e) + ia^2 \sin(4e) - 2ab \sin(4e) - ib^2 \sin(4e)} \right)$$

$$+ \frac{b^2c^3 \sin(fx) + 3b^2c^2dx \sin(fx) + 3b^2cd^2x^2 \sin(fx) + b^2d^3x^3 \sin(fx)}{(a-ib)(a+ib)f(b \cos(e) + a \sin(e))(b \cos(e + fx) + a \sin(e + fx))}$$

```
[In] Integrate[(c + d*x)^3/(a + b*Cot[e + f*x])^2,x]
```

```
[Out] (b*((4*c^2*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e))))*(-3*b*d + 2*a*c*f
)*x)/(a^2 + b^2) - (4*b*(c + d*x)^3)/(a + I*b) + (2*a*f*(c + d*x)^4)/((a +
I*b)*d) + (12*c*d*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(-b*d) +
a*c*f)*x*Log[1 + (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]/((a - I*b)*((
-I)*a + b)*f) - (6*d^2*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*b*d
- 2*a*c*f)*x^2*Log[1 + (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]/((a - I
*b)*((-I)*a + b)*f) + (4*a*d^3*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e
)))*x^3*Log[1 + (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]/((a - I*b)*((-I
)*a + b)) + (2*c^2*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(-3*b*d +
2*a*c*f)*Log[a - I*b - (a + I*b)*E^((2*I)*(e + f*x))]/((a - I*b)*((-I)*a
+ b)*f) - (6*c*d*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(-b*d) + a
*c*f)*PolyLog[2, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]/((a^2 + b^2)*f
^2) + (3*d^2*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*b*d - 2*a*c*f)
*(2*f*x*PolyLog[2, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))] - I*PolyLog[3
, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]/((a^2 + b^2)*f^3) - (3*a*d^3
*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*2*f^2*x^2*PolyLog[2, (a -
I*b)/((a + I*b)*E^((2*I)*(e + f*x)))] - (2*I)*f*x*PolyLog[3, (a - I*b)/((a
+ I*b)*E^((2*I)*(e + f*x)))] - PolyLog[4, (a - I*b)/((a + I*b)*E^((2*I)*(e
```

$$\begin{aligned} &+ f*x)))])))/((a^2 + b^2)*f^3))/((2*(a - I*b)*(a + I*b)*((-I)*a*(-1 + E^((2*I)*e)) + b*(1 + E^((2*I)*e)))*f) + (3*x^2*(-(a*c^2*d) - I*b*c^2*d + a*c^2*d \\ &*Cos[2*e] - I*b*c^2*d*Cos[2*e] + I*a*c^2*d*Sin[2*e] + b*c^2*d*Sin[2*e]))/(2 \\ &*(a - I*b)*(a + I*b)*(-a + I*b + a*Cos[2*e] + I*b*Cos[2*e] + I*a*Sin[2*e] - \\ &b*Sin[2*e])) + (x^3*(-(a*c*d^2) - I*b*c*d^2 + a*c*d^2*Cos[2*e] - I*b*c*d^2 \\ &*Cos[2*e] + I*a*c*d^2*Sin[2*e] + b*c*d^2*Sin[2*e]))/((a - I*b)*(a + I*b)*(- \\ &a + I*b + a*Cos[2*e] + I*b*Cos[2*e] + I*a*Sin[2*e] - b*Sin[2*e])) + (x^4*(- \\ &(a*d^3) - I*b*d^3 + a*d^3*Cos[2*e] - I*b*d^3*Cos[2*e] + I*a*d^3*Sin[2*e] + \\ &b*d^3*Sin[2*e]))/(4*(a - I*b)*(a + I*b)*(-a + I*b + a*Cos[2*e] + I*b*Cos[2* \\ &e] + I*a*Sin[2*e] - b*Sin[2*e])) + x*(c^3/(a^2 - (2*I)*a*b - b^2 + a^2*Cos[\\ &4*e] + (2*I)*a*b*Cos[4*e] - b^2*Cos[4*e] + I*a^2*Sin[4*e] - 2*a*b*Sin[4*e] \\ &- I*b^2*Sin[4*e]) + (((-I)*a - b - I*a*Cos[2*e] + b*Cos[2*e] + a*Sin[2*e] + \\ &I*b*Sin[2*e])*(4*a*b*c^3*Cos[2*e] + (4*I)*a*b*c^3*Sin[2*e]))/((a - I*b)*(a \\ &+ I*b)*(-a + I*b + a*Cos[2*e] + I*b*Cos[2*e] + I*a*Sin[2*e] - b*Sin[2*e]))*(\\ &a^2 - (2*I)*a*b - b^2 + a^2*Cos[4*e] + (2*I)*a*b*Cos[4*e] - b^2*Cos[4*e] + \\ &I*a^2*Sin[4*e] - 2*a*b*Sin[4*e] - I*b^2*Sin[4*e])) + (c^3*Cos[4*e] + I*c^3 \\ &*Sin[4*e])/(a^2 - (2*I)*a*b - b^2 + a^2*Cos[4*e] + (2*I)*a*b*Cos[4*e] - b^2 \\ &*Cos[4*e] + I*a^2*Sin[4*e] - 2*a*b*Sin[4*e] - I*b^2*Sin[4*e])) + (b^2*c^3*S \\ &in[f*x] + 3*b^2*c^2*d*x*Sin[f*x] + 3*b^2*c*d^2*x^2*Sin[f*x] + b^2*d^3*x^3*S \\ &in[f*x])/((a - I*b)*(a + I*b)*f*(b*Cos[e] + a*Sin[e])*(b*Cos[e + f*x] + a*S \\ &in[e + f*x])) \end{aligned}$$

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5930 vs. $2(754) = 1508$.

Time = 0.78 (sec) , antiderivative size = 5931, normalized size of antiderivative = 7.07

method	result	size
risch	Expression too large to display	5931

[In] `int((d*x+c)^3/(a+b*cot(f*x+e))^2,x,method=_RETURNVERBOSE)`

[Out] result too large to display

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3363 vs. $2(687) = 1374$.

Time = 0.38 (sec) , antiderivative size = 3363, normalized size of antiderivative = 4.01

$$\int \frac{(c + dx)^3}{(a + b \cot(e + fx))^2} dx = \text{Too large to display}$$

[In] `integrate((d*x+c)^3/(a+b*cot(f*x+e))^2,x, algorithm="fricas")`

```
[Out] 1/4*((a^2*b - b^3)*d^3*f^4*x^4 - 4*a*b^2*c^3*f^3 - 4*(a*b^2*d^3*f^3 - (a^2*b - b^3)*c*d^2*f^4)*x^3 - 6*(2*a*b^2*c*d^2*f^3 - (a^2*b - b^3)*c^2*d*f^4)*x^2 - 4*(3*a*b^2*c^2*d*f^3 - (a^2*b - b^3)*c^3*f^4)*x + ((a^2*b - b^3)*d^3*f^4*x^4 - 4*a*b^2*c^3*f^3 - 4*(a*b^2*d^3*f^3 - (a^2*b - b^3)*c*d^2*f^4)*x^3 - 6*(2*a*b^2*c*d^2*f^3 - (a^2*b - b^3)*c^2*d*f^4)*x^2 - 4*(3*a*b^2*c^2*d*f^3 - (a^2*b - b^3)*c^3*f^4)*x)*cos(2*f*x + 2*e) - 6*(-I*a*b^2*d^3*f^2*x^2 - I*a*b^2*c^2*d*f^2 + I*b^3*c*d^2*f - I*(2*a*b^2*c*d^2*f^2 - b^3*d^3*f)*x + (-I*a*b^2*d^3*f^2*x^2 - I*a*b^2*c^2*d*f^2 + I*b^3*c*d^2*f - I*(2*a*b^2*c*d^2*f^2 - b^3*d^3*f)*x)*cos(2*f*x + 2*e) + (-I*a^2*b*d^3*f^2*x^2 - I*a^2*b*c^2*d*f^2 + I*a*b^2*c*d^2*f - I*(2*a^2*b*c*d^2*f^2 - a*b^2*d^3*f)*x)*sin(2*f*x + 2*e))*dilog(-(a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - 6*(I*a*b^2*d^3*f^2*x^2 + I*a*b^2*c^2*d*f^2 - I*b^3*c*d^2*f + I*(2*a*b^2*c*d^2*f^2 - b^3*d^3*f)*x + (I*a*b^2*d^3*f^2*x^2 + I*a*b^2*c^2*d*f^2 - I*b^3*c*d^2*f + I*(2*a*b^2*c*d^2*f^2 - b^3*d^3*f)*x)*cos(2*f*x + 2*e) + (I*a^2*b*d^3*f^2*x^2 + I*a^2*b*c^2*d*f^2 - I*a*b^2*c*d^2*f + I*(2*a^2*b*c*d^2*f^2 - a*b^2*d^3*f)*x)*sin(2*f*x + 2*e))*dilog(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) + 2*(2*a*b^2*d^3*e^3 - 2*a*b^2*c^3*f^3 + 3*b^3*d^3*e^2 + 3*(2*a*b^2*c^2*d*e + b^3*c^2*d)*f^2 - 6*(a*b^2*c*d^2*e^2 + b^3*c*d^2*e)*f + (2*a*b^2*d^3*e^3 - 2*a*b^2*c^3*f^3 + 3*b^3*d^3*e^2 + 3*(2*a*b^2*c^2*d*e + b^3*c^2*d)*f^2 - 6*(a*b^2*c*d^2*e^2 + b^3*c*d^2*e)*f)*cos(2*f*x + 2*e) + (2*a^2*b*d^3*e^3 - 2*a^2*b*c^3*f^3 + 3*a*b^2*d^3*e^2 + 3*(2*a^2*b*c^2*d*e + a*b^2*c^2*d)*f^2 - 6*(a^2*b*c*d^2*e^2 + a*b^2*c*d^2*e)*f)*sin(2*f*x + 2*e))*log(1/2*a^2 + I*a*b - 1/2*b^2 - 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) + 2*(2*a*b^2*d^3*e^3 - 2*a*b^2*c^3*f^3 + 3*b^3*d^3*e^2 + 3*(2*a*b^2*c^2*d*e + b^3*c^2*d)*f^2 - 6*(a*b^2*c*d^2*e^2 + b^3*c*d^2*e)*f + (2*a*b^2*d^3*e^3 - 2*a*b^2*c^3*f^3 + 3*b^3*d^3*e^2 + 3*(2*a*b^2*c^2*d*e + b^3*c^2*d)*f^2 - 6*(a*b^2*c*d^2*e^2 + b^3*c*d^2*e)*f)*cos(2*f*x + 2*e) + (2*a^2*b*d^3*e^3 - 2*a^2*b*c^3*f^3 + 3*a*b^2*d^3*e^2 + 3*(2*a^2*b*c^2*d*e + a*b^2*c^2*d)*f^2 - 6*(a^2*b*c*d^2*e^2 + a*b^2*c*d^2*e)*f)*sin(2*f*x + 2*e))*log(-1/2*a^2 + I*a*b + 1/2*b^2 + 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 2*(2*a*b^2*d^3*f^3*x^3 + 2*a*b^2*d^3*e^3 + 6*a*b^2*c^2*d*e*f^2 + 3*b^3*d^3*e^2 + 3*(2*a*b^2*c*d^2*f^3 - b^3*d^3*f^2)*x^2 - 6*(a*b^2*c*d^2*e^2 + b^3*c*d^2*e)*f + 6*(a*b^2*c^2*d*f^3 - b^3*c*d^2*f^2)*x + (2*a*b^2*d^3*f^3*x^3 + 2*a*b^2*d^3*e^3 + 6*a*b^2*c^2*d*e*f^2 + 3*b^3*d^3*e^2 + 3*(2*a*b^2*c*d^2*f^3 - b^3*d^3*f^2)*x^2 - 6*(a*b^2*c*d^2*e^2 + b^3*c*d^2*e)*f + 6*(a*b^2*c^2*d*f^3 - b^3*c*d^2*f^2)*x)*cos(2*f*x + 2*e) + (2*a^2*b*d^3*f^3*x^3 + 2*a^2*b*d^3*e^3 + 6*a^2*b*c^2*d*e*f^2 + 3*a*b^2*d^3*e^2 + 3*(2*a^2*b*c*d^2*f^3 - a*b^2*d^3*f^2)*x^2 - 6*(a^2*b*c*d^2*e^2 + a*b^2*c*d^2*e)*f + 6*(a^2*b*c^2*d*f^3 - a*b^2*c*d^2*f^2)*x)*sin(2*f*x + 2*e))*log((a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 2*(2*a*b^2*d^3*f^3*x^3 + 2*a*b^2*d^3*e^3 + 6*a*b^2*c^2*d*e*f^2 + 3*b^3*d^3*e^2 + 3*(2*a*b^2*c*d^2*f^3 - b^3*d^3*f^2)*x^2 - 6*(a*b^2*c*d^2*e^2 + b^3*c*d^2*e)*f + 6*(a*b^2*c^2*d*f^3 - b^3*c*d^2*f^2)*x + (2*a*
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b^2*d^3*f^3*x^3 + 2*a*b^2*d^3*e^3 + 6*a*b^2*c^2*d*e*f^2 + 3*b^3*d^3*e^2 + 3
*(2*a*b^2*c*d^2*f^3 - b^3*d^3*f^2)*x^2 - 6*(a*b^2*c*d^2*e^2 + b^3*c*d^2*e)*
f + 6*(a*b^2*c^2*d*f^3 - b^3*c*d^2*f^2)*x*cos(2*f*x + 2*e) + (2*a^2*b*d^3*
f^3*x^3 + 2*a^2*b*d^3*e^3 + 6*a^2*b*c^2*d*e*f^2 + 3*a*b^2*d^3*e^2 + 3*(2*a^
2*b*c*d^2*f^3 - a*b^2*d^3*f^2)*x^2 - 6*(a^2*b*c*d^2*e^2 + a*b^2*c*d^2*e)*f
+ 6*(a^2*b*c^2*d*f^3 - a*b^2*c*d^2*f^2)*x)*sin(2*f*x + 2*e))*log((a^2 + b^2
- (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f
*x + 2*e))/(a^2 + b^2)) - 3*(I*a*b^2*d^3*cos(2*f*x + 2*e) + I*a^2*b*d^3*sin
(2*f*x + 2*e) + I*a*b^2*d^3)*polylog(4, ((a^2 + 2*I*a*b - b^2)*cos(2*f*x +
2*e) + (I*a^2 - 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 3*(-I*a*b^2
*d^3*cos(2*f*x + 2*e) - I*a^2*b*d^3*sin(2*f*x + 2*e) - I*a*b^2*d^3)*polylog
(4, ((a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 - 2*a*b + I*b^2)*sin(
2*f*x + 2*e))/(a^2 + b^2)) - 3*(2*a*b^2*d^3*f*x + 2*a*b^2*c*d^2*f - b^3*d^3
+ (2*a*b^2*d^3*f*x + 2*a*b^2*c*d^2*f - b^3*d^3)*cos(2*f*x + 2*e) + (2*a^2*
b*d^3*f*x + 2*a^2*b*c*d^2*f - a*b^2*d^3)*sin(2*f*x + 2*e))*polylog(3, ((a^2
+ 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 - 2*a*b - I*b^2)*sin(2*f*x + 2*
e))/(a^2 + b^2)) - 3*(2*a*b^2*d^3*f*x + 2*a*b^2*c*d^2*f - b^3*d^3 + (2*a*b^
2*d^3*f*x + 2*a*b^2*c*d^2*f - b^3*d^3)*cos(2*f*x + 2*e) + (2*a^2*b*d^3*f*x
+ 2*a^2*b*c*d^2*f - a*b^2*d^3)*sin(2*f*x + 2*e))*polylog(3, ((a^2 - 2*I*a*b
- b^2)*cos(2*f*x + 2*e) + (-I*a^2 - 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2
+ b^2)) + ((a^3 - a*b^2)*d^3*f^4*x^4 + 4*b^3*c^3*f^3 + 4*(b^3*d^3*f^3 + (a^
3 - a*b^2)*c*d^2*f^4)*x^3 + 6*(2*b^3*c*d^2*f^3 + (a^3 - a*b^2)*c^2*d*f^4)*x
^2 + 4*(3*b^3*c^2*d*f^3 + (a^3 - a*b^2)*c^3*f^4)*x)*sin(2*f*x + 2*e))/((a^4
*b + 2*a^2*b^3 + b^5)*f^4*cos(2*f*x + 2*e) + (a^5 + 2*a^3*b^2 + a*b^4)*f^4*
sin(2*f*x + 2*e) + (a^4*b + 2*a^2*b^3 + b^5)*f^4)

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Sympy [F]

$$\int \frac{(c + dx)^3}{(a + b \cot(e + fx))^2} dx = \int \frac{(c + dx)^3}{(a + b \cot(e + fx))^2} dx$$

```
[In] integrate((d*x+c)**3/(a+b*cot(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**3/(a + b*cot(e + f*x))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4641 vs. $2(687) = 1374$.

Time = 3.18 (sec) , antiderivative size = 4641, normalized size of antiderivative = 5.53

$$\int \frac{(c + dx)^3}{(a + b \cot(e + fx))^2} dx = \text{Too large to display}$$

[In] integrate((d*x+c)^3/(a+b*cot(f*x+e))^2,x, algorithm="maxima")

[Out] $\frac{1}{12} \cdot (36 \cdot (b^2 / ((a^4 + a^2 \cdot b^2) \cdot f \cdot \tan(fx + e) + (a^3 \cdot b + a \cdot b^3) \cdot f) + 2 \cdot a \cdot b \cdot \log(a \cdot \tan(fx + e) + b) / ((a^4 + 2 \cdot a^2 \cdot b^2 + b^4) \cdot f) - a \cdot b \cdot \log(\tan(fx + e)^2 + 1) / ((a^4 + 2 \cdot a^2 \cdot b^2 + b^4) \cdot f) - (a^2 - b^2) \cdot (fx + e) / ((a^4 + 2 \cdot a^2 \cdot b^2 + b^4) \cdot f)) \cdot c^2 \cdot d \cdot e - 12 \cdot (2 \cdot a \cdot b \cdot \log(a \cdot \tan(fx + e) + b) / (a^4 + 2 \cdot a^2 \cdot b^2 + b^4) - a \cdot b \cdot \log(\tan(fx + e)^2 + 1) / (a^4 + 2 \cdot a^2 \cdot b^2 + b^4) - (a^2 - b^2) \cdot (fx + e) / (a^4 + 2 \cdot a^2 \cdot b^2 + b^4) + b^2 / (a^3 \cdot b + a \cdot b^3 + (a^4 + a^2 \cdot b^2) \cdot \tan(fx + e))) \cdot c^3 - (3 \cdot (a^3 + I \cdot a^2 \cdot b + a \cdot b^2 + I \cdot b^3) \cdot (fx + e)^4 \cdot d^3 + 24 \cdot (-I \cdot a \cdot b^2 - b^3) \cdot d^3 \cdot e^3 + 72 \cdot (I \cdot a \cdot b^2 + b^3) \cdot c \cdot d^2 \cdot e^2 \cdot f - 12 \cdot ((a^3 + I \cdot a^2 \cdot b + a \cdot b^2 + I \cdot b^3) \cdot d^3 \cdot e - (a^3 + I \cdot a^2 \cdot b + a \cdot b^2 + I \cdot b^3) \cdot c \cdot d^2 \cdot f) \cdot (fx + e)^3 + 18 \cdot ((a^3 + I \cdot a^2 \cdot b + a \cdot b^2 + I \cdot b^3) \cdot d^3 \cdot e^2 - 2 \cdot (a^3 + I \cdot a^2 \cdot b + a \cdot b^2 + I \cdot b^3) \cdot c \cdot d^2 \cdot e \cdot f + (a^3 + I \cdot a^2 \cdot b + a \cdot b^2 + I \cdot b^3) \cdot c^2 \cdot d \cdot f^2) \cdot (fx + e)^2 - 12 \cdot ((a^3 + I \cdot a^2 \cdot b + a \cdot b^2 + I \cdot b^3) \cdot d^3 \cdot e^3 - 3 \cdot (a^3 + I \cdot a^2 \cdot b + a \cdot b^2 + I \cdot b^3) \cdot c \cdot d^2 \cdot e^2 \cdot f) \cdot (fx + e) + 12 \cdot (2 \cdot (I \cdot a^2 \cdot b + a \cdot b^2) \cdot d^3 \cdot e^3 + 3 \cdot (I \cdot a \cdot b^2 + b^3) \cdot d^3 \cdot e^2 + 3 \cdot (I \cdot a \cdot b^2 + b^3) \cdot c^2 \cdot d \cdot f^2 + 6 \cdot ((-I \cdot a^2 \cdot b - a \cdot b^2) \cdot c \cdot d^2 \cdot e^2 + (-I \cdot a \cdot b^2 - b^3) \cdot c \cdot d^2 \cdot e) \cdot f + (2 \cdot (-I \cdot a^2 \cdot b + a \cdot b^2) \cdot d^3 \cdot e^3 + 3 \cdot (-I \cdot a \cdot b^2 + b^3) \cdot d^3 \cdot e^2 + 3 \cdot (-I \cdot a \cdot b^2 + b^3) \cdot c^2 \cdot d \cdot f^2 + 6 \cdot ((I \cdot a^2 \cdot b - a \cdot b^2) \cdot c \cdot d^2 \cdot e^2 + (I \cdot a \cdot b^2 - b^3) \cdot c \cdot d^2 \cdot e) \cdot f) \cdot \cos(2 \cdot fx + 2 \cdot e) + (2 \cdot (a^2 \cdot b + I \cdot a \cdot b^2) \cdot d^3 \cdot e^3 + 3 \cdot (a \cdot b^2 + I \cdot b^3) \cdot d^3 \cdot e^2 + 3 \cdot (a \cdot b^2 + I \cdot b^3) \cdot c^2 \cdot d \cdot f^2 - 6 \cdot ((a^2 \cdot b + I \cdot a \cdot b^2) \cdot c \cdot d^2 \cdot e^2 + (a \cdot b^2 + I \cdot b^3) \cdot c \cdot d^2 \cdot e) \cdot f) \cdot \sin(2 \cdot fx + 2 \cdot e)) \cdot \arctan2(b \cdot \cos(2 \cdot fx + 2 \cdot e) + a \cdot \sin(2 \cdot fx + 2 \cdot e) + b, a \cdot \cos(2 \cdot fx + 2 \cdot e) - b \cdot \sin(2 \cdot fx + 2 \cdot e) - a) + 4 \cdot (8 \cdot (-I \cdot a^2 \cdot b - a \cdot b^2) \cdot (fx + e)^3 \cdot d^3 + 9 \cdot (2 \cdot (I \cdot a^2 \cdot b + a \cdot b^2) \cdot d^3 \cdot e + 2 \cdot (-I \cdot a^2 \cdot b - a \cdot b^2) \cdot c \cdot d^2 \cdot f + (I \cdot a \cdot b^2 + b^3) \cdot d^3) \cdot (fx + e)^2 + 18 \cdot ((-I \cdot a^2 \cdot b - a \cdot b^2) \cdot d^3 \cdot e^2 + (-I \cdot a^2 \cdot b - a \cdot b^2) \cdot c^2 \cdot d \cdot f^2 + (-I \cdot a \cdot b^2 - b^3) \cdot d^3 \cdot e + (2 \cdot (I \cdot a^2 \cdot b + a \cdot b^2) \cdot c \cdot d^2 \cdot e + (I \cdot a \cdot b^2 + b^3) \cdot c \cdot d^2) \cdot f) \cdot (fx + e) + (8 \cdot (I \cdot a^2 \cdot b - a \cdot b^2) \cdot (fx + e)^3 \cdot d^3 + 9 \cdot (2 \cdot (-I \cdot a^2 \cdot b + a \cdot b^2) \cdot d^3 \cdot e + 2 \cdot (I \cdot a^2 \cdot b - a \cdot b^2) \cdot c \cdot d^2 \cdot f + (-I \cdot a \cdot b^2 + b^3) \cdot d^3) \cdot (fx + e)^2 + 18 \cdot ((I \cdot a^2 \cdot b - a \cdot b^2) \cdot d^3 \cdot e^2 + (I \cdot a^2 \cdot b - a \cdot b^2) \cdot c^2 \cdot d \cdot f^2 + (I \cdot a \cdot b^2 - b^3) \cdot d^3 \cdot e + (2 \cdot (-I \cdot a^2 \cdot b + a \cdot b^2) \cdot c \cdot d^2 \cdot e + (-I \cdot a \cdot b^2 + b^3) \cdot c \cdot d^2) \cdot f) \cdot (fx + e)) \cdot \cos(2 \cdot fx + 2 \cdot e) - (8 \cdot (a^2 \cdot b + I \cdot a \cdot b^2) \cdot (fx + e)^3 \cdot d^3 - 9 \cdot (2 \cdot (a^2 \cdot b + I \cdot a \cdot b^2) \cdot d^3 \cdot e - 2 \cdot (a^2 \cdot b + I \cdot a \cdot b^2) \cdot c \cdot d^2 \cdot f + (a \cdot b^2 + I \cdot b^3) \cdot d^3) \cdot (fx + e)^2 + 18 \cdot ((a^2 \cdot b + I \cdot a \cdot b^2) \cdot d^3 \cdot e^2 + (a^2 \cdot b + I \cdot a \cdot b^2) \cdot c^2 \cdot d \cdot f^2 + (a \cdot b^2 + I \cdot b^3) \cdot d^3 \cdot e - (2 \cdot (a^2 \cdot b + I \cdot a \cdot b^2) \cdot c \cdot d^2 \cdot e + (a \cdot b^2 + I \cdot b^3) \cdot c \cdot d^2) \cdot f) \cdot (fx + e)) \cdot \sin(2 \cdot fx + 2 \cdot e)) \cdot \arctan2(-(2 \cdot a \cdot b \cdot \cos(2 \cdot fx + 2 \cdot e) + (a^2 - b^2) \cdot \sin(2 \cdot fx + 2 \cdot e)) / (a^2 + b^2), (2 \cdot a \cdot b \cdot \sin(2 \cdot fx + 2 \cdot e)$

$$\begin{aligned}
& + a^2 + b^2 - (a^2 - b^2)\cos(2f*x + 2e))/(a^2 + b^2)) - 3*((a^3 + 3I*a \\
& ^2*b - 3*a*b^2 - I*b^3)*(f*x + e)^4*d^3 - 4*((a^3 + 3I*a^2*b - 3*a*b^2 - I \\
& *b^3)*d^3*e - (a^3 + 3I*a^2*b - 3*a*b^2 - I*b^3)*c*d^2*f + 2*(I*a*b^2 - b^ \\
& 3)*d^3)*(f*x + e)^3 + 6*((a^3 + 3I*a^2*b - 3*a*b^2 - I*b^3)*d^3*e^2 + (a^3 \\
& + 3I*a^2*b - 3*a*b^2 - I*b^3)*c^2*d*f^2 - 4*(-I*a*b^2 + b^3)*d^3*e - 2*((\\
& a^3 + 3I*a^2*b - 3*a*b^2 - I*b^3)*c*d^2*e + 2*(I*a*b^2 - b^3)*c*d^2)*f)*(f \\
& *x + e)^2 - 4*((a^3 + 3I*a^2*b - 3*a*b^2 - I*b^3)*d^3*e^3 + 6*(I*a*b^2 - b \\
& ^3)*d^3*e^2 + 6*(I*a*b^2 - b^3)*c^2*d*f^2 - 3*((a^3 + 3I*a^2*b - 3*a*b^2 - \\
& I*b^3)*c*d^2*e^2 - 4*(-I*a*b^2 + b^3)*c*d^2*e)*f)*(f*x + e))*\cos(2f*x + 2 \\
& *e) + 12*(4*(I*a^2*b + a*b^2)*(f*x + e)^2*d^3 + 3*(I*a^2*b + a*b^2)*d^3*e^2 \\
& + 3*(I*a^2*b + a*b^2)*c^2*d*f^2 + 3*(I*a*b^2 + b^3)*d^3*e + 3*(2*(-I*a^2*b \\
& - a*b^2)*d^3*e + 2*(I*a^2*b + a*b^2)*c*d^2*f + (-I*a*b^2 - b^3)*d^3)*(f*x \\
& + e) + 3*(2*(-I*a^2*b - a*b^2)*c*d^2*e + (-I*a*b^2 - b^3)*c*d^2)*f + (4*(-I \\
& *a^2*b + a*b^2)*(f*x + e)^2*d^3 + 3*(-I*a^2*b + a*b^2)*d^3*e^2 + 3*(-I*a^2* \\
& b + a*b^2)*c^2*d*f^2 + 3*(-I*a*b^2 + b^3)*d^3*e + 3*(2*(I*a^2*b - a*b^2)*d^ \\
& 3*e + 2*(-I*a^2*b + a*b^2)*c*d^2*f + (I*a*b^2 - b^3)*d^3)*(f*x + e) + 3*(2* \\
& (I*a^2*b - a*b^2)*c*d^2*e + (I*a*b^2 - b^3)*c*d^2)*f)*\cos(2f*x + 2e) + (4 \\
& *(a^2*b + I*a*b^2)*(f*x + e)^2*d^3 + 3*(a^2*b + I*a*b^2)*d^3*e^2 + 3*(a^2*b \\
& + I*a*b^2)*c^2*d*f^2 + 3*(a*b^2 + I*b^3)*d^3*e - 3*(2*(a^2*b + I*a*b^2)*d^ \\
& 3*e - 2*(a^2*b + I*a*b^2)*c*d^2*f + (a*b^2 + I*b^3)*d^3)*(f*x + e) - 3*(2*(\\
& a^2*b + I*a*b^2)*c*d^2*e + (a*b^2 + I*b^3)*c*d^2)*f)*\sin(2f*x + 2e))*dilo \\
& g((I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) + 6*(2*(a^2*b - I*a*b^2)*d^3*e^3 \\
& + 3*(a*b^2 - I*b^3)*d^3*e^2 + 3*(a*b^2 - I*b^3)*c^2*d*f^2 - 6*((a^2*b - I* \\
& a*b^2)*c*d^2*e^2 + (a*b^2 - I*b^3)*c*d^2*e)*f - (2*(a^2*b + I*a*b^2)*d^3*e^ \\
& 3 + 3*(a*b^2 + I*b^3)*d^3*e^2 + 3*(a*b^2 + I*b^3)*c^2*d*f^2 - 6*((a^2*b + I \\
& *a*b^2)*c*d^2*e^2 + (a*b^2 + I*b^3)*c*d^2*e)*f)*\cos(2f*x + 2e) + (2*(-I*a \\
& ^2*b + a*b^2)*d^3*e^3 + 3*(-I*a*b^2 + b^3)*d^3*e^2 + 3*(-I*a*b^2 + b^3)*c^2 \\
& *d*f^2 + 6*((I*a^2*b - a*b^2)*c*d^2*e^2 + (I*a*b^2 - b^3)*c*d^2*e)*f)*\sin(2 \\
& *f*x + 2e))*\log((a^2 + b^2)*\cos(2f*x + 2e)^2 + 4*a*b*\sin(2f*x + 2e) + \\
& (a^2 + b^2)*\sin(2f*x + 2e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*\cos(2f*x + 2e) \\
&) - 2*(8*(a^2*b - I*a*b^2)*(f*x + e)^3*d^3 - 9*(2*(a^2*b - I*a*b^2)*d^3*e - \\
& 2*(a^2*b - I*a*b^2)*c*d^2*f + (a*b^2 - I*b^3)*d^3)*(f*x + e)^2 + 18*((a^2* \\
& b - I*a*b^2)*d^3*e^2 + (a^2*b - I*a*b^2)*c^2*d*f^2 + (a*b^2 - I*b^3)*d^3*e \\
& - (2*(a^2*b - I*a*b^2)*c*d^2*e + (a*b^2 - I*b^3)*c*d^2)*f)*(f*x + e) - (8*(\\
& a^2*b + I*a*b^2)*(f*x + e)^3*d^3 - 9*(2*(a^2*b + I*a*b^2)*d^3*e - 2*(a^2*b \\
& + I*a*b^2)*c*d^2*f + (a*b^2 + I*b^3)*d^3)*(f*x + e)^2 + 18*((a^2*b + I*a*b^ \\
& 2)*d^3*e^2 + (a^2*b + I*a*b^2)*c^2*d*f^2 + (a*b^2 + I*b^3)*d^3*e - (2*(a^2* \\
& b + I*a*b^2)*c*d^2*e + (a*b^2 + I*b^3)*c*d^2)*f)*(f*x + e))*\cos(2f*x + 2e \\
&) - (8*(I*a^2*b - a*b^2)*(f*x + e)^3*d^3 + 9*(2*(-I*a^2*b + a*b^2)*d^3*e + \\
& 2*(I*a^2*b - a*b^2)*c*d^2*f + (-I*a*b^2 + b^3)*d^3)*(f*x + e)^2 + 18*((I*a^ \\
& 2*b - a*b^2)*d^3*e^2 + (I*a^2*b - a*b^2)*c^2*d*f^2 + (I*a*b^2 - b^3)*d^3*e \\
& + (2*(-I*a^2*b + a*b^2)*c*d^2*e + (-I*a*b^2 + b^3)*c*d^2)*f)*(f*x + e))*\sin \\
& (2f*x + 2e))*\log(((a^2 + b^2)*\cos(2f*x + 2e)^2 + 4*a*b*\sin(2f*x + 2e) \\
& + (a^2 + b^2)*\sin(2f*x + 2e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*\cos(2f*x + 2 \\
& *e))/(a^2 + b^2)) + 24*((I*a^2*b - a*b^2)*d^3*\cos(2f*x + 2e) - (a^2*b + I
\end{aligned}$$

```

*a*b^2)*d^3*sin(2*f*x + 2*e) + (-I*a^2*b - a*b^2)*d^3)*polylog(4, (I*a - b)
*e^(2*I*f*x + 2*I*e)/(I*a + b)) - 6*(8*(a^2*b - I*a*b^2)*(f*x + e)*d^3 - 6*
(a^2*b - I*a*b^2)*d^3*e + 6*(a^2*b - I*a*b^2)*c*d^2*f - 3*(a*b^2 - I*b^3)*d
^3 - (8*(a^2*b + I*a*b^2)*(f*x + e)*d^3 - 6*(a^2*b + I*a*b^2)*d^3*e + 6*(a^
2*b + I*a*b^2)*c*d^2*f - 3*(a*b^2 + I*b^3)*d^3)*cos(2*f*x + 2*e) - (8*(I*a^
2*b - a*b^2)*(f*x + e)*d^3 + 6*(-I*a^2*b + a*b^2)*d^3*e + 6*(I*a^2*b - a*b^
2)*c*d^2*f + 3*(-I*a*b^2 + b^3)*d^3)*sin(2*f*x + 2*e))*polylog(3, (I*a - b)
*e^(2*I*f*x + 2*I*e)/(I*a + b)) + 3*((-I*a^3 + 3*a^2*b + 3*I*a*b^2 - b^3)*(
f*x + e)^4*d^3 + 4*((I*a^3 - 3*a^2*b - 3*I*a*b^2 + b^3)*d^3*e + (-I*a^3 + 3
*a^2*b + 3*I*a*b^2 - b^3)*c*d^2*f - 2*(a*b^2 + I*b^3)*d^3)*(f*x + e)^3 + 6*
((-I*a^3 + 3*a^2*b + 3*I*a*b^2 - b^3)*d^3*e^2 + (-I*a^3 + 3*a^2*b + 3*I*a*b
^2 - b^3)*c^2*d*f^2 + 4*(a*b^2 + I*b^3)*d^3*e + 2*((I*a^3 - 3*a^2*b - 3*I*a
*b^2 + b^3)*c*d^2*e - 2*(a*b^2 + I*b^3)*c*d^2)*f)*(f*x + e)^2 + 4*((I*a^3 -
3*a^2*b - 3*I*a*b^2 + b^3)*d^3*e^3 - 6*(a*b^2 + I*b^3)*d^3*e^2 - 6*(a*b^2
+ I*b^3)*c^2*d*f^2 + 3*((-I*a^3 + 3*a^2*b + 3*I*a*b^2 - b^3)*c*d^2*e^2 + 4*
(a*b^2 + I*b^3)*c*d^2*e)*f)*(f*x + e))*sin(2*f*x + 2*e))/((a^5 + I*a^4*b +
2*a^3*b^2 + 2*I*a^2*b^3 + a*b^4 + I*b^5)*f^3*cos(2*f*x + 2*e) + (I*a^5 - a^
4*b + 2*I*a^3*b^2 - 2*a^2*b^3 + I*a*b^4 - b^5)*f^3*sin(2*f*x + 2*e) - (a^5
- I*a^4*b + 2*a^3*b^2 - 2*I*a^2*b^3 + a*b^4 - I*b^5)*f^3))/f

```

Giac [F]

$$\int \frac{(c + dx)^3}{(a + b \cot(e + fx))^2} dx = \int \frac{(dx + c)^3}{(b \cot(fx + e) + a)^2} dx$$

```
[In] integrate((d*x+c)^3/(a+b*cot(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*cot(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx)^3}{(a + b \cot(e + fx))^2} dx = \int \frac{(c + dx)^3}{(a + b \cot(e + fx))^2} dx$$

```
[In] int((c + d*x)^3/(a + b*cot(e + f*x))^2,x)
```

```
[Out] int((c + d*x)^3/(a + b*cot(e + f*x))^2, x)
```


3.58 $\int \frac{(c+dx)^2}{(a+b \cot(e+fx))^2} dx$

Optimal result	409
Rubi [A] (verified)	410
Mathematica [A] (verified)	415
Maple [B] (verified)	416
Fricas [B] (verification not implemented)	418
Sympy [F]	419
Maxima [B] (verification not implemented)	419
Giac [F]	421
Mupad [F(-1)]	421

Optimal result

Integrand size = 20, antiderivative size = 650

$$\begin{aligned}
 \int \frac{(c+dx)^2}{(a+b \cot(e+fx))^2} dx = & -\frac{2ib^2(c+dx)^2}{(a^2+b^2)^2 f} - \frac{2b^2(c+dx)^2}{(a-ib)(a+ib)^2 (ia+b-(ia-b)e^{2ie+2ifx}) f} \\
 & + \frac{(c+dx)^3}{3(a+ib)^2 d} - \frac{4b(c+dx)^3}{3(a+ib)^2 (ia+b)d} - \frac{4b^2(c+dx)^3}{3(a^2+b^2)^2 d} \\
 & + \frac{2b^2 d(c+dx) \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} \\
 & - \frac{2b(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a-ib)(a+ib)^2 f} \\
 & - \frac{2ib^2(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f} \\
 & - \frac{ib^2 d^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^3} \\
 & - \frac{2bd(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)^2 (ia+b) f^2} \\
 & - \frac{2b^2 d(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} \\
 & - \frac{bd^2 \text{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a-ib)(a+ib)^2 f^3} \\
 & - \frac{ib^2 d^2 \text{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^3}
 \end{aligned}$$

[Out] $-2*I*b^2*(d*x+c)^2/(a^2+b^2)^2/f-2*b^2*(d*x+c)^2/(a-I*b)/(a+I*b)^2/(I*a+b-(I*a-b)*\exp(2*I*e+2*I*f*x))/f+1/3*(d*x+c)^3/(a+I*b)^2/d-4/3*b*(d*x+c)^3/(a+I*b)^2/(I*a+b)/d-4/3*b^2*(d*x+c)^3/(a^2+b^2)^2/d+2*b^2*d*(d*x+c)*\ln(1-(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^2-2*b*(d*x+c)^2*\ln(1-(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a-I*b)/(a+I*b)^2/f-2*I*b^2*(d*x+c)^2*\ln(1-(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f-I*b^2*d^2*\text{polylog}(2,(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^3-2*b*d*(d*x+c)*\text{polylog}(2,(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a+I*b)^2/(I*a+b)/f^2-2*b^2*d*(d*x+c)*\text{polylog}(2,(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^2-b*d^2*\text{polylog}(3,(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a-I*b)/(a+I*b)^2/f^3-I*b^2*d^2*\text{polylog}(3,(a+I*b)*\exp(2*I*e+2*I*f*x)/(a-I*b))/(a^2+b^2)^2/f^3$

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3815, 2216, 2215, 2221, 2611, 2320, 6724, 2222, 2317, 2438}

$$\int \frac{(c+dx)^2}{(a+b \cot(e+fx))^2} dx = -\frac{2b^2d(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^2(a^2+b^2)^2} + \frac{2b^2d(c+dx) \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^2(a^2+b^2)^2} - \frac{2ib^2(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f(a^2+b^2)^2} - \frac{2ib^2(c+dx)^2}{f(a^2+b^2)^2} - \frac{4b^2(c+dx)^3}{3d(a^2+b^2)^2} - \frac{ib^2d^2 \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^3(a^2+b^2)^2} - \frac{ib^2d^2 \text{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^3(a^2+b^2)^2} - \frac{2b^2(c+dx)^2}{f(a-ib)(a+ib)^2(-(-b+ia)e^{2ie+2ifx}+ia+b)} - \frac{2bd(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^2(a+ib)^2(b+ia)} - \frac{2b(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f(a-ib)(a+ib)^2} - \frac{4b(c+dx)^3}{3d(a+ib)^2(b+ia)} + \frac{(c+dx)^3}{3d(a+ib)^2} - \frac{bd^2 \text{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{f^3(a-ib)(a+ib)^2}$$

[In] Int[(c + d*x)^2/(a + b*Cot[e + f*x])^2,x]

```
[Out] ((-2*I)*b^2*(c + d*x)^2)/((a^2 + b^2)^2*f) - (2*b^2*(c + d*x)^2)/((a - I*b)
*(a + I*b)^2*(I*a + b - (I*a - b)*E^((2*I)*e + (2*I)*f*x))*f) + (c + d*x)^3
/(3*(a + I*b)^2*d) - (4*b*(c + d*x)^3)/(3*(a + I*b)^2*(I*a + b)*d) - (4*b^2
*(c + d*x)^3)/(3*(a^2 + b^2)^2*d) + (2*b^2*d*(c + d*x)*Log[1 - ((a + I*b)*E
^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^2) - (2*b*(c + d*x)^2*
Log[1 - ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a - I*b)*(a + I*b)
^2*f) - ((2*I)*b^2*(c + d*x)^2*Log[1 - ((a + I*b)*E^((2*I)*e + (2*I)*f*x))
/(a - I*b)])/((a^2 + b^2)^2*f) - (I*b^2*d^2*PolyLog[2, ((a + I*b)*E^((2*I)*
e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^3) - (2*b*d*(c + d*x)*PolyLog[
2, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a + I*b)^2*(I*a + b)*f
^2) - (2*b^2*d*(c + d*x)*PolyLog[2, ((a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a
- I*b)])/((a^2 + b^2)^2*f^2) - (b*d^2*PolyLog[3, ((a + I*b)*E^((2*I)*e + (2
*I)*f*x))/(a - I*b)])/((a - I*b)*(a + I*b)^2*f^3) - (I*b^2*d^2*PolyLog[3, (
(a + I*b)*E^((2*I)*e + (2*I)*f*x))/(a - I*b)])/((a^2 + b^2)^2*f^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2216

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] := Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :=
Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 3815

```
Int[((c_.) + (d_.)*(x_)^(m_.))*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - 2*I*(b/(a^2 +
b^2 + (a - I*b)^2)*E^(2*I*(e + f*x))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(c + dx)^2}{(a + ib)^2} - \frac{4b^2(c + dx)^2}{(ia - b)^2 (ia(1 - \frac{ib}{a}) - ia(1 + \frac{ib}{a})e^{2ie+2ifx})^2} + \frac{4b(c + dx)^2}{(a + ib)^2 (-ia(1 - \frac{ib}{a}) + ia(1 + \frac{ib}{a})e^{2ie+2ifx})} \right) dx$$

$$\begin{aligned}
&= \frac{(c+dx)^3}{3(a+ib)^2 d} + \frac{(4b) \int \frac{(c+dx)^2}{-ia\left(1-\frac{ib}{a}\right)+ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}} dx}{(a+ib)^2} - \frac{(4b^2) \int \frac{(c+dx)^2}{\left(ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}\right)^2} dx}{(ia-b)^2} \\
&= \frac{(c+dx)^3}{3(a+ib)^2 d} - \frac{4b(c+dx)^3}{3(a+ib)^2(ia+b)d} + \frac{(4b^2) \int \frac{(c+dx)^2}{ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}} dx}{(a+ib)^2(ia+b)} \\
&\quad + \frac{(4b) \int \frac{e^{2ie+2ifx}(c+dx)^2}{-ia\left(1-\frac{ib}{a}\right)+ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}} dx}{a^2+b^2} + \frac{(4b^2) \int \frac{e^{2ie+2ifx}(c+dx)^2}{\left(ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}\right)^2} dx}{a^2+b^2} \\
&= -\frac{2b^2(c+dx)^2}{(a+ib)(a^2+b^2)(ia+b-(ia-b)e^{2ie+2ifx})f} + \frac{(c+dx)^3}{3(a+ib)^2 d} - \frac{4b(c+dx)^3}{3(a+ib)^2(ia+b)d} \\
&\quad - \frac{4b^2(c+dx)^3}{3(a^2+b^2)^2 d} - \frac{2b(c+dx)^2 \log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)(a^2+b^2)f} + \frac{(4b^2) \int \frac{e^{2ie+2ifx}(c+dx)^2}{ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}} dx}{(ia-b)(a-ib)^2} \\
&\quad + \frac{(4bd) \int (c+dx) \log\left(1-\frac{\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}{1-\frac{ib}{a}}\right) dx}{(a+ib)(a^2+b^2)f} + \frac{(4b^2 d) \int \frac{c+dx}{ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}} dx}{(a+ib)(a^2+b^2)f} \\
&= -\frac{2ib^2(c+dx)^2}{(a^2+b^2)^2 f} - \frac{2b^2(c+dx)^2}{(a+ib)(a^2+b^2)(ia+b-(ia-b)e^{2ie+2ifx})f} + \frac{(c+dx)^3}{3(a+ib)^2 d} \\
&\quad - \frac{4b(c+dx)^3}{3(a+ib)^2(ia+b)d} - \frac{4b^2(c+dx)^3}{3(a^2+b^2)^2 d} - \frac{2ib^2(c+dx)^2 \log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f} \\
&\quad - \frac{2b(c+dx)^2 \log\left(1-\frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)(a^2+b^2)f} - \frac{2bd(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)^2(ia+b)f^2} \\
&\quad + \frac{(2bd^2) \int \text{PolyLog}\left(2, \frac{\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}{1-\frac{ib}{a}}\right) dx}{(a+ib)^2(ia+b)f^2} + \frac{(4b^2 d) \int \frac{e^{2ie+2ifx}(c+dx)}{ia\left(1-\frac{ib}{a}\right)-ia\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}} dx}{(a-ib)^2(a+ib)f} \\
&\quad + \frac{(4ib^2 d) \int (c+dx) \log\left(1-\frac{\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}{1-\frac{ib}{a}}\right) dx}{(a^2+b^2)^2 f}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2ib^2(c+dx)^2}{(a^2+b^2)^2 f} - \frac{2b^2(c+dx)^2}{(a+ib)(a^2+b^2)(ia+b-(ia-b)e^{2ie+2ifx})f} \\
&+ \frac{(c+dx)^3}{3(a+ib)^2 d} - \frac{4b(c+dx)^3}{3(a+ib)^2(ia+b)d} - \frac{4b^2(c+dx)^3}{3(a^2+b^2)^2 d} \\
&+ \frac{2b^2 d(c+dx) \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} - \frac{2ib^2(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f} \\
&- \frac{2b(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)(a^2+b^2)f} - \frac{2bd(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)^2(ia+b)f^2} \\
&- \frac{2b^2 d(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} \\
&- \frac{(bd^2) \text{Subst}\left(\int \frac{\text{PolyLog}\left(2, \frac{(a+ib)x}{a-ib}\right)}{x} dx, x, e^{2ie+2ifx}\right)}{(a-ib)(a+ib)^2 f^3} \\
&- \frac{(2b^2 d^2) \int \log\left(1 - \frac{\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}{1-\frac{ib}{a}}\right) dx}{(a^2+b^2)^2 f^2} \\
&+ \frac{(2b^2 d^2) \int \text{PolyLog}\left(2, \frac{\left(1+\frac{ib}{a}\right)e^{2ie+2ifx}}{1-\frac{ib}{a}}\right) dx}{(a^2+b^2)^2 f^2} \\
&= -\frac{2ib^2(c+dx)^2}{(a^2+b^2)^2 f} - \frac{2b^2(c+dx)^2}{(a+ib)(a^2+b^2)(ia+b-(ia-b)e^{2ie+2ifx})f} \\
&+ \frac{(c+dx)^3}{3(a+ib)^2 d} - \frac{4b(c+dx)^3}{3(a+ib)^2(ia+b)d} - \frac{4b^2(c+dx)^3}{3(a^2+b^2)^2 d} \\
&+ \frac{2b^2 d(c+dx) \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} - \frac{2ib^2(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f} \\
&- \frac{2b(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)(a^2+b^2)f} - \frac{2bd(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)^2(ia+b)f^2} \\
&- \frac{2b^2 d(c+dx) \text{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} - \frac{bd^2 \text{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a-ib)(a+ib)^2 f^3} \\
&+ \frac{(ib^2 d^2) \text{Subst}\left(\int \frac{\log\left(1 - \frac{\left(1+\frac{ib}{a}\right)x}{1-\frac{ib}{a}}\right)}{x} dx, x, e^{2ie+2ifx}\right)}{(a^2+b^2)^2 f^3} \\
&+ \frac{(ib^2 d^2) \text{Subst}\left(\int \frac{\text{PolyLog}\left(2, \frac{(a+ib)x}{a-ib}\right)}{x} dx, x, e^{2ie+2ifx}\right)}{(a^2+b^2)^2 f^3}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2ib^2(c+dx)^2}{(a^2+b^2)^2 f} - \frac{2b^2(c+dx)^2}{(a+ib)(a^2+b^2)(ia+b-(ia-b)e^{2ie+2ifx})f} + \frac{(c+dx)^3}{3(a+ib)^2 d} \\
&\quad - \frac{4b(c+dx)^3}{3(a+ib)^2(ia+b)d} - \frac{4b^2(c+dx)^3}{3(a^2+b^2)^2 d} + \frac{2b^2 d(c+dx) \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} \\
&\quad - \frac{2ib^2(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f} - \frac{2b(c+dx)^2 \log\left(1 - \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)(a^2+b^2)f} \\
&\quad - \frac{ib^2 d^2 \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^3} - \frac{2bd(c+dx) \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a+ib)^2(ia+b)f^2} \\
&\quad - \frac{2b^2 d(c+dx) \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^2} \\
&\quad - \frac{bd^2 \operatorname{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a-ib)(a+ib)^2 f^3} - \frac{ib^2 d^2 \operatorname{PolyLog}\left(3, \frac{(a+ib)e^{2ie+2ifx}}{a-ib}\right)}{(a^2+b^2)^2 f^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 9.73 (sec) , antiderivative size = 969, normalized size of antiderivative = 1.49

$$\begin{aligned}
&\int \frac{(c+dx)^2}{(a+b \cot(e+fx))^2} dx \\
&= \frac{b \left(\frac{12c(-bd+acf)x}{a+ib} + \frac{12c(a(-1+e^{2ie})+ib(1+e^{2ie}))(-bd+acf)x}{a^2+b^2} + \frac{6d(-bd+2acf)x^2}{a+ib} + \frac{4ad^2fx^3}{a+ib} - \frac{6d(a(-1+e^{2ie})+ib(1+e^{2ie}))(bd-2c)}{(a-ib)(-} \right. \\
&\quad \left. + \frac{3a^2c^2fx \cos(fx) - 3b^2c^2fx \cos(fx) + 3a^2cdfx^2 \cos(fx) - 3b^2cdfx^2 \cos(fx) + a^2d^2fx^3 \cos(fx) - b^2d^2fx^3 \cos(fx)}{(a^2+b^2)^2 f^3} \right)}{(a^2+b^2)^2 f^3}
\end{aligned}$$

[In] Integrate[(c + d*x)^2/(a + b*Cot[e + f*x])^2,x]

[Out] (b*((12*c*(-(b*d) + a*c*f)*x)/(a + I*b) + (12*c*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(-(b*d) + a*c*f)*x)/(a^2 + b^2) + (6*d*(-(b*d) + 2*a*c*f)*x^2)/(a + I*b) + (4*a*d^2*f*x^3)/(a + I*b) - (6*d*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(b*d - 2*a*c*f)*x*Log[1 + (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]/((a - I*b)*((-I)*a + b)*f) + (6*a*d^2*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*x^2*Log[1 + (-a + I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]/((a - I*b)*((-I)*a + b)) + (6*c*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))*(-(b*d) + a*c*f)*Log[a - I*b - (a + I*b)*E^((2*I)*(e + f*x))]/((a - I*b)*((-I)*a + b)*f) + (3*d*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))* (b*d - 2*a*c*f)*PolyLog[2, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))]/((a^2 + b^2)*f^2) - (3*a*d^2*(a*(-1 + E^((2*I)*e)) + I*b*(1 + E^((2*I)*e)))* (2*f*x)*PolyLog[2, (a - I*b)/((a + I*b)*E^((2*I)*(e + f*x)))] - I*Po

$$\frac{\text{lyLog}\left[3, \frac{(a - I*b)}{((a + I*b)*E^{((2*I)*(e + f*x))})}\right]}{((a^2 + b^2)*f^2)} \cdot \frac{3*(a - I*b)*(a + I*b)*((-I)*a*(-1 + E^{((2*I)*e)}) + b*(1 + E^{((2*I)*e)})) * f}{(3*a^2*c^2*f*x*\text{Cos}[f*x] - 3*b^2*c^2*f*x*\text{Cos}[f*x] + 3*a^2*c*d*f*x^2*\text{Cos}[f*x] - 3*b^2*c*d*f*x^2*\text{Cos}[f*x] + a^2*d^2*f*x^3*\text{Cos}[f*x] - b^2*d^2*f*x^3*\text{Cos}[f*x] - 3*a^2*c^2*f*x*\text{Cos}[2*e + f*x] - 3*b^2*c^2*f*x*\text{Cos}[2*e + f*x] - 3*a^2*c*d*f*x^2*\text{Cos}[2*e + f*x] - 3*b^2*c*d*f*x^2*\text{Cos}[2*e + f*x] - a^2*d^2*f*x^3*\text{Cos}[2*e + f*x] - b^2*d^2*f*x^3*\text{Cos}[2*e + f*x] + 6*b^2*c^2*\text{Sin}[f*x] + 12*b^2*c*d*f*x*\text{Sin}[f*x] - 6*a*b*c^2*f*x*\text{Sin}[f*x] + 6*b^2*d^2*x^2*\text{Sin}[f*x] - 6*a*b*c*d*f*x^2*\text{Sin}[f*x] - 2*a*b*d^2*f*x^3*\text{Sin}[f*x])}{(6*(a - I*b)*(a + I*b)*f*(b*\text{Cos}[e] + a*\text{Sin}[e]))*(b*\text{Cos}[e + f*x] + a*\text{Sin}[e + f*x])}$$

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3885 vs. $2(584) = 1168$.

Time = 0.66 (sec) , antiderivative size = 3886, normalized size of antiderivative = 5.98

method	result	size
risch	Expression too large to display	3886

[In] `int((d*x+c)^2/(a+b*cot(f*x+e))^2,x,method=_RETURNVERBOSE)`

[Out] $\frac{4}{(b-I*a)^2/f^2/(I*a+b)*b*e*d*a^2*c/(I*b-a)/(a+I*b)*\arctan(1/2/a*b*\exp(2*I*(f*x+e))+1/2/a*b+1/2/b*\exp(2*I*(f*x+e))*a-1/2/b*a)+4/(b-I*a)^2/f^2/(I*a+b)*b*e*d*a^2*c/(I*b-a)/(a+I*b)*\arctan(1/b*a)+4*I/(b-I*a)^2/f/(I*a+b)*b/(a-I*b)*d*a*c*\ln(1-(a+I*b)*\exp(2*I*(f*x+e))/(a-I*b))*x-I/(b-I*a)^2/f^3/(I*a+b)*b*e^2*d^2*a^2/(I*b-a)/(a+I*b)*\ln(a^2*\exp(4*I*(f*x+e))+b^2*\exp(4*I*(f*x+e))-2*a^2*\exp(2*I*(f*x+e))+2*b^2*\exp(2*I*(f*x+e))+a^2+b^2)-2*I/(b-I*a)^2/f^3/(I*a+b)*b^2*e^2*d^2*a/(I*b-a)/(a+I*b)*\arctan(1/2/a*b*\exp(2*I*(f*x+e))+1/2/a*b+1/2/b*\exp(2*I*(f*x+e))*a-1/2/b*a)-2*I/(b-I*a)^2/f^3/(I*a+b)*b^2*e^2*d^2*a/(I*b-a)/(a+I*b)*\arctan(1/b*a)-I/(b-I*a)^2/f^3/(I*a+b)*b^2*e*d^2/(I*b-a)/(a+I*b)*\ln(a^2*\exp(4*I*(f*x+e))+b^2*\exp(4*I*(f*x+e))-2*a^2*\exp(2*I*(f*x+e))+2*b^2*\exp(2*I*(f*x+e))+a^2+b^2)*a-8*I/(b-I*a)^2/f^2/(I*a+b)*b*e*d*a*c/(I*b-a)*\ln(\exp(I*(f*x+e)))+4*I/(b-I*a)^2/f^2/(I*a+b)*b/(a-I*b)*d*a*c*\ln(1-(a+I*b)*\exp(2*I*(f*x+e))/(a-I*b))*e+2*I/(b-I*a)^2/f/(I*a+b)*b/(a-I*b)*d^2*a*\ln(1-(a+I*b)*\exp(2*I*(f*x+e))/(a-I*b))*x^2-2*I/(b-I*a)^2/f^3/(I*a+b)*b^3*e*d^2/(I*b-a)/(a+I*b)*\arctan(1/2/a*b*\exp(2*I*(f*x+e))+1/2/a*b+1/2/b*\exp(2*I*(f*x+e))*a-1/2/b*a)-2*I/(b-I*a)^2/f^3/(I*a+b)*b^3*e*d^2/(I*b-a)/(a+I*b)*\arctan(1/b*a)-2*I/(b-I*a)^2/f/(I*a+b)*b^2*a*c^2/(I*b-a)/(a+I*b)*\arctan(1/2/a*b*\exp(2*I*(f*x+e))+1/2/a*b+1/2/b*\exp(2*I*(f*x+e))*a-1/2/b*a)-2*I/(b-I*a)^2/f/(I*a+b)*b^2*a*c^2/(I*b-a)/(a+I*b)*\arctan(1/b*a)-2*I/(b-I*a)^2/f^3/(I*a+b)*b/(a-I*b)*e^2*d^2*a*\ln(1-(a+I*b)*\exp(2*I*(f*x+e))/(a-I*b))+I/(b-I*a)^2/f^2/(I*a+b)*b^2*c*d/(I*b-a)/(a+I*b)*\ln(a^2*\exp(4*I*(f*x+e))+b^2*\exp(4*I*(f*x+e))-2*a^2*\exp(2*I*(f*x+e))+2*b^2*\exp(2*I*(f*x+e))+a^2+b^2)*a-2/(b-I*a)^2/f^2/(I*a+b)*b^2*e*d*a*c/(I*b-a)/(a+I*b)*\ln(a^2*\exp(4*I*(f*x+e))+b^2*\exp(4*I*(f*x+e))-2*a^2$

$$\begin{aligned}
& * \exp(2*I*(f*x+e))+2*b^2*\exp(2*I*(f*x+e))+a^2+b^2-4*I/(b-I*a)^2/f^2/(I*a+b) \\
& *b^2*c*d/(I*b-a)*\ln(\exp(I*(f*x+e)))+4*I/(b-I*a)^2/f/(I*a+b)*b*a*c^2/(I*b-a) \\
& *\ln(\exp(I*(f*x+e)))+4*I/(b-I*a)^2/f^3/(I*a+b)*b^2*e*d^2/(I*b-a)*\ln(\exp(I*(f \\
& *x+e)))-2*I/(b-I*a)^2/f^2/(I*a+b)*b^2/(a-I*b)*d^2*\ln(1-(a+I*b)*\exp(2*I*(f*x \\
& +e))/(a-I*b))*x-2*I/(b-I*a)^2/f^3/(I*a+b)*b^2/(a-I*b)*d^2*\ln(1-(a+I*b)*\exp(\\
& 2*I*(f*x+e))/(a-I*b))*e+4*I/(b-I*a)^2/f^2/(I*a+b)*b^2*e*d*a*c/(I*b-a)/(a+I* \\
& b)*\arctan(1/2/a*b*\exp(2*I*(f*x+e))+1/2/a*b+1/2/b*\exp(2*I*(f*x+e))*a-1/2/b*a \\
&)+4*I/(b-I*a)^2/f^2/(I*a+b)*b^2*e*d*a*c/(I*b-a)/(a+I*b)*\arctan(1/b*a)+2*I/(\\
& b-I*a)^2/f^2/(I*a+b)*b*e*d*a^2*c/(I*b-a)/(a+I*b)*\ln(a^2*\exp(4*I*(f*x+e))+b^ \\
& 2*\exp(4*I*(f*x+e))-2*a^2*\exp(2*I*(f*x+e))+2*b^2*\exp(2*I*(f*x+e))+a^2+b^2)+d \\
& /(2*I*a*b+a^2-b^2)*c*x^2-2/(b-I*a)^2/f/(I*a+b)*b^2/(a-I*b)*d^2*x^2-2/(b-I*a \\
&)^2/f^3/(I*a+b)*b^2/(a-I*b)*d^2*e^2-1/(b-I*a)^2/f^3/(I*a+b)*b^2/(a-I*b)*d^2 \\
& *polylog(2,(a+I*b)*\exp(2*I*(f*x+e))/(a-I*b))+4/3/(b-I*a)^2/(I*a+b)*b/(a-I*b \\
&)*d^2*a*x^3+1/3*d^2/(2*I*a*b+a^2-b^2)*x^3+1/(2*I*a*b+a^2-b^2)*c^2*x+1/3/d/(\\
& 2*I*a*b+a^2-b^2)*c^3-2/(b-I*a)^2/f^3/(I*a+b)*b*e^2*d^2*a^2/(I*b-a)/(a+I*b)* \\
& \arctan(1/2/a*b*\exp(2*I*(f*x+e))+1/2/a*b+1/2/b*\exp(2*I*(f*x+e))*a-1/2/b*a)+2 \\
& /(b-I*a)^2/f^2/(I*a+b)*b^2*c*d/(I*b-a)/(a+I*b)*\arctan(1/2/a*b*\exp(2*I*(f*x+ \\
& e))+1/2/a*b+1/2/b*\exp(2*I*(f*x+e))*a-1/2/b*a)*a+2/(b-I*a)^2/f^2/(I*a+b)*b^2 \\
& *c*d/(I*b-a)/(a+I*b)*\arctan(1/b*a)*a-2/(b-I*a)^2/f^3/(I*a+b)*b*e^2*d^2*a^2/ \\
& (I*b-a)/(a+I*b)*\arctan(1/b*a)-2/(b-I*a)^2/f^3/(I*a+b)*b^2*e*d^2/(I*b-a)/(a+ \\
& I*b)*\arctan(1/2/a*b*\exp(2*I*(f*x+e))+1/2/a*b+1/2/b*\exp(2*I*(f*x+e))*a-1/2/b \\
& *a)*a-2/(b-I*a)^2/f^3/(I*a+b)*b^2*e*d^2/(I*b-a)/(a+I*b)*\arctan(1/b*a)*a+1/(\\
& b-I*a)^2/f^3/(I*a+b)*b^2*e^2*d^2*a/(I*b-a)/(a+I*b)*\ln(a^2*\exp(4*I*(f*x+e))+ \\
& b^2*\exp(4*I*(f*x+e))-2*a^2*\exp(2*I*(f*x+e))+2*b^2*\exp(2*I*(f*x+e))+a^2+b^2) \\
& +8/(b-I*a)^2/f/(I*a+b)*b/(a-I*b)*d*a*c*e*x+4*I/(b-I*a)^2/f^3/(I*a+b)*b*e^2* \\
& d^2*a/(I*b-a)*\ln(\exp(I*(f*x+e)))+2*I/(b-I*a)^2/f^2/(I*a+b)*b^3*c*d/(I*b-a)/ \\
& (a+I*b)*\arctan(1/2/a*b*\exp(2*I*(f*x+e))+1/2/a*b+1/2/b*\exp(2*I*(f*x+e))*a-1/ \\
& 2/b*a)-I/(b-I*a)^2/f/(I*a+b)*b*a^2*c^2/(I*b-a)/(a+I*b)*\ln(a^2*\exp(4*I*(f*x+ \\
& e))+b^2*\exp(4*I*(f*x+e))-2*a^2*\exp(2*I*(f*x+e))+2*b^2*\exp(2*I*(f*x+e))+a^2+ \\
& b^2)+2*I/(b-I*a)^2/f^2/(I*a+b)*b^3*c*d/(I*b-a)/(a+I*b)*\arctan(1/b*a)+4/(b-I \\
& *a)^2/(I*a+b)*b/(a-I*b)*d*a*c*x^2-4/(b-I*a)^2/f^2/(I*a+b)*b^2/(a-I*b)*d^2*e \\
& *x-8/3/(b-I*a)^2/f^3/(I*a+b)*b/(a-I*b)*e^3*d^2*a-4/(b-I*a)^2/f^2/(I*a+b)*b/ \\
& (a-I*b)*e^2*d^2*a*x-1/(b-I*a)^2/f^2/(I*a+b)*b^3*c*d/(I*b-a)/(a+I*b)*\ln(a^2* \\
& \exp(4*I*(f*x+e))+b^2*\exp(4*I*(f*x+e))-2*a^2*\exp(2*I*(f*x+e))+2*b^2*\exp(2*I* \\
& (f*x+e))+a^2+b^2)+4/(b-I*a)^2/f^2/(I*a+b)*b/(a-I*b)*d*a*c*e^2+2/(b-I*a)^2/f \\
& ^2/(I*a+b)*b/(a-I*b)*d*a*c*polylog(2,(a+I*b)*\exp(2*I*(f*x+e))/(a-I*b))+1/(b \\
& -I*a)^2/f/(I*a+b)*b^2*a*c^2/(I*b-a)/(a+I*b)*\ln(a^2*\exp(4*I*(f*x+e))+b^2*\exp \\
& (4*I*(f*x+e))-2*a^2*\exp(2*I*(f*x+e))+2*b^2*\exp(2*I*(f*x+e))+a^2+b^2)+2/(b-I \\
& *a)^2/f^2/(I*a+b)*b/(a-I*b)*d^2*a*polylog(2,(a+I*b)*\exp(2*I*(f*x+e))/(a-I*b \\
&))*x+I/(b-I*a)^2/f^3/(I*a+b)*b/(a-I*b)*d^2*a*polylog(3,(a+I*b)*\exp(2*I*(f*x \\
& +e))/(a-I*b))-2/(b-I*a)^2/f/(I*a+b)*b*a^2*c^2/(I*b-a)/(a+I*b)*\arctan(1/2/a* \\
& b*\exp(2*I*(f*x+e))+1/2/a*b+1/2/b*\exp(2*I*(f*x+e))*a-1/2/b*a)-2/(b-I*a)^2/f/ \\
& (I*a+b)*b*a^2*c^2/(I*b-a)/(a+I*b)*\arctan(1/b*a)+1/(b-I*a)^2/f^3/(I*a+b)*b^3 \\
& *e*d^2/(I*b-a)/(a+I*b)*\ln(a^2*\exp(4*I*(f*x+e))+b^2*\exp(4*I*(f*x+e))-2*a^2*e \\
& xp(2*I*(f*x+e))+2*b^2*\exp(2*I*(f*x+e))+a^2+b^2)+2*I*b^2*(d^2*x^2+2*c*d*x+c^
\end{aligned}$$

2)/(I*a+b)/f/(b-I*a)^2/(b*exp(2*I*(f*x+e))-I*a*exp(2*I*(f*x+e))+b+I*a)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2038 vs. 2(531) = 1062.

Time = 0.36 (sec) , antiderivative size = 2038, normalized size of antiderivative = 3.14

$$\int \frac{(c + dx)^2}{(a + b \cot(e + fx))^2} dx = \text{Too large to display}$$

[In] integrate((d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="fricas")

[Out] 1/6*(2*(a^2*b - b^3)*d^2*f^3*x^3 - 6*a*b^2*c^2*f^2 - 6*(a*b^2*d^2*f^2 - (a^2*b - b^3)*c*d*f^3)*x^2 - 6*(2*a*b^2*c*d*f^2 - (a^2*b - b^3)*c^2*f^3)*x + 2*((a^2*b - b^3)*d^2*f^3*x^3 - 3*a*b^2*c^2*f^2 - 3*(a*b^2*d^2*f^2 - (a^2*b - b^3)*c*d*f^3)*x^2 - 3*(2*a*b^2*c*d*f^2 - (a^2*b - b^3)*c^2*f^3)*x)*cos(2*f*x + 2*e) - 3*(-2*I*a*b^2*d^2*f*x - 2*I*a*b^2*c*d*f + I*b^3*d^2 + (-2*I*a*b^2*d^2*f*x - 2*I*a*b^2*c*d*f + I*b^3*d^2)*cos(2*f*x + 2*e) + (-2*I*a^2*b*d^2*f*x - 2*I*a^2*b*c*d*f + I*a*b^2*d^2)*sin(2*f*x + 2*e))*dilog(-(a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - 3*(2*I*a*b^2*d^2*f*x + 2*I*a*b^2*c*d*f - I*b^3*d^2 + (2*I*a*b^2*d^2*f*x + 2*I*a*b^2*c*d*f - I*b^3*d^2)*cos(2*f*x + 2*e) + (2*I*a^2*b*d^2*f*x + 2*I*a^2*b*c*d*f - I*a*b^2*d^2)*sin(2*f*x + 2*e))*dilog(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2) + 1) - 6*(a*b^2*d^2*e^2 + a*b^2*c^2*f^2 + b^3*d^2*e - (2*a*b^2*c*d*e + b^3*c*d)*f + (a*b^2*d^2*e^2 + a*b^2*c^2*f^2 + b^3*d^2*e - (2*a*b^2*c*d*e + b^3*c*d)*f)*cos(2*f*x + 2*e) + (a^2*b*d^2*e^2 + a^2*b*c^2*f^2 + a*b^2*d^2*e - (2*a^2*b*c*d*e + a*b^2*c*d)*f)*sin(2*f*x + 2*e))*log(1/2*a^2 + I*a*b - 1/2*b^2 - 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 6*(a*b^2*d^2*e^2 + a*b^2*c^2*f^2 + b^3*d^2*e - (2*a*b^2*c*d*e + b^3*c*d)*f + (a*b^2*d^2*e^2 + a*b^2*c^2*f^2 + b^3*d^2*e - (2*a*b^2*c*d*e + b^3*c*d)*f)*cos(2*f*x + 2*e) + (a^2*b*d^2*e^2 + a^2*b*c^2*f^2 + a*b^2*d^2*e - (2*a^2*b*c*d*e + a*b^2*c*d)*f)*sin(2*f*x + 2*e))*log(-1/2*a^2 + I*a*b + 1/2*b^2 + 1/2*(a^2 + b^2)*cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*sin(2*f*x + 2*e)) - 6*(a*b^2*d^2*f^2*x^2 - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f - b^3*d^2*e + (2*a*b^2*c*d*f^2 - b^3*d^2*f)*x + (a*b^2*d^2*f^2*x^2 - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f - b^3*d^2*e + (2*a*b^2*c*d*f^2 - b^3*d^2*f)*x)*cos(2*f*x + 2*e) + (a^2*b*d^2*f^2*x^2 - a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f - a*b^2*d^2*e + (2*a^2*b*c*d*f^2 - a*b^2*d^2*f)*x)*sin(2*f*x + 2*e))*log((a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*sin(2*f*x + 2*e))/(a^2 + b^2)) - 6*(a*b^2*d^2*f^2*x^2 - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f - b^3*d^2*e + (2*a*b^2*c*d*f^2 - b^3*d^2*f)*x + (a*b^2*d^2*f^2*x^2 - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f - b^3*d^2*e + (2*a*b^2*c*d*f^2 - b^3*d^2*f)*x)*cos(2*f*x + 2*e) + (a^2*b*d^2*f^2*x^2 - a^2*b

$$d^2e^2 + 2a^2b^2cd^2ef - ab^2d^2e + (2a^2b^2cd^2f^2 - ab^2d^2f)x \\ \sin(2fx + 2e) \log((a^2 + b^2 - (a^2 - 2Iab - b^2)\cos(2fx + 2e) \\ + (Ia^2 + 2ab - Ib^2)\sin(2fx + 2e))/(a^2 + b^2)) - 3(ab^2d^2\cos \\ (2fx + 2e) + a^2bd^2\sin(2fx + 2e) + ab^2d^2)\text{polylog}(3, ((a^2 + \\ 2Iab - b^2)\cos(2fx + 2e) + (Ia^2 - 2ab - Ib^2)\sin(2fx + 2e) \\)/(a^2 + b^2)) - 3(ab^2d^2\cos(2fx + 2e) + a^2bd^2\sin(2fx + 2e) \\ + ab^2d^2)\text{polylog}(3, ((a^2 - 2Iab - b^2)\cos(2fx + 2e) + (-Ia^2 \\ - 2ab + Ib^2)\sin(2fx + 2e))/(a^2 + b^2)) + 2((a^3 - ab^2)d^2f^3x^3 \\ + 3b^3c^2f^2 + 3(b^3d^2f^2 + (a^3 - ab^2)cd^2f^3)x^2 + 3(2b^3 \\ cd^2f^2 + (a^3 - ab^2)c^2f^3)x)\sin(2fx + 2e))/((a^4b + 2a^2b^3 \\ + b^5)f^3\cos(2fx + 2e) + (a^5 + 2a^3b^2 + ab^4)f^3\sin(2fx + 2e) \\ + (a^4b + 2a^2b^3 + b^5)f^3)$$

Sympy [F]

$$\int \frac{(c + dx)^2}{(a + b \cot(e + fx))^2} dx = \int \frac{(c + dx)^2}{(a + b \cot(e + fx))^2} dx$$

[In] integrate((d*x+c)**2/(a+b*cot(f*x+e))**2,x)

[Out] Integral((c + d*x)**2/(a + b*cot(e + f*x))**2, x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2552 vs. 2(531) = 1062.

Time = 1.26 (sec) , antiderivative size = 2552, normalized size of antiderivative = 3.93

$$\int \frac{(c + dx)^2}{(a + b \cot(e + fx))^2} dx = \text{Too large to display}$$

[In] integrate((d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="maxima")

[Out] 1/3*(6*(b^2/((a^4 + a^2*b^2)*f*tan(f*x + e) + (a^3*b + a*b^3)*f) + 2*a*b*log(a*tan(f*x + e) + b)/((a^4 + 2*a^2*b^2 + b^4)*f) - a*b*log(tan(f*x + e)^2 + 1)/((a^4 + 2*a^2*b^2 + b^4)*f) - (a^2 - b^2)*(f*x + e)/((a^4 + 2*a^2*b^2 + b^4)*f))*c*d*e - 3*(2*a*b*log(a*tan(f*x + e) + b)/(a^4 + 2*a^2*b^2 + b^4) - a*b*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) + b^2/(a^3*b + a*b^3 + (a^4 + a^2*b^2)*tan(f*x + e)))*c^2 - ((a^3 + I*a^2*b + a*b^2 + I*b^3)*(f*x + e)^3*d^2 + 3*(a^3 + I*a^2*b + a*b^2 + I*b^3)*(f*x + e)*d^2*e^2 + 6*(I*a*b^2 + b^3)*d^2*e^2 - 3*((a^3 + I*a^2*b + a*b^2 + I*b^3)*d^2*e - (a^3 + I*a^2*b + a*b^2 + I*b^3)*c*d*f)*(f*x + e)^2 + 6*((-I*a^2*b - a*b^2)*d^2*e^2 + (-I*a*b^2 - b^3)*d^2*e + (

$$\begin{aligned}
& I*a*b^2 + b^3)*c*d*f + ((I*a^2*b - a*b^2)*d^2*e^2 + (I*a*b^2 - b^3)*d^2*e + \\
& (-I*a*b^2 + b^3)*c*d*f)*\cos(2*f*x + 2*e) - ((a^2*b + I*a*b^2)*d^2*e^2 + (a \\
& *b^2 + I*b^3)*d^2*e - (a*b^2 + I*b^3)*c*d*f)*\sin(2*f*x + 2*e))*\arctan2(b*co \\
& s(2*f*x + 2*e) + a*\sin(2*f*x + 2*e) + b, a*\cos(2*f*x + 2*e) - b*\sin(2*f*x + \\
& 2*e) - a) + 6*((-I*a^2*b - a*b^2)*(f*x + e)^2*d^2 + (2*(I*a^2*b + a*b^2)*d \\
& ^2*e + 2*(-I*a^2*b - a*b^2)*c*d*f + (I*a*b^2 + b^3)*d^2)*(f*x + e) + ((I*a^ \\
& 2*b - a*b^2)*(f*x + e)^2*d^2 + (2*(-I*a^2*b + a*b^2)*d^2*e + 2*(I*a^2*b - a \\
& *b^2)*c*d*f + (-I*a*b^2 + b^3)*d^2)*(f*x + e))*\cos(2*f*x + 2*e) - ((a^2*b + \\
& I*a*b^2)*(f*x + e)^2*d^2 - (2*(a^2*b + I*a*b^2)*d^2*e - 2*(a^2*b + I*a*b^2 \\
&)*c*d*f + (a*b^2 + I*b^3)*d^2)*(f*x + e))*\sin(2*f*x + 2*e))*\arctan2(-(2*a*b \\
& *cos(2*f*x + 2*e) + (a^2 - b^2)*sin(2*f*x + 2*e))/(a^2 + b^2), (2*a*b*sin(2 \\
& *f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)*cos(2*f*x + 2*e))/(a^2 + b^2)) - ((a^ \\
& 3 + 3*I*a^2*b - 3*a*b^2 - I*b^3)*(f*x + e)^3*d^2 - 3*((a^3 + 3*I*a^2*b - 3* \\
& a*b^2 - I*b^3)*d^2*e - (a^3 + 3*I*a^2*b - 3*a*b^2 - I*b^3)*c*d*f + 2*(I*a*b \\
& ^2 - b^3)*d^2)*(f*x + e)^2 + 3*((a^3 + 3*I*a^2*b - 3*a*b^2 - I*b^3)*d^2*e^2 \\
& - 4*(-I*a*b^2 + b^3)*d^2*e - 4*(I*a*b^2 - b^3)*c*d*f)*(f*x + e))*\cos(2*f*x \\
& + 2*e) + 3*(2*(I*a^2*b + a*b^2)*(f*x + e)*d^2 + 2*(-I*a^2*b - a*b^2)*d^2*e \\
& + 2*(I*a^2*b + a*b^2)*c*d*f + (-I*a*b^2 - b^3)*d^2 + (2*(-I*a^2*b + a*b^2) \\
& *(f*x + e)*d^2 + 2*(I*a^2*b - a*b^2)*d^2*e + 2*(-I*a^2*b + a*b^2)*c*d*f + (\\
& I*a*b^2 - b^3)*d^2)*\cos(2*f*x + 2*e) + (2*(a^2*b + I*a*b^2)*(f*x + e)*d^2 - \\
& 2*(a^2*b + I*a*b^2)*d^2*e + 2*(a^2*b + I*a*b^2)*c*d*f - (a*b^2 + I*b^3)*d^ \\
& 2)*\sin(2*f*x + 2*e))*\operatorname{dilog}((I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) - 3*((a \\
& ^2*b - I*a*b^2)*d^2*e^2 + (a*b^2 - I*b^3)*d^2*e - (a*b^2 - I*b^3)*c*d*f - (\\
& (a^2*b + I*a*b^2)*d^2*e^2 + (a*b^2 + I*b^3)*d^2*e - (a*b^2 + I*b^3)*c*d*f)* \\
& \cos(2*f*x + 2*e) - ((I*a^2*b - a*b^2)*d^2*e^2 + (I*a*b^2 - b^3)*d^2*e + (-I \\
& *a*b^2 + b^3)*c*d*f)*\sin(2*f*x + 2*e))*\log((a^2 + b^2)*\cos(2*f*x + 2*e)^2 + \\
& 4*a*b*\sin(2*f*x + 2*e) + (a^2 + b^2)*\sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a \\
& ^2 - b^2)*\cos(2*f*x + 2*e)) - 3*((a^2*b - I*a*b^2)*(f*x + e)^2*d^2 - (2*(a^ \\
& 2*b - I*a*b^2)*d^2*e - 2*(a^2*b - I*a*b^2)*c*d*f + (a*b^2 - I*b^3)*d^2)*(f* \\
& x + e) - ((a^2*b + I*a*b^2)*(f*x + e)^2*d^2 - (2*(a^2*b + I*a*b^2)*d^2*e - \\
& 2*(a^2*b + I*a*b^2)*c*d*f + (a*b^2 + I*b^3)*d^2)*(f*x + e))*\cos(2*f*x + 2*e \\
&) - ((I*a^2*b - a*b^2)*(f*x + e)^2*d^2 + (2*(-I*a^2*b + a*b^2)*d^2*e + 2*(I \\
& *a^2*b - a*b^2)*c*d*f + (-I*a*b^2 + b^3)*d^2)*(f*x + e))*\sin(2*f*x + 2*e))* \\
& \log(((a^2 + b^2)*\cos(2*f*x + 2*e)^2 + 4*a*b*\sin(2*f*x + 2*e) + (a^2 + b^2)* \\
& \sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*\cos(2*f*x + 2*e))/(a^2 + b^2 \\
&)) + 3*((a^2*b + I*a*b^2)*d^2*\cos(2*f*x + 2*e) + (I*a^2*b - a*b^2)*d^2*\sin(\\
& 2*f*x + 2*e) - (a^2*b - I*a*b^2)*d^2)*\operatorname{polylog}(3, (I*a - b)*e^(2*I*f*x + 2*I \\
& *e)/(I*a + b)) - ((I*a^3 - 3*a^2*b - 3*I*a*b^2 + b^3)*(f*x + e)^3*d^2 - 3*(\\
& (I*a^3 - 3*a^2*b - 3*I*a*b^2 + b^3)*d^2*e + (-I*a^3 + 3*a^2*b + 3*I*a*b^2 - \\
& b^3)*c*d*f - 2*(a*b^2 + I*b^3)*d^2)*(f*x + e)^2 - 3*((-I*a^3 + 3*a^2*b + 3 \\
& *I*a*b^2 - b^3)*d^2*e^2 + 4*(a*b^2 + I*b^3)*d^2*e - 4*(a*b^2 + I*b^3)*c*d*f \\
& *(f*x + e))*\sin(2*f*x + 2*e))/((a^5 + I*a^4*b + 2*a^3*b^2 + 2*I*a^2*b^3 + \\
& a*b^4 + I*b^5)*f^2*\cos(2*f*x + 2*e) + (I*a^5 - a^4*b + 2*I*a^3*b^2 - 2*a^2* \\
& b^3 + I*a*b^4 - b^5)*f^2*\sin(2*f*x + 2*e) - (a^5 - I*a^4*b + 2*a^3*b^2 - 2* \\
& I*a^2*b^3 + a*b^4 - I*b^5)*f^2))/f
\end{aligned}$$

Giac [F]

$$\int \frac{(c + dx)^2}{(a + b \cot(e + fx))^2} dx = \int \frac{(dx + c)^2}{(b \cot(fx + e) + a)^2} dx$$

[In] integrate((d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2/(b*cot(f*x + e) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx)^2}{(a + b \cot(e + fx))^2} dx = \int \frac{(c + dx)^2}{(a + b \cot(e + fx))^2} dx$$

[In] int((c + d*x)^2/(a + b*cot(e + f*x))^2,x)

[Out] int((c + d*x)^2/(a + b*cot(e + f*x))^2, x)

3.59 $\int \frac{c+dx}{(a+b \cot(e+fx))^2} dx$

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Optimal result

Integrand size = 18, antiderivative size = 213

$$\int \frac{c+dx}{(a+b \cot(e+fx))^2} dx = -\frac{(c+dx)^2}{2(a^2+b^2)d} + \frac{(bd-2acf-2adf x)^2}{4a(a-ib)^2(a+ib)df^2}$$

$$+ \frac{b(c+dx)}{(a^2+b^2)f(a+b \cot(e+fx))}$$

$$+ \frac{b(bd-2acf-2adf x) \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)^2 f^2}$$

$$+ \frac{iabd \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{(a^2+b^2)^2 f^2}$$

```
[Out] -1/2*(d*x+c)^2/(a^2+b^2)/d+1/4*(-2*a*d*f*x-2*a*c*f+b*d)^2/a/(a-I*b)^2/(a+I*b)/d/f^2+b*(d*x+c)/(a^2+b^2)/f/(a+b*cot(f*x+e))+b*(-2*a*d*f*x-2*a*c*f+b*d)*ln(1-(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)^2/f^2+I*a*b*d*polylog(2,(a+I*b)*exp(2*I*(f*x+e))/(a-I*b))/(a^2+b^2)^2/f^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used

= {3814, 3812, 2221, 2317, 2438}

$$\int \frac{c + dx}{(a + b \cot(e + fx))^2} dx = \frac{b(-2acf - 2adfx + bd) \log\left(1 - \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f^2(a^2 + b^2)^2} + \frac{b(c + dx)}{f(a^2 + b^2)(a + b \cot(e + fx))} - \frac{(c + dx)^2}{2d(a^2 + b^2)} + \frac{iabd \operatorname{PolyLog}\left(2, \frac{(a+ib)e^{2i(e+fx)}}{a-ib}\right)}{f^2(a^2 + b^2)^2} + \frac{(-2acf - 2adfx + bd)^2}{4adf^2(a - ib)^2(a + ib)}$$

[In] Int[(c + d*x)/(a + b*Cot[e + f*x])^2,x]

[Out] -1/2*(c + d*x)^2/((a^2 + b^2)*d) + (b*d - 2*a*c*f - 2*a*d*f*x)^2/(4*a*(a - I*b)^2*(a + I*b)*d*f^2) + (b*(c + d*x))/((a^2 + b^2)*f*(a + b*Cot[e + f*x])) + (b*(b*d - 2*a*c*f - 2*a*d*f*x)*Log[1 - ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/((a^2 + b^2)^2*f^2) + (I*a*b*d*PolyLog[2, ((a + I*b)*E^((2*I)*(e + f*x)))/(a - I*b)])/((a^2 + b^2)^2*f^2)

Rule 2221

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2317

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2438

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 3812

Int[(((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*tan[(e_) + Pi*(k_) + (f_)*(x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist[2*I*b, Int[(c + d*x)^m*E^(2*I*k*Pi)*(E^Simp[2*I*(e + f*x), x])/((a + I*b)^2 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

Rule 3814

```

Int[((c_.) + (d_.)*(x_))/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol
] := Simp[-(c + d*x)^2/(2*d*(a^2 + b^2)), x] + (Dist[1/(f*(a^2 + b^2)), Int
[(b*d + 2*a*c*f + 2*a*d*f*x)/(a + b*Tan[e + f*x]), x], x] - Simp[b*(c + d*
x)/(f*(a^2 + b^2)*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[a^2 + b^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(c + dx)^2}{2(a^2 + b^2)d} + \frac{b(c + dx)}{(a^2 + b^2)f(a + b \cot(e + fx))} + \frac{\int \frac{-bd + 2acf + 2adf x}{a + b \cot(e + fx)} dx}{(a^2 + b^2)f} \\
&= -\frac{(c + dx)^2}{2(a^2 + b^2)d} + \frac{(bd - 2acf - 2adf x)^2}{4a(a - ib)^2(a + ib)df^2} \\
&\quad + \frac{b(c + dx)}{(a^2 + b^2)f(a + b \cot(e + fx))} + \frac{(2ib) \int \frac{e^{2i(e+fx)}(-bd + 2acf + 2adf x)}{(a - ib)^2 + (-a^2 - b^2)e^{2i(e+fx)}} dx}{(a^2 + b^2)f} \\
&= -\frac{(c + dx)^2}{2(a^2 + b^2)d} + \frac{(bd - 2acf - 2adf x)^2}{4a(a - ib)^2(a + ib)df^2} + \frac{b(c + dx)}{(a^2 + b^2)f(a + b \cot(e + fx))} \\
&\quad + \frac{b(bd - 2acf - 2adf x) \log\left(1 - \frac{(a + ib)e^{2i(e+fx)}}{a - ib}\right)}{(a^2 + b^2)^2 f^2} \\
&\quad + \frac{(2abd) \int \log\left(1 + \frac{(-a^2 - b^2)e^{2i(e+fx)}}{(a - ib)^2}\right) dx}{(a^2 + b^2)^2 f} \\
&= -\frac{(c + dx)^2}{2(a^2 + b^2)d} + \frac{(bd - 2acf - 2adf x)^2}{4a(a - ib)^2(a + ib)df^2} + \frac{b(c + dx)}{(a^2 + b^2)f(a + b \cot(e + fx))} \\
&\quad + \frac{b(bd - 2acf - 2adf x) \log\left(1 - \frac{(a + ib)e^{2i(e+fx)}}{a - ib}\right)}{(a^2 + b^2)^2 f^2} \\
&\quad - \frac{(iabd) \text{Subst}\left(\int \frac{\log\left(1 + \frac{(-a^2 - b^2)x}{(a - ib)^2}\right)}{x} dx, x, e^{2i(e+fx)}\right)}{(a^2 + b^2)^2 f^2} \\
&= -\frac{(c + dx)^2}{2(a^2 + b^2)d} + \frac{(bd - 2acf - 2adf x)^2}{4a(a - ib)^2(a + ib)df^2} + \frac{b(c + dx)}{(a^2 + b^2)f(a + b \cot(e + fx))} \\
&\quad + \frac{b(bd - 2acf - 2adf x) \log\left(1 - \frac{(a + ib)e^{2i(e+fx)}}{a - ib}\right)}{(a^2 + b^2)^2 f^2} + \frac{iabd \text{PolyLog}\left(2, \frac{(a + ib)e^{2i(e+fx)}}{a - ib}\right)}{(a^2 + b^2)^2 f^2}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 730 vs. $2(213) = 426$.

Time = 8.55 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.43

$$\int \frac{c + dx}{(a + b \cot(e + fx))^2} dx$$

$$= -\frac{(e + fx)(-2de + 2cf + d(e + fx)) \csc^2(e + fx)(b \cos(e + fx) + a \sin(e + fx))^2}{2(-ia + b)(ia + b)f^2(a + b \cot(e + fx))^2}$$

$$+ \frac{bd \csc^2(e + fx)(-a(e + fx) + b \log(b \cos(e + fx) + a \sin(e + fx)))(b \cos(e + fx) + a \sin(e + fx))^2}{(-ia + b)(ia + b)(a^2 + b^2)f^2(a + b \cot(e + fx))^2}$$

$$+ \frac{2ade \csc^2(e + fx)(-a(e + fx) + b \log(b \cos(e + fx) + a \sin(e + fx)))(b \cos(e + fx) + a \sin(e + fx))^2}{(-ia + b)(ia + b)(a^2 + b^2)f^2(a + b \cot(e + fx))^2}$$

$$- \frac{2ac \csc^2(e + fx)(-a(e + fx) + b \log(b \cos(e + fx) + a \sin(e + fx)))(b \cos(e + fx) + a \sin(e + fx))^2}{(-ia + b)(ia + b)(a^2 + b^2)f(a + b \cot(e + fx))^2}$$

$$+ \frac{d \csc^2(e + fx) \left(e^{i \arctan\left(\frac{b}{a}\right)} (e + fx)^2 + \frac{b \left(i(e + fx) (-\pi + 2 \arctan\left(\frac{b}{a}\right)) - \pi \log(1 + e^{-2i(e + fx)}) - 2(e + fx + \arctan\left(\frac{b}{a}\right)) \log(1 - e^{-2i(e + fx)}) \right)}{(-ia + b)(ia + b)(a^2 + b^2)} \right)}{(-ia + b)(ia + b)(a^2 + b^2)f^2(a + b \cot(e + fx))^2}$$

$$+ \frac{\csc^2(e + fx)(b \cos(e + fx) + a \sin(e + fx))(-bde \sin(e + fx) + bcf \sin(e + fx) + bd(e + fx) \sin(e + fx))}{(-ia + b)(ia + b)f^2(a + b \cot(e + fx))^2}$$

[In] Integrate[(c + d*x)/(a + b*Cot[e + f*x])^2,x]

[Out] $-1/2*((e + f*x)*(-2*d*e + 2*c*f + d*(e + f*x))*\text{Csc}[e + f*x]^2*(b*\text{Cos}[e + f*x] + a*\text{Sin}[e + f*x])^2)/(((-I)*a + b)*(I*a + b)*f^2*(a + b*\text{Cot}[e + f*x])^2) + (b*d*\text{Csc}[e + f*x]^2*(-(a*(e + f*x)) + b*\text{Log}[b*\text{Cos}[e + f*x] + a*\text{Sin}[e + f*x]])*(b*\text{Cos}[e + f*x] + a*\text{Sin}[e + f*x])^2)/(((-I)*a + b)*(I*a + b)*(a^2 + b^2)*f^2*(a + b*\text{Cot}[e + f*x])^2) + (2*a*d*e*\text{Csc}[e + f*x]^2*(-(a*(e + f*x)) + b*\text{Log}[b*\text{Cos}[e + f*x] + a*\text{Sin}[e + f*x]])*(b*\text{Cos}[e + f*x] + a*\text{Sin}[e + f*x])^2)/(((-I)*a + b)*(I*a + b)*(a^2 + b^2)*f^2*(a + b*\text{Cot}[e + f*x])^2) - (2*a*c*\text{Csc}[e + f*x]^2*(-(a*(e + f*x)) + b*\text{Log}[b*\text{Cos}[e + f*x] + a*\text{Sin}[e + f*x]])*(b*\text{Cos}[e + f*x] + a*\text{Sin}[e + f*x])^2)/(((-I)*a + b)*(I*a + b)*(a^2 + b^2)*f*(a + b*\text{Cot}[e + f*x])^2) + (d*\text{Csc}[e + f*x]^2*(E^(I*ArcTan[b/a])*(e + f*x)^2 + (b*(I*(e + f*x)*(-Pi + 2*ArcTan[b/a]) - Pi*Log[1 + E^((-2*I)*(e + f*x))]) - 2*(e + f*x + ArcTan[b/a])*Log[1 - E^((2*I)*(e + f*x + ArcTan[b/a]))]) + Pi*Log[Cos[e + f*x]] + 2*ArcTan[b/a]*Log[Sin[e + f*x + ArcTan[b/a]]] + I*PolyLog[2, E^((2*I)*(e + f*x + ArcTan[b/a]))]))/(a*sqrt[1 + b^2/a^2]))*(b*\text{Cos}[e + f*x] + a*\text{Sin}[e + f*x])^2)/(((-I)*a + b)*(I*a + b)*sqrt[(a^2 + b^2)/a^2]*f^2*(a + b*\text{Cot}[e + f*x])^2) + (\text{Csc}[e + f*x]^2*(b*\text{Cos}[e + f*x] + a*\text{Sin}[e + f*x])*(-(b*d*e*\text{Sin}[e + f*x]) + b*c*f*\text{Sin}[e + f*x] + b*d*(e + f*x)*\text{Sin}[e + f*x]))/(((-I)*a + b)*(I*a + b)*f^2*(a + b*\text{Cot}[e + f*x])^2)$

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1996 vs. 2(198) = 396.

Time = 0.63 (sec) , antiderivative size = 1997, normalized size of antiderivative = 9.38

method	result	size
risch	Expression too large to display	1997

[In] `int((d*x+c)/(a+b*cot(f*x+e))^2,x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & I/(b-Ia)^2/f^2/(Ia+b)*b*a^2*d*e/(I*b-a)/(a+I*b)*\ln(a^2*\exp(4*I*(f*x+e))+b \\ & ^2*\exp(4*I*(f*x+e))-2*a^2*\exp(2*I*(f*x+e))+2*b^2*\exp(2*I*(f*x+e))+a^2+b^2)+ \\ & 2*I/(b-Ia)^2/f^2/(Ia+b)*b^2*a*d*e/(I*b-a)/(a+I*b)*\arctan(1/2/a*b*\exp(2*I* \\ & (f*x+e))+1/2/a*b+1/2/b*\exp(2*I*(f*x+e))*a-1/2/b*a)+2*I/(b-Ia)^2/f^2/(Ia+b) \\ &)*b^2*a*d*e/(I*b-a)/(a+I*b)*\arctan(1/b*a)-1/(b-Ia)^2/f^2/(Ia+b)*b^2*a*d*e \\ & /(I*b-a)/(a+I*b)*\ln(a^2*\exp(4*I*(f*x+e))+b^2*\exp(4*I*(f*x+e))-2*a^2*\exp(2*I \\ & *(f*x+e))+2*b^2*\exp(2*I*(f*x+e))+a^2+b^2)+1/(b-Ia)^2/f^2/(Ia+b)*b^2*d/(I* \\ & b-a)/(a+I*b)*\arctan(1/2/a*b*\exp(2*I*(f*x+e))+1/2/a*b+1/2/b*\exp(2*I*(f*x+e)) \\ & *a-1/2/b*a)*a+I/(b-Ia)^2/f^2/(Ia+b)*b^3*d/(I*b-a)/(a+I*b)*\arctan(1/2/a*b* \\ & \exp(2*I*(f*x+e))+1/2/a*b+1/2/b*\exp(2*I*(f*x+e))*a-1/2/b*a)+4/(b-Ia)^2/f/(I \\ & *a+b)*b/(a-I*b)*a*d*e*x+1/(b-Ia)^2/f/(Ia+b)*b^2*a*c/(I*b-a)/(a+I*b)*\ln(a^ \\ & 2*\exp(4*I*(f*x+e))+b^2*\exp(4*I*(f*x+e))-2*a^2*\exp(2*I*(f*x+e))+2*b^2*\exp(2* \\ & I*(f*x+e))+a^2+b^2)-2/(b-Ia)^2/f/(Ia+b)*b*a^2*c/(I*b-a)/(a+I*b)*\arctan(1/ \\ & 2/a*b*\exp(2*I*(f*x+e))+1/2/a*b+1/2/b*\exp(2*I*(f*x+e))*a-1/2/b*a)-2/(b-Ia)^ \\ & 2/f/(Ia+b)*b*a^2*c/(I*b-a)/(a+I*b)*\arctan(1/b*a)+1/(b-Ia)^2/f^2/(Ia+b)*b \\ & ^2*d/(I*b-a)/(a+I*b)*\arctan(1/b*a)*a+I/(b-Ia)^2/f^2/(Ia+b)*b^3*d/(I*b-a)/ \\ & (a+I*b)*\arctan(1/b*a)+4*I/(b-Ia)^2/f/(Ia+b)*b*a*c/(I*b-a)*\ln(\exp(I*(f*x+e) \\ &))+2/(b-Ia)^2/f^2/(Ia+b)*b/(a-I*b)*a*d*e^2+1/(b-Ia)^2/f^2/(Ia+b)*b/(a- \\ & I*b)*a*d*polylog(2,(a+I*b)*\exp(2*I*(f*x+e))/(a-I*b))-1/2/(b-Ia)^2/f^2/(Ia \\ & +b)*b^3*d/(I*b-a)/(a+I*b)*\ln(a^2*\exp(4*I*(f*x+e))+b^2*\exp(4*I*(f*x+e))-2*a^ \\ & 2*\exp(2*I*(f*x+e))+2*b^2*\exp(2*I*(f*x+e))+a^2+b^2)-2*I/(b-Ia)^2/f^2/(Ia+b) \\ &)*b^2*d/(I*b-a)*\ln(\exp(I*(f*x+e)))+2*I*b^2*(d*x+c)/(Ia+b)/f/(b-Ia)^2/(b*\exp(2*I*(f*x+e))-I*a*\exp(2*I*(f*x+e))+b+I*a)+2/(b-Ia)^2/f^2/(Ia+b)*b*a^2*d \\ & *e/(I*b-a)/(a+I*b)*\arctan(1/2/a*b*\exp(2*I*(f*x+e))+1/2/a*b+1/2/b*\exp(2*I*(f \\ & *x+e))*a-1/2/b*a)+2/(b-Ia)^2/f^2/(Ia+b)*b*a^2*d*e/(I*b-a)/(a+I*b)*\arctan(\\ & 1/b*a)-2*I/(b-Ia)^2/f/(Ia+b)*b^2*a*c/(I*b-a)/(a+I*b)*\arctan(1/b*a)-2*I/(b \\ & -Ia)^2/f/(Ia+b)*b^2*a*c/(I*b-a)/(a+I*b)*\arctan(1/2/a*b*\exp(2*I*(f*x+e))+1 \\ & /2/a*b+1/2/b*\exp(2*I*(f*x+e))*a-1/2/b*a)-4*I/(b-Ia)^2/f^2/(Ia+b)*b*a*d*e/ \\ & (I*b-a)*\ln(\exp(I*(f*x+e)))+2*I/(b-Ia)^2/f^2/(Ia+b)*b/(a-I*b)*a*d*\ln(1-(a+ \\ & I*b)*\exp(2*I*(f*x+e))/(a-I*b))*e+1/2*I/(b-Ia)^2/f^2/(Ia+b)*b^2*d/(I*b-a)/ \\ & (a+I*b)*\ln(a^2*\exp(4*I*(f*x+e))+b^2*\exp(4*I*(f*x+e))-2*a^2*\exp(2*I*(f*x+e)) \\ & +2*b^2*\exp(2*I*(f*x+e))+a^2+b^2)*a-I/(b-Ia)^2/f/(Ia+b)*b*a^2*c/(I*b-a)/(a \\ & +I*b)*\ln(a^2*\exp(4*I*(f*x+e))+b^2*\exp(4*I*(f*x+e))-2*a^2*\exp(2*I*(f*x+e))+2 \\ & *b^2*\exp(2*I*(f*x+e))+a^2+b^2)+2*I/(b-Ia)^2/f/(Ia+b)*b/(a-I*b)*a*d*\ln(1-(\end{aligned}$$

$a+I*b)*\exp(2*I*(f*x+e))/(a-I*b))*x+1/2/(2*I*a*b+a^2-b^2)*d*x^2+1/(2*I*a*b+a^2-b^2)*x*c+2/(b-I*a)^2/(I*a+b)*b/(a-I*b)*a*d*x^2$

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(193) = 386$.

Time = 0.33 (sec) , antiderivative size = 1053, normalized size of antiderivative = 4.94

$$\int \frac{c + dx}{(a + b \cot(e + fx))^2} dx = \text{Too large to display}$$

[In] integrate((d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="fricas")

[Out] $\frac{1}{2}*((a^2*b - b^3)*d*f^2*x^2 - 2*a*b^2*c*f - 2*(a*b^2*d*f - (a^2*b - b^3)*c*f^2)*x + ((a^2*b - b^3)*d*f^2*x^2 - 2*a*b^2*c*f - 2*(a*b^2*d*f - (a^2*b - b^3)*c*f^2)*x)*\cos(2*f*x + 2*e) + (I*a*b^2*d*\cos(2*f*x + 2*e) + I*a^2*b*d*\sin(2*f*x + 2*e) + I*a*b^2*d)*\operatorname{dilog}(-(a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*\cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*\sin(2*f*x + 2*e))/(a^2 + b^2) + 1) + (-I*a*b^2*d*\cos(2*f*x + 2*e) - I*a^2*b*d*\sin(2*f*x + 2*e) - I*a*b^2*d)*\operatorname{dilog}(-(a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*\cos(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*\sin(2*f*x + 2*e))/(a^2 + b^2) + 1) + (2*a*b^2*d*e - 2*a*b^2*c*f + b^3*d + (2*a*b^2*d*e - 2*a*b^2*c*f + b^3*d)*\cos(2*f*x + 2*e) + (2*a^2*b*d*e - 2*a^2*b*c*f + a*b^2*d)*\sin(2*f*x + 2*e))*\log(1/2*a^2 + I*a*b - 1/2*b^2 - 1/2*(a^2 + b^2)*\cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*\sin(2*f*x + 2*e)) + (2*a*b^2*d*e - 2*a*b^2*c*f + b^3*d + (2*a*b^2*d*e - 2*a*b^2*c*f + b^3*d)*\cos(2*f*x + 2*e) + (2*a^2*b*d*e - 2*a^2*b*c*f + a*b^2*d)*\sin(2*f*x + 2*e))*\log(-1/2*a^2 + I*a*b + 1/2*b^2 + 1/2*(a^2 + b^2)*\cos(2*f*x + 2*e) + 1/2*(I*a^2 + I*b^2)*\sin(2*f*x + 2*e)) - 2*(a*b^2*d*f*x + a*b^2*d*e + (a*b^2*d*f*x + a*b^2*d*e)*\cos(2*f*x + 2*e) + (a^2*b*d*f*x + a^2*b*d*e)*\sin(2*f*x + 2*e))*\log((a^2 + b^2 - (a^2 + 2*I*a*b - b^2)*\cos(2*f*x + 2*e) + (-I*a^2 + 2*a*b + I*b^2)*\sin(2*f*x + 2*e))/(a^2 + b^2)) - 2*(a*b^2*d*f*x + a*b^2*d*e + (a*b^2*d*f*x + a*b^2*d*e)*\cos(2*f*x + 2*e) + (a^2*b*d*f*x + a^2*b*d*e)*\sin(2*f*x + 2*e))*\log((a^2 + b^2 - (a^2 - 2*I*a*b - b^2)*\cos(2*f*x + 2*e) + (I*a^2 + 2*a*b - I*b^2)*\sin(2*f*x + 2*e))/(a^2 + b^2)) + ((a^3 - a*b^2)*d*f^2*x^2 + 2*b^3*c*f + 2*(b^3*d*f + (a^3 - a*b^2)*c*f^2)*x)*\sin(2*f*x + 2*e))/((a^4*b + 2*a^2*b^3 + b^5)*f^2*\cos(2*f*x + 2*e) + (a^5 + 2*a^3*b^2 + a*b^4)*f^2*\sin(2*f*x + 2*e) + (a^4*b + 2*a^2*b^3 + b^5)*f^2)$

SymPy [F]

$$\int \frac{c + dx}{(a + b \cot(e + fx))^2} dx = \int \frac{c + dx}{(a + b \cot(e + fx))^2} dx$$

[In] integrate((d*x+c)/(a+b*cot(f*x+e))**2,x)

[Out] Integral((c + d*x)/(a + b*cot(e + f*x))**2, x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1171 vs. 2(193) = 386.

Time = 0.87 (sec) , antiderivative size = 1171, normalized size of antiderivative = 5.50

$$\int \frac{c + dx}{(a + b \cot(e + fx))^2} dx = \text{Too large to display}$$

[In] integrate((d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/2*((a^3 + I*a^2*b + a*b^2 + I*b^3)*d*f^2*x^2 + 2*(a^3 + I*a^2*b + a*b^2 \\ & + I*b^3)*c*f^2*x + 4*(I*a*b^2 + b^3)*c*f + 2*(2*(-I*a^2*b - a*b^2)*c*f + (I \\ & *a*b^2 + b^3)*d + (2*(I*a^2*b - a*b^2)*c*f + (-I*a*b^2 + b^3)*d)*\cos(2*f*x \\ & + 2*e) - (2*(a^2*b + I*a*b^2)*c*f - (a*b^2 + I*b^3)*d)*\sin(2*f*x + 2*e))*\ar \\ & \text{ctan2}(b*\cos(2*f*x + 2*e) + a*\sin(2*f*x + 2*e) + b, a*\cos(2*f*x + 2*e) - b*s \\ & \text{in}(2*f*x + 2*e) - a) + 4*((I*a^2*b - a*b^2)*d*f*x*\cos(2*f*x + 2*e) - (a^2*b \\ & + I*a*b^2)*d*f*x*\sin(2*f*x + 2*e) + (-I*a^2*b - a*b^2)*d*f*x)*\arctan2(- (2* \\ & a*b*\cos(2*f*x + 2*e) + (a^2 - b^2)*\sin(2*f*x + 2*e))/(a^2 + b^2), (2*a*b*\si \\ & n(2*f*x + 2*e) + a^2 + b^2 - (a^2 - b^2)*\cos(2*f*x + 2*e))/(a^2 + b^2)) - (\\ & (a^3 + 3*I*a^2*b - 3*a*b^2 - I*b^3)*d*f^2*x^2 + 2*((a^3 + 3*I*a^2*b - 3*a*b \\ & ^2 - I*b^3)*c*f^2 - 2*(I*a*b^2 - b^3)*d*f)*x*\cos(2*f*x + 2*e) + 2*((-I*a^2 \\ & *b + a*b^2)*d*\cos(2*f*x + 2*e) + (a^2*b + I*a*b^2)*d*\sin(2*f*x + 2*e) + (I* \\ & a^2*b + a*b^2)*d)*\text{dilog}((I*a - b)*e^(2*I*f*x + 2*I*e)/(I*a + b)) - (2*(a^2*b \\ & - I*a*b^2)*c*f - (a*b^2 - I*b^3)*d - (2*(a^2*b + I*a*b^2)*c*f - (a*b^2 + \\ & I*b^3)*d)*\cos(2*f*x + 2*e) - (2*(I*a^2*b - a*b^2)*c*f - (I*a*b^2 - b^3)*d)* \\ & \sin(2*f*x + 2*e))*\log((a^2 + b^2)*\cos(2*f*x + 2*e)^2 + 4*a*b*\sin(2*f*x + 2* \\ & e) + (a^2 + b^2)*\sin(2*f*x + 2*e)^2 + a^2 + b^2 - 2*(a^2 - b^2)*\cos(2*f*x + \\ & 2*e)) + 2*((a^2*b + I*a*b^2)*d*f*x*\cos(2*f*x + 2*e) + (I*a^2*b - a*b^2)*d* \\ & f*x*\sin(2*f*x + 2*e) - (a^2*b - I*a*b^2)*d*f*x)*\log(((a^2 + b^2)*\cos(2*f*x \\ & + 2*e)^2 + 4*a*b*\sin(2*f*x + 2*e) + (a^2 + b^2)*\sin(2*f*x + 2*e)^2 + a^2 + \\ & b^2 - 2*(a^2 - b^2)*\cos(2*f*x + 2*e))/(a^2 + b^2)) - ((I*a^3 - 3*a^2*b - 3* \\ & I*a*b^2 + b^3)*d*f^2*x^2 - 2*((-I*a^3 + 3*a^2*b + 3*I*a*b^2 - b^3)*c*f^2 - \\ & 2*(a*b^2 + I*b^3)*d*f)*x*\sin(2*f*x + 2*e))/((a^5 + I*a^4*b + 2*a^3*b^2 + 2* \\ & *I*a^2*b^3 + a*b^4 + I*b^5)*f^2*\cos(2*f*x + 2*e) + (I*a^5 - a^4*b + 2*I*a^3 \\ & *b^2 - 2*a^2*b^3 + I*a*b^4 - b^5)*f^2*\sin(2*f*x + 2*e) - (a^5 - I*a^4*b + 2 \\ & *a^3*b^2 - 2*I*a^2*b^3 + a*b^4 - I*b^5)*f^2) \end{aligned}$$

Giac [F]

$$\int \frac{c + dx}{(a + b \cot(e + fx))^2} dx = \int \frac{dx + c}{(b \cot(fx + e) + a)^2} dx$$

[In] integrate((d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="giac")

[Out] integrate((d*x + c)/(b*cot(f*x + e) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx}{(a + b \cot(e + fx))^2} dx = \int \frac{c + dx}{(a + b \cot(e + fx))^2} dx$$

[In] int((c + d*x)/(a + b*cot(e + f*x))^2,x)

[Out] int((c + d*x)/(a + b*cot(e + f*x))^2, x)

3.60 $\int \frac{1}{(c+dx)(a+b \cot(e+fx))^2} dx$

Optimal result	430
Rubi [N/A]	430
Mathematica [N/A]	431
Maple [N/A] (verified)	431
Fricas [N/A]	431
Sympy [N/A]	432
Maxima [N/A]	432
Giac [N/A]	433
Mupad [N/A]	433

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b \cot(e+fx))^2} dx = \text{Int}\left(\frac{1}{(c+dx)(a+b \cot(e+fx))^2}, x\right)$$

[Out] Unintegrable(1/(d*x+c)/(a+b*cot(f*x+e))^2,x)

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b \cot(e+fx))^2} dx = \int \frac{1}{(c+dx)(a+b \cot(e+fx))^2} dx$$

[In] Int[1/((c + d*x)*(a + b*Cot[e + f*x])^2),x]

[Out] Defer[Int][1/((c + d*x)*(a + b*Cot[e + f*x])^2), x]

Rubi steps

$$\text{integral} = \int \frac{1}{(c+dx)(a+b \cot(e+fx))^2} dx$$

Mathematica [N/A]

Not integrable

Time = 22.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c + dx)(a + b \cot(e + fx))^2} dx = \int \frac{1}{(c + dx)(a + b \cot(e + fx))^2} dx$$

[In] Integrate[1/((c + d*x)*(a + b*Cot[e + f*x])^2), x]

[Out] Integrate[1/((c + d*x)*(a + b*Cot[e + f*x])^2), x]

Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx + c)(a + b \cot(fx + e))^2} dx$$

[In] int(1/(d*x+c)/(a+b*cot(f*x+e))^2,x)

[Out] int(1/(d*x+c)/(a+b*cot(f*x+e))^2,x)

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \frac{1}{(c + dx)(a + b \cot(e + fx))^2} dx = \int \frac{1}{(dx + c)(b \cot(fx + e) + a)^2} dx$$

[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="fricas")

[Out] integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*cot(f*x + e)^2 + 2*(a*b*d*x + a*b*c)*cot(f*x + e)), x)

Sympy [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c + dx)(a + b \cot(e + fx))^2} dx = \int \frac{1}{(a + b \cot(e + fx))^2 (c + dx)} dx$$

[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e))**2,x)

[Out] Integral(1/((a + b*cot(e + f*x))**2*(c + d*x)), x)

Maxima [N/A]

Not integrable

Time = 10.99 (sec) , antiderivative size = 1494, normalized size of antiderivative = 74.70

$$\int \frac{1}{(c + dx)(a + b \cot(e + fx))^2} dx = \int \frac{1}{(dx + c)(b \cot(fx + e) + a)^2} dx$$

[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="maxima")

[Out] (((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*cos(2*f*x + 2*e)^2*log(d*x + c) + ((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*log(d*x + c)*sin(2*f*x + 2*e)^2 - 2*(2*a*b^3*d + ((a^4 - 2*a^2*b^2 + b^4)*d*f*x + (a^4 - 2*a^2*b^2 + b^4)*c*f)*log(d*x + c))*cos(2*f*x + 2*e) + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f)*cos(2*f*x + 2*e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f)*sin(2*f*x + 2*e)^2 - 2*((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d^2*f*x + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d*f)*cos(2*f*x + 2*e) + 4*((a^5*b + 2*a^3*b^3 + a*b^5)*d^2*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*d*f)*sin(2*f*x + 2*e))*integrate(-2*(2*(2*a^2*b^2*d*f*x + 2*a^2*b^2*c*f + a*b^3*d)*cos(2*f*x + 2*e) + (2*(a^3*b - a*b^3)*d*f*x + 2*(a^3*b - a*b^3)*c*f + (a^2*b^2 - b^4)*d)*sin(2*f*x + 2*e))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*f + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*f)*cos(2*f*x + 2*e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*f*x^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*f)*sin(2*f*x + 2*e)^2 - 2*((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d^2*f*x^2 + 2*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d*f*x + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*c^2*f)*cos(2*f*x + 2*e) + 4*((a^5*b + 2*a^3*b^3 + a*b^5)*d^2*f*x^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c*d*f*x + (a^5*b + 2*a^3*b^3 + a

$b^5 * c^2 * f * \sin(2 * f * x + 2 * e)), x) + ((a^4 - b^4) * d * f * x + (a^4 - b^4) * c * f) * \log(dx + c) - 2 * ((a^2 * b^2 - b^4) * d - 2 * ((a^3 * b - a * b^3) * d * f * x + (a^3 * b - a * b^3) * c * f) * \log(dx + c)) * \sin(2 * f * x + 2 * e)) / ((a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) * d^2 * f * x + (a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) * c * d * f + ((a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) * d^2 * f * x + (a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) * c * d * f) * \cos(2 * f * x + 2 * e)^2 + ((a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) * d^2 * f * x + (a^6 + 3 * a^4 * b^2 + 3 * a^2 * b^4 + b^6) * c * d * f) * \sin(2 * f * x + 2 * e)^2 - 2 * ((a^6 + a^4 * b^2 - a^2 * b^4 - b^6) * d^2 * f * x + (a^6 + a^4 * b^2 - a^2 * b^4 - b^6) * c * d * f) * \cos(2 * f * x + 2 * e) + 4 * ((a^5 * b + 2 * a^3 * b^3 + a * b^5) * d^2 * f * x + (a^5 * b + 2 * a^3 * b^3 + a * b^5) * c * d * f) * \sin(2 * f * x + 2 * e))$

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c + dx)(a + b \cot(e + fx))^2} dx = \int \frac{1}{(dx + c)(b \cot(fx + e) + a)^2} dx$$

[In] integrate(1/(d*x+c)/(a+b*cot(f*x+e))^2,x, algorithm="giac")

[Out] integrate(1/((d*x + c)*(b*cot(f*x + e) + a)^2), x)

Mupad [N/A]

Not integrable

Time = 13.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c + dx)(a + b \cot(e + fx))^2} dx = \int \frac{1}{(a + b \cot(e + fx))^2 (c + dx)} dx$$

[In] int(1/((a + b*cot(e + f*x))^2*(c + d*x)),x)

[Out] int(1/((a + b*cot(e + f*x))^2*(c + d*x)), x)

3.61 $\int \frac{1}{(c+dx)^2(a+b \cot(e+fx))^2} dx$

Optimal result	434
Rubi [N/A]	434
Mathematica [N/A]	435
Maple [N/A] (verified)	435
Fricas [N/A]	435
Sympy [N/A]	436
Maxima [N/A]	436
Giac [N/A]	437
Mupad [N/A]	437

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b \cot(e+fx))^2} dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+b \cot(e+fx))^2}, x\right)$$

[Out] Unintegrable(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x)

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b \cot(e+fx))^2} dx = \int \frac{1}{(c+dx)^2(a+b \cot(e+fx))^2} dx$$

[In] Int[1/((c + d*x)^2*(a + b*Cot[e + f*x])^2), x]

[Out] Defer[Int][1/((c + d*x)^2*(a + b*Cot[e + f*x])^2), x]

Rubi steps

$$\text{integral} = \int \frac{1}{(c+dx)^2(a+b \cot(e+fx))^2} dx$$

Mathematica [N/A]

Not integrable

Time = 21.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c + dx)^2(a + b \cot(e + fx))^2} dx = \int \frac{1}{(c + dx)^2(a + b \cot(e + fx))^2} dx$$

[In] Integrate[1/((c + d*x)^2*(a + b*Cot[e + f*x])^2), x]

[Out] Integrate[1/((c + d*x)^2*(a + b*Cot[e + f*x])^2), x]

Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx + c)^2(a + b \cot(fx + e))^2} dx$$

[In] int(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x)

[Out] int(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x)

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.80

$$\int \frac{1}{(c + dx)^2(a + b \cot(e + fx))^2} dx = \int \frac{1}{(dx + c)^2(b \cot(fx + e) + a)^2} dx$$

[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="fricas")

```
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cot(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*cot(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 3.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))^2} dx = \int \frac{1}{(a+b\cot(e+fx))^2(c+dx)^2} dx$$

[In] integrate(1/(d*x+c)**2/(a+b*cot(f*x+e))**2,x)

[Out] Integral(1/((a + b*cot(e + f*x))**2*(c + d*x)**2), x)

Maxima [N/A]

Not integrable

Time = 28.61 (sec) , antiderivative size = 1975, normalized size of antiderivative = 98.75

$$\int \frac{1}{(c+dx)^2(a+b\cot(e+fx))^2} dx = \int \frac{1}{(dx+c)^2(b\cot(fx+e)+a)^2} dx$$

[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="maxima")

```
[Out] -(a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f + ((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*cos(2*f*x + 2*e)^2 + ((a^4 - b^4)*d*f*x + (a^4 - b^4)*c*f)*sin(2*f*x + 2*e)^2 + 2*(2*a*b^3*d - (a^4 - 2*a^2*b^2 + b^4)*d*f*x - (a^4 - 2*a^2*b^2 + b^4)*c*f)*cos(2*f*x + 2*e) - ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d*f + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d*f)*cos(2*f*x + 2*e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^2 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d*f)*sin(2*f*x + 2*e)^2 - 2*((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d^3*f*x^2 + 2*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*c*d^2*f*x + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*c^2*d*f)*cos(2*f*x + 2*e) + 4*((a^5*b + 2*a^3*b^3 + a*b^5)*d^3*f*x^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c*d^2*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c^2*d*f)*sin(2*f*x + 2*e))*integrate(-4*(2*(a^2*b^2*d*f*x + a^2*b^2*c*f + a*b^3*d)*cos(2*f*x + 2*e) + ((a^3*b - a*b^3)*d*f*x + (a^3*b - a*b^3)*c*f + (a^2*b^2 - b^4)*d)*sin(2*f*x + 2*e))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3*f + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3*f)*cos(2*f*x + 2*e)^2 + ((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^3*f*x^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c*d^2*f*x^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d*f*x + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^3*f)*sin(2*f*x + 2*e)^2
```

$$\begin{aligned}
& a^4 b^2 + 3 a^2 b^4 + b^6) c d^2 f x^2 + 3(a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) c^2 d f x + (a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) c^3 f) \sin(2 f x + 2 e) \\
& ^2 - 2((a^6 + a^4 b^2 - a^2 b^4 - b^6) d^3 f x^3 + 3(a^6 + a^4 b^2 - a^2 b^4 - b^6) c d^2 f x^2 + 3(a^6 + a^4 b^2 - a^2 b^4 - b^6) c^2 d f x + (a^6 \\
& + a^4 b^2 - a^2 b^4 - b^6) c^3 f) \cos(2 f x + 2 e) + 4((a^5 b + 2 a^3 b^3 + a b^5) d^3 f x^3 + 3(a^5 b + 2 a^3 b^3 + a b^5) c d^2 f x^2 + 3(a^5 b \\
& + 2 a^3 b^3 + a b^5) c^2 d f x + (a^5 b + 2 a^3 b^3 + a b^5) c^3 f) \sin(2 f x + 2 e), x) + 2(2(a^3 b - a b^3) d f x + 2(a^3 b - a b^3) c f + (a^2 b^2 - b^4) d) \sin(2 f x + 2 e) / ((a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) d^3 f x^2 + 2(a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) c d^2 f x + (a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) c^2 d f + ((a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) d^3 f x^2 + 2(a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) c d^2 f x + (a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) c^2 d f) \cos(2 f x + 2 e)^2 + ((a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) d^3 f x^2 + 2(a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) c d^2 f x + (a^6 + 3 a^4 b^2 + 3 a^2 b^4 + b^6) c^2 d f) \sin(2 f x + 2 e)^2 - 2((a^6 + a^4 b^2 - a^2 b^4 - b^6) d^3 f x^2 + 2(a^6 + a^4 b^2 - a^2 b^4 - b^6) c d^2 f x + (a^6 + a^4 b^2 - a^2 b^4 - b^6) c^2 d f) \cos(2 f x + 2 e) + 4((a^5 b + 2 a^3 b^3 + a b^5) d^3 f x^2 + 2(a^5 b + 2 a^3 b^3 + a b^5) c d^2 f x + (a^5 b + 2 a^3 b^3 + a b^5) c^2 d f) \sin(2 f x + 2 e)
\end{aligned}$$

Giac [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c + dx)^2 (a + b \cot(e + fx))^2} dx = \int \frac{1}{(dx + c)^2 (b \cot(fx + e) + a)^2} dx$$

[In] integrate(1/(d*x+c)^2/(a+b*cot(f*x+e))^2,x, algorithm="giac")

[Out] integrate(1/((d*x + c)^2*(b*cot(f*x + e) + a)^2), x)

Mupad [N/A]

Not integrable

Time = 13.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c + dx)^2 (a + b \cot(e + fx))^2} dx = \int \frac{1}{(a + b \cot(e + fx))^2 (c + dx)^2} dx$$

[In] int(1/((a + b*cot(e + f*x))^2*(c + d*x)^2),x)

[Out] int(1/((a + b*cot(e + f*x))^2*(c + d*x)^2), x)

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 439

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7, 2022. add second output which gives reason for the grade *)
(*      Small rewrite of logic in main function to make it*)
(*      match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```

(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCo
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A",""}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count is
        ]
      ,(*ELSE*)
      finalresult={"C","Result contains complex when optimal does not."}
    ]
    ,(*ELSE*)(*result does not contains complex*)
    If[leafCountResult<=2*leafCountOptimal,
      finalresult={"A",""}
      ,(*ELSE*)
      finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"}
    ]
  ]
  ,(*ELSE*) (*expnResult>expnOptimal*)
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    finalresult={"C","Result contains higher order function than in optimal. Order "<>
    ,
    finalresult={"F","Contains unresolved integral."}
  ]
];

  finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)

```



```

(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType, expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]], 2]],
            Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
          If[ElementaryFunctionQ[Head[expn]],
            Max[3, ExpnType[expn[[1]]],
          If[SpecialFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
          If[HypergeometricFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
          If[AppellFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
          If[Head[expn]===RootSum,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
          If[Head[expn]===Integrate || Head[expn]===Int,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
          9]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,

```

```

    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1}, func]

```

Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result, optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

      leaf_count_result:=leafcount(result);

```

```

#do NOT call ExpnType() if leaf size is too large. Recursion problem
if leaf_count_result > 500000 then
    return "B","result has leaf size over 500,000. Avoiding possible recursion issues
fi;

leaf_count_optimal := leafcount(optimal);
ExpnType_result := ExpnType(result);
ExpnType_optimal := ExpnType(optimal);

if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 ("
```

```

                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
    end if
else #result contains complex but optimal is not
    if debug then
        print("result contains complex but optimal is not");
    fi;
    return "C","Result contains complex when optimal does not.";
fi;
else # result do not contain complex
    # this assumes optimal do not as well. No check is needed here.
    if debug then
        print("result do not contain complex, this assumes optimal do not as well")
    fi;
    if leaf_count_result<=2*leaf_count_optimal then
        if debug then
            print("leaf_count_result<=2*leaf_count_optimal");
        fi;
        return "A"," ";
    else
        if debug then
            print("leaf_count_result>2*leaf_count_optimal");
        fi;
        return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                convert(leaf_count_result,string)," $ vs. $2(",
                                convert(leaf_count_optimal,string)," )=",convert(2*leaf_count
    fi;
fi;
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
fi;
end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

```

```

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+`) or type(expn,'*`) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else

```

```

9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func, [
    exp, log, ln,
    sin, cos, tan, cot, sec, csc,
    arcsin, arccos, arctan, arccot, arcsec, arccsc,
    sinh, cosh, tanh, coth, sech, csch,
    arcsinh, arccosh, arctanh, arccoth, arcsech, arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf, erfc, erfi,
    FresnelS, FresnelC,
    Ei, Ei, Li, Si, Ci, Shi, Chi,
    GAMMA, lnGAMMA, Psi, Zeta, polylog, dilog, LambertW,
    EllipticF, EllipticE, EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1, hypergeom, HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u), op(2..nops(u), u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                    asinh,acosh,atanh,acoth,asech,acsch
                    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                    gamma,loggamma,digamma,zeta,polylog,LambertW,
                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
                    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

```

```

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+') or type(expn,'*')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:

```



```

    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation = ""
                    else:
                        grade = "B"
                        grade_annotation = "Both result and optimal contain complex but leaf count of result is large"
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation = "Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result)
            else:
                grade = "C"
                grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType_result)

```

```

# print("Before returning. grade=", grade, " grade_annotation=", grade_annotation)

return grade, grade_annotation

```

SageMath grading function

```

# Dec 24, 2019. Nasser: Ported original Maple grading function by
# Albert Rich to use with Sagemath. This is used to
# grade Fricas, Giac and Maxima results.
# Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
# 'arctan2', 'floor', 'abs', 'log_integral'
# June 4, 2022 Made default grade_annotation "none" instead of "" due
# issue later when reading the file.
# July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    # print("Enter tree_size, expr is ", expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr, Pow):
        if expr.operands()[1] == 1/2: # expr.args[1] == Rational(1,2):
            if debug: print("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func , " is special_function")
        else:
            print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

```

```

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in is_atom")
        print ("caught exception" , type(error).__name__ )
        return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)

```

```

    return 1
  else:
    return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
  else:
    return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger than"
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. " + str(leaf_c

else:
    grade = "C"
    grade_annotation = "Result contains higher order function than in optimal. Order " + str(expnType_result)

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```